

# Few More and Many Fewer: Complex Quantifiers Based on *Many* and *Few*

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## 1. Introduction

### 1.1 The Data

- ◆ **A well-known pattern:** *many* and *few* occur both in bare ‘positive’ form (1) and in combination with a range of degree modifiers (2) – the same modifiers that occur with gradable adjectives (3):
  - (1) a. Many students attended the lecture  
b. Few students attended the lecture
  - (2) a. Very many/very few students attended the lecture  
b. Professor Jones was worried that too many/too few students would attend the lecture  
c. The speaker was astonished that so many/so few students attended the lecture  
d. More than 100/fewer than 100 students attended the lecture
  - (3) a. very tall  
b. too tall  
c. so tall  
d. taller than 6 feet

→ Captured by degree-based analyses of *many/few* & gradable adjectives (Hackl 2000, to appear; Kennedy 2001; Kennedy & McNally 2005; Rett 2006; Takahashi 2006)
- ◆ **Some little-studied patterns:**
  - (4) a. Frank’s good qualities are many/few  
b. The many/few archeologists I know  
c. A few students (vs. \*a many students)  
d. A long few years  
e. Every few years  
f. Professor Jones’ many and important/few and trivial contributions ....

→ Not accounted for by existing theories

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- ◆ **Focus of today’s talk:** the ‘differential’ uses of *many/few*, where they occur as modifiers in comparatives:
  - (5) a. **Many more:** There were 100 seats in the lecture hall, but unfortunately many more than 100 students showed up for the lecture  
b. **Few more:** The lecture hall has 500 seats, but few more than 100 students attended the lecture  
c. **Many fewer:** The whole class of 100 was supposed to attend the lecture, but many fewer than 100 students actually came  
d. **Few fewer:** ??

- (6) a. **Many more:** Most homes have many more than 3 televisions  
b. **Few more:** Few more than 400 Sumatran tigers survive in the wild  
c. **Many fewer:** The latest attempt to count the number of transient vacation rentals on Maui finds many fewer than previously estimated

→ This sort of recursive degree modification discussed in the earlier syntax literature (Bresnan 1973; Jackendoff 1977), but the facts in (5) - (6) have received little attention in the semantics literature (though see Schwarzschild 2006)

### 1.2 Goals of Today’s Talk

- ❖ Propose a semantic analysis of *many/few* as gradable predicates of scalar intervals, which allows a compositional analysis of the complex quantifiers in (5) as well as the simpler constructions in (1)-(2)
- ❖ Discuss some possible extensions to other data
- ❖ Explore consequences for degree modification in the quantificational vs. adjectival domains

## 2. The Analysis

### 2.1 What Doesn’t Work

- ◆ The differential constructions exemplified in (5) and (6) are problematic for...
    - ... an analysis of *many/few* as the ‘quantifying determiners’ of Generalized Quantifier Theory -- (type  $\langle et, \langle et, t \rangle \rangle$ ) - Barwise & Cooper 1981) or their gradable counterparts (type  $\langle d, \langle et, \langle et, t \rangle \rangle \rangle$ ) - Hackl 2000; Takahashi 2006)
- In (5) - (6), what 2 sets could serve as the arguments for a quantificational *many/few*?

... an analysis of *many* and *few* as (gradable) cardinality predicates or noun modifiers (type  $\langle et, \rangle$ ,  $\langle d, et \rangle$  or  $\langle d, \langle et, et \rangle \rangle$  - Milsark 1977; Partee 1989; McNally 1998; Kennedy & McNally 2005; Hackl to appear)

→ In (5c) in particular, *many* cannot be predicated of a group of individuals

(5a) Many more than 100 students attended the lecture

✓ 'there was a group X composed of 100 students & there was a separate group Y composed of many students &  $X \cup Y$  attended the lecture'

(5c) Many fewer than 100 students attended the lecture

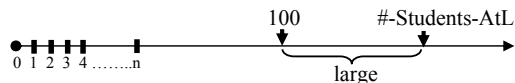
✗ 'there was a group X composed of 100 students and a separate group Y composed of many students ....'

cf. 'many fewer than 400 Sumatran tigers survive today'

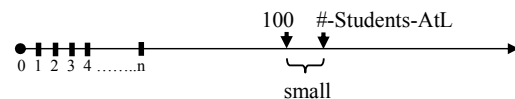
## 2.2 The Proposal - Intuitively

◆ In (5), *many* and *few* express properties of an interval on the scale of natural numbers – the interval between the point 100 and that corresponding to the number of students who attended the lecture (#-Students-AtL)

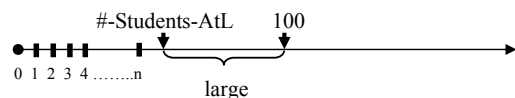
(7) a. Many more than 100 students attended the lecture



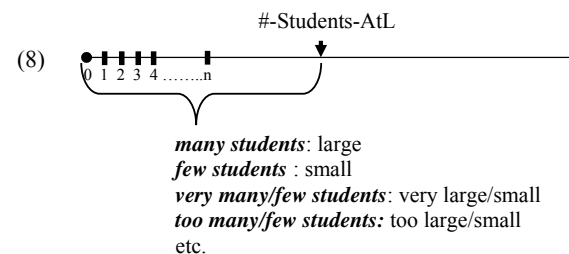
b. Few more than 100 students attended the lecture



c. Many fewer than 100 students attended the lecture



◆ The account can be extended to the simpler constructions in (1)-(2) by considering a interval beginning at 0:



**Central Claim:** *Many and few denote gradable predicates of scalar intervals* (cf. Schwarzschild 2006, Rett 2006 for similar proposals based on different data).

## Advantages:

◆ Parallel treatment of *many/few* and *much/little* as mass quantifiers (9a,b) and modifiers in adjectival comparatives (9c) – the latter of which long been analyzed as degree constructions (e.g. Klein 1982)

- (9) a. much/little wine  
 b. much/little more than a gallon of wine  
 c. much/little taller than Fred

◆ Separation *many/few* from predication or quantification over individuals provides path to account for broader range of data

## 2.3 The Proposal – Formally

### Preliminaries:

◆ I assume an ontology that includes degrees as a primitive type (type  $d$ )

▪ **Degrees**  $d$  are points on a scale  $S$  associated with some dimension  $DIM$ , ordered by the 'greater than' relationship ( $>$ )

▪ In the case of *many/few*, the relevant dimension is 'cardinality', and the resulting scale is that of the natural numbers

▪ I use the term **interval** to denote a continuous set of degrees. Formally:

- (10) A set of degrees  $I \in D_{\langle dt \rangle}$  is an **interval** iff  
 $\forall d, d', d'' \in D_d$  such that  $d > d'' > d'$ ,  $(d \in I \wedge d' \in I) \rightarrow d'' \in I$

- ◆ I assume the framework of Heim (2000, 2006) in which degree modifiers (*very*, *too*, *-er than n*, etc.) are analyzed as generalized quantifiers over degrees (type  $\langle dt, t \rangle$ )

- Provisionally: The positive forms (bare *many/few*) involve null degree quantifier POS with the following definition (Heim 2006; von Stechow 2006)

$$(11) \quad \|\text{POS}\| = \lambda I_{\langle dt, t \rangle} . \forall d \in N_S [d \in I]$$

- Here  $N_S$  is the ‘neutral range’ on the scale  $S$  – the range of degrees that would be considered neither large nor small w.r.t the given context (see also Cresswell 1977; von Stechow 1984; Kennedy 2007 for other views of POS)

### Formal Proposal:

- 1) *Many* and *few* denote gradable predicates of scalar intervals (type  $\langle d, \langle dt, t \rangle \rangle$ ), with *few* defined relative to the join complementary interval of the original interval (cf. Kennedy 2001; Heim 2006):

$$(12) \quad \text{a.} \quad \|\text{many}\| = \lambda d \lambda I_{\langle dt, t \rangle} . d \in I$$

$$\text{b.} \quad \|\text{few}\| = \lambda d \lambda I_{\langle dt, t \rangle} . d \in \text{INV}(I),$$

where for  $I = [0, d]$ ,  $\text{INV}(I) = [d, \infty]$

- 2) Quantification over individuals arises via function of existential closure ( $\exists$ ) associated with a node high within the DP projection (cf. Krifka 1999)

$$(13) \quad \|\exists\| = \lambda P_{\langle et \rangle} \lambda Q_{\langle et \rangle} \exists x [P(x) \wedge Q(x)]$$

- 3) *Many* and *few* (predicates of scalar intervals) are linked to common nouns (predicates of individuals) via covert measure function COUNT (cf. Schwarzschild’s 2006 Mon; Rett’s 2006 COUNT; Kayne’s 2005 null noun NUMBER)

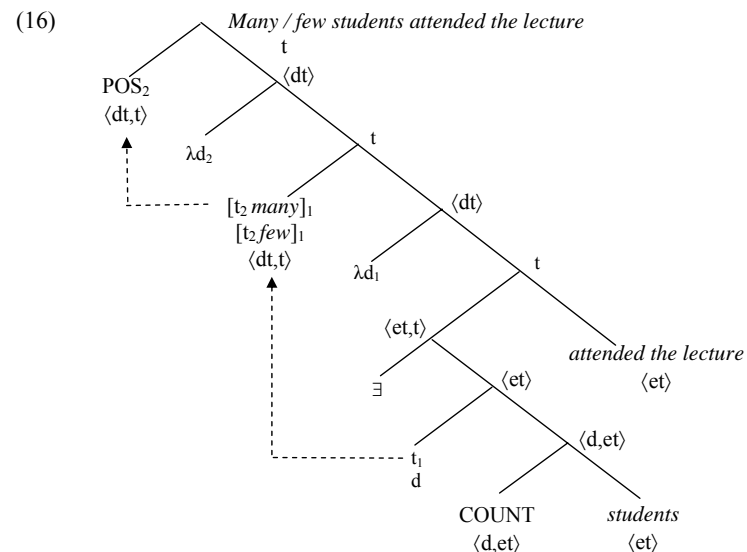
$$(14) \quad \|\text{COUNT}\| = \lambda d \lambda X . \text{COUNT}(X) \geq d$$

- The role of COUNT is to introduce a degree argument and link it to an individual argument
- Being of type  $\langle d, et \rangle$ , COUNT may combine with a common noun (type  $\langle et \rangle$ ) via variable identification (Kratzer 1996)

$$(15) \quad \|\text{students}\| = \lambda X . * \text{student}(X) \\ \|\text{COUNT students}\| = \lambda d \lambda X . * \text{student}(X) \wedge \text{COUNT}(X) \geq d$$

### The Positive Forms:

- ◆ The sentences in (1) then have the derivation shown schematically in (16).



- ◆ The resulting structure has the following interpretation (give first formally, then more simply):

$$(17) \quad \text{a.} \quad \|1a\|: \quad \text{Many students attended the lecture} \\ [[\text{POS}_2 [t_2\text{-many}_1 [t_1\text{-COUNT students attended the lecture}}]]]$$

$$\forall d \in N_S [\exists X . * \text{student}(X) \wedge \text{AtL}(X) \wedge \text{COUNT}(X) \geq d]$$

$$N_S \subseteq [0, \# \text{-Students-AtL}]$$

‘the number of students attending the lecture exceeds the neutral range’

$$\text{b.} \quad \|1b\|: \quad \text{Few students attended the lecture} \\ [[\text{POS}_2 [t_2\text{-few}_1 [t_1\text{-COUNT students attended the lecture}}]]]$$

$$\forall d \in N_S [\neg \exists X . * \text{student}(X) \wedge \text{AtL}(X) \wedge \text{COUNT}(X) > d]$$

$$N_S \subseteq [\# \text{-Students-AtL}, \infty]$$

‘the number of students attending the lecture falls short of the neutral range’

◆ Advantages of degree-quantifier analysis of *few* (not fully captured by other accounts):

- Existential quantification without ‘van Benthem’s problem’ (van Benthem 1986)
- Scope splitting

(18) They need few reasons to fire you  
‘it is not the case that they need a large number of reasons...’

**The Comparative:**

◆ Comparatives must provide intervals to serve as the arguments of *many/few*. For example, for (5a) – repeated as (19a) -- we require the interval in (19b) or, equivalently, that in (19c):

- (19) a. many more than 100 students attended the lecture  
b. **[100, #-Students-AtL]**  
c. **[0, #-Students-AtL- 100]**

◆ Can be achieved by defining comparative to include an additional degree argument that can be saturated or bound by a degree expression (von Stechow 1984; Kennedy 2001; Schwarzschild & Wilkinson 2002):

$$(20) \quad \|\text{-er than } d\| = \lambda d' \lambda I_{dir}. [d \in I \wedge 0 \leq d' \leq |MAX_{dir}(I) - d|]$$

where  $|n|$  = absolute value of  $n$ ,

$MAX_{dir}$  is a maximality operator sensitive to interval direction

◆ Yields (21) - (23) as the logical forms and resulting semantic interpretations for (5a-c):

(21) Many more than 100 students attended the lecture  
[POS<sub>4</sub> [t<sub>4</sub>-many<sub>3</sub> [(t<sub>3</sub>-er than 100)<sub>2</sub> [t<sub>2</sub>-many<sub>1</sub> [t<sub>1</sub>-COUNT students attended]]]]]  
 $100 \in [0, \#-Student-AtL] \wedge N_S \subseteq [0, \#-Student-AtL - 100]$

(22) Few more than 100 students attended the lecture  
[POS<sub>4</sub> [t<sub>4</sub>-few<sub>3</sub> [(t<sub>3</sub>-er than 100)<sub>2</sub> [t<sub>2</sub>-many<sub>1</sub> [t<sub>1</sub>-COUNT students attended]]]]]  
 $100 \in [0, \#-Student-AtL] \wedge N_S \subseteq [\#-Student-AtL - 100, \infty]$

(23) Many fewer than 100 students attended the lecture  
[POS<sub>4</sub> [t<sub>4</sub>-many<sub>3</sub> [(t<sub>3</sub>-er than 100)<sub>2</sub> [t<sub>2</sub>-few<sub>1</sub> [t<sub>1</sub>-COUNT students attended]]]]]  
 $100 \in [\#-Student-AtL, \infty] \wedge N_S \subseteq [0, 100 - \#-Student-AtL]$

◆ Computation of logical forms in (21)-(23) involves successive creation and manipulation of scalar intervals.

(24) Many fewer than 100 students attended the lecture (5c)

d-COUNT students attended	<b>[0, #-Students-AtL]</b>	
<i>few</i>	<b>[-#-Students-AtL, ∞]</b>	by INV in (12b)
<i>-er than 100</i>	<b>[0, 100 - #-Student-AtL]</b>	by (20)
<i>many</i>	<b>[0, 100 - #-Student-AtL]</b>	by (12a)
POS	$N_S \in [0, 100 - \#-Student-AtL]$	by (11)

→ Ungrammaticality of \**few fewer*: successive application of INV disallowed??

◆ Unmodified comparatives: I propose the degree argument is existentially bound by an existential operator that picks out a non-zero degree:

(25) More than 100 students attended the lecture  
[(d-er than 100)<sub>2</sub> [t<sub>2</sub>-many<sub>1</sub> [t<sub>1</sub>-COUNT students attended]]]  
 $100 \in [0, \#-Student-AtL] \wedge \exists d \neq 0. d \in [0, \#-Student-AtL - 100]$

**Interim Summary**

- ◆ Present proposal yields correct semantics for complex quantifiers based on *many/few* (5), as well as simpler constructions (e.g. (1), (2))
- ◆ Below I show it has the potential to account for other facts (as in (4)) – but requires revisiting some assumptions about degree modifiers

**3. Extensions**

**3.1 Predicative *many/few***

◆ *Many/few* predicated of set of degrees associated with DP

(26) John’s friends are many  
[POS<sub>2</sub> [t<sub>2</sub>-many<sub>1</sub> [t<sub>1</sub>-COUNT(john’s friends) ≥ d’]]]  
 $\forall d \in N_S [COUNT(john’s friends) \geq d]$   
 $N_S \subseteq [0, \# \text{ of John’s friends}]$   
‘the number of John’s friends exceeds the neutral range’

◆ Note also:

- (27) a. Mary considers John tall (Hackl 2000)  
b. \*Mary considers the guests many "  
c. Mary considers 20 students many (R. Fiengo, p.c.)

### 3.2 A (*very*) *few*

- ◆ In bare form and when modified by some degree modifiers (e.g. *very*), *few* may follow *a*; in combination with other degree modifiers (e.g. comparative; *too*) it may not.
  - (28) a. A few students attended the lecture
  - b. A very few students attended the lecture
  - c. \*A fewer than 100 students attended the lecture
  - d. \*A too few students attended the lecture
- ◆ *Few* vs. *a few* (see also Solt 2006)
  - *A few* is not an idiom: other elements can intervene between *a* and *few*:
    - (29) a very few students
    - an incredibly few collectors
    - a long few days
  - The difference between *few* and *a few* is scope related – they can be given paraphrases that differ only in scope of negation (30), and *few* but not *a few* shows characteristics of sentential negation, e.g. NPI licensing (31):
    - (30) a. Few students attended the lecture  
          ‘it is not the case that there was a large group of students who attended the lecture’
    - b. A few students attended the lecture  
          ‘there was a not-large group of students who attended the lecture’
    - (31) a. Few students have ever been to one of Prof. Jones’ lectures
    - b. \*A few students have ever been to one of Prof. Jones’ lectures
- ◆ These differences can be accounted for with an analysis under which the negative expression *few* in *a (very) few* does not raise to take sentential scope (as *few* does in (16)), but rather is interpreted in its base position, below the existential quantifier.
  - (32) a. A few students attended the lecture  
           $\exists X[*\text{student}(X) \wedge \text{AtL}(X) \wedge \exists \exists d[\text{POS-few}(I) \wedge d \in I \wedge \text{COUNT}(X) \geq d]]$
  - b. A very few students attended the lecture  
           $\exists X[*\text{student}(X) \wedge \text{AtL}(X) \wedge \exists \exists d[\text{very-few}(I) \wedge d \in I \wedge \text{COUNT}(X) \geq d]]$
  - If the presence of *a* creates an environment from which *few* cannot raise, it should block the raising of degree modifiers as well. Furthermore, and crucially, this suggests an explanation for the contrast in (28), on the assumption that *-er than 100* and *too* must raise for interpretability
    - ➔ Implies some degree modifiers (POS, *very*) can be interpreted in situ

### 4. Implications for Degree Modification in Quantificational vs. Adjectival Domains

- ◆ *Many/few* vs. gradable adjectives – both gradable, but different semantic types:
  - (12) a.  $\| \text{many} \| = \lambda d \lambda I_{\langle dt \rangle}. d \in I \quad \langle d, \langle dt, t \rangle \rangle$
  - b.  $\| \text{few} \| = \lambda d \lambda I_{\langle dt \rangle}. d \in \text{INV}(I)$
  - (33)  $\| \text{tall} \| = \lambda d \lambda x_e. x \text{ is } d\text{-tall} \quad \langle d, et \rangle$
- ◆ Immaterial if degree modifiers interpreted as generalized quantifiers over degrees
  - Semantics of comparative given in (20) can be extended to adjectival comparatives such as *-er than 6 feet in taller than 6 feet*
    - (34) a. Fred is taller than 6 feet
    - b.  $6\text{ft} \in [\mathbf{0}, \text{Fred's height}] \wedge \exists d \neq 0. d \in [\mathbf{0}, \text{Fred's height} - 6\text{ft}]$
- ◆ If degree modifiers can combine locally with gradable expressions (as proposed above for POS, *very*), raises questions:
  - Gradable expression as argument of degree modifier (e.g. Kennedy 2007)?
    - ➔ Would require degree modifier to have different semantic type in case of *many/few* and gradable adjectives, an unwelcome result
  - Degree modifier as type-d argument of gradable expression?
    - ➔ Consistent with referential/quantificational duality in both individual and degree domains
    - ➔ Do POS/*very* (sometimes) have a ‘referential’ type-d interpretation?
- (35)  $\| \text{POS}_d \| = d_{\text{standard}}$   
 As support, *a few* has a context-insensitive interpretation: while *few* is context-dependent, *a few* is always  $\approx 3\text{-}4$  (cf. *every few days*)

### 5. Conclusions

- ❖ *Many* and *few* are neither quantificational determiners nor cardinality predicates, but are rather predicates of sets of degrees (intervals)
- ❖ This proposal accounts for both complex quantifiers such as *few more than 100* as well as simpler constructions, and shows potential to extent to occurrence of *many/few* in other contexts
- ❖ Consequence is that some degree modifiers cannot always be analyzed as generalized quantifiers over degrees

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