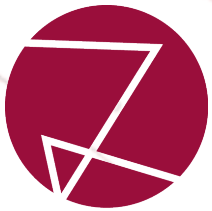
Granularity and Weak Disjunction

Stephanie Solt, ZAS Berlin



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Numerical Approximation

Much recent work on the semantics of approximation and precision regulators:

(1) What John cooked was exactly/approximately fifty tapas.
(Sauerland and Stateva, 2007)

(2) John arrived at be-gadol ("basically") / more or less 3
(Greenberg and Ronen, 2013)

(3) Let's meet at Starbuck's at 3-ish. (Bochnak and Csipak, 2014)

(4) Some twenty people attended the party. (Anderson, 2014)

(5) Mary waited for forty-five minutes. (Krifka, 2009)

Weak Disjunction

Today's topic: a lesser-studied means of expressing approximation, which I refer to as Weak Disjunction (WD)
(Pollmann and Jansen, 1996; Langendoen, 2006; Eriksson et al., 2010)

(6) a. There were forty or fifty people at the party.
b. John bought fifteen or twenty books.

- Have the form of disjunctions
- Convey approximate ranges

Objectives

- Propose a semantic analysis of Weak Disjunction in terms of scale granularity, construed here in terms of sets of alternatives
- Argue on the basis of these (and other) data for a more general granularity-based theory of numerical (im)precision

Outline

Characteristics of Weak Disjunction

Theories of Imprecision

An Alternative Theory of Granularity

Back to Weak Disjunction

Conclusions and Open Questions

Outline

Characteristics of WD

WD constructions are superficially similar to other range-based approximative expressions:

(7) a. There were 40 or 50 people at the meeting.
b. There were between 40 and 50 people at the meeting.
c. There were 40 to 50 people at the meeting.

But closer examination reveals important differences.

#1. Only some pairs form acceptable WDs

The pairs in (8) are felicitous on an approximation reading; those in (9) are infelicitous or allow only a 'true disjunction' reading.

(8) a. There were 5 or 6 people at the public meeting.
b. …10 or 12 …
c. …15 or 20 …
d. …30 or 40 …

(9) a. #…10 or 13 …
b. #…18 or 21 …
c. #…15 or 25 …
d. #…30 or 50 …
e. #…12 or 10 …
f. #…500 or 502 …

#1. Only some pairs form acceptable WDs

These constraints are not observed for *between* sentences (except: sequential values, declining sequences)

(10)
a. #There were between 5 and 6 people at the public meeting.
b. …between 10 and 12 …
c. …between 15 and 20 …
d. …between 30 and 40 …
e. …between 10 and 13 …
f. …between 18 and 21 …
g. …between 15 and 25 …
h. …between 30 and 50 …
i. #…between 12 and 10 …
j. …between 500and 502 …

#1. Only some pairs form acceptable WDs

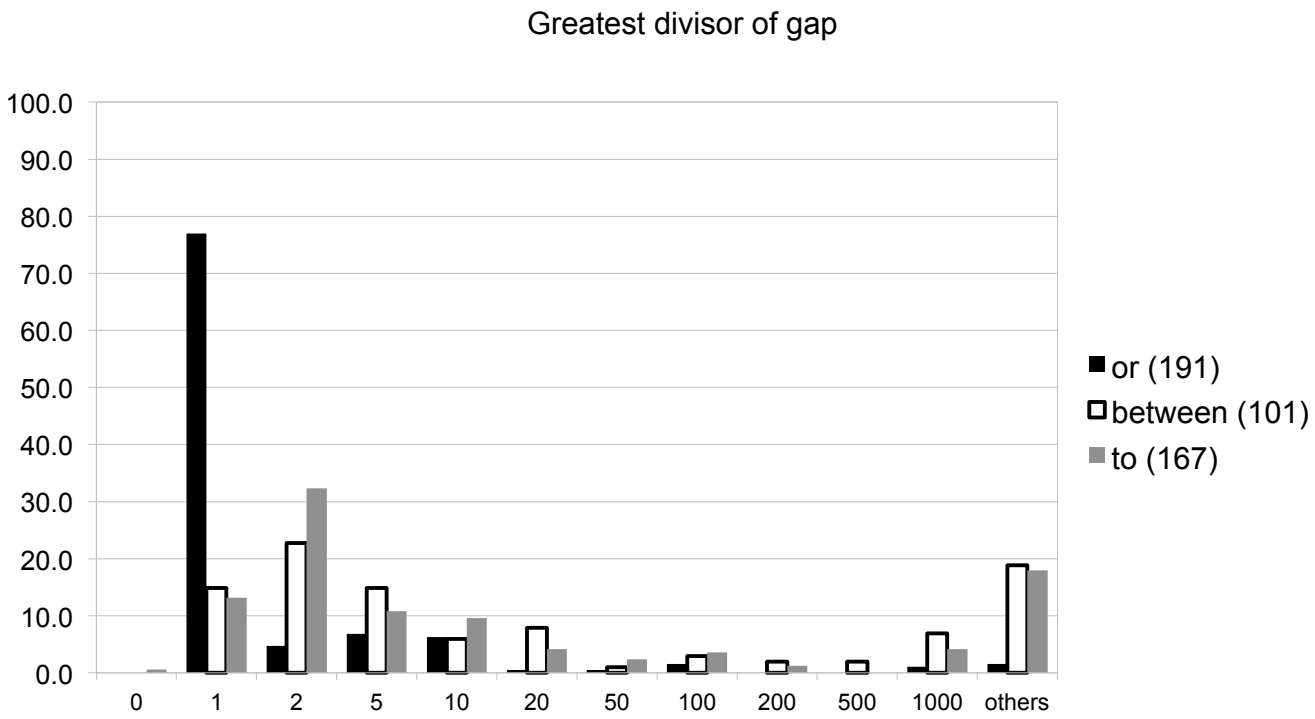
Pollmann and Jansen (1996) and Eriksson et al. (2010) identify the following 'rules' for the WD construction:

1. the two numbers must be in ascending order;
2. the gap between them must be a divisor of both values;
3. the gap must be a so-called favored number, being of the form $\{1/2/2.5/5\} * 10^n$
4. the two numbers cannot be too close ('5% rule').

#1. Only some pairs form acceptable WDs

Corpus of Contemporary American English (COCA) (Davies, 2008-)

- Random samples of WDs, *between*, *to* (n=200 each)



Gap not a divisor of both values:

- WD - 2%, *between* - 59 %, *to* - 53 %

#2. WDs express approximations

Intuitively, (11) expresses an approximation and signals speaker uncertainty, while (12) is a precise expression of a range.

Relatedly, (12) would be false if 49 people attended, while (11) is at least marginally acceptable:

(11) There were 50 or 60 people at the public meeting.

(12) There were between 50 and 60 people at the public meeting.

#2. WDs express approximations

Experimental support

- Amazon MTurk; two scenarios
  - We've sold 40 or 50 / between 40 and 50 tickets
  - We still need to deliver 20 or 30 / between 20 and 30 newspapers
- Two questions (7-point Likert scale)
  - Acceptability in 'one under' situation
  - Speaker certainty
- n=25 per scenario/question

	Mean		
	WD	*between*	
Acceptability	3.8	2.6	$z=3.25$, $p<0.01$
Certainty	3.7	5.0	$z=-4.02$, $p<0.001$

Outline

Facts to be explained

Round numbers allow – or even favor – approximate interpretations:

(13)
a. Mabel owns one hundred sheep.
b. Jane arrived at three o'clock.
c. The meeting lasted forty-five minutes.
d. The rope is fifty meters long.

Non-round numbers must be interpreted precisely:

(14)
a. Mabel owns ninety nine sheep.
b. Jane arrived at three-oh-one.
c. The meeting lasted forty-three minutes.
d. The rope is fifty-one meters long.

(Im)precision can be conveyed explicitly via approximators (*about*, *roughly*, *exactly*, etc.)

Pragmatic Halos

(Lasersohn, 1999)

Core idea: Speakers may utter sentences that, while strictly speaking false, are 'close enough' to true for practical purposes.

Pragmatic Halos

Each expression of the language is assigned a denotation as usual, but this denotation is also associated with a set of entities of the same semantic type–its halo–which represent the values that differ from the denotation in only pragmatically ignorable ways.

(15) a. $[\![\text{three o'clock}]\!] = i$

b. $H_C(\text{three o'clock}) = \{\ldots, g, h, i, k, l, \ldots\}$

Halos of complex expressions derived compositionally. A sentence is felicitous if some element in its halo is true.

Approximators manipulate halos.

Pragmatic Halos

(Lasersohn, 1999)

What it gets right:

- Account of imprecise interpretations, including WD:

(16) There were 50 or 60 people at the public meeting.

$\approx$ 'There were n people at the meeting, for some n different from 50 or from 60 in only pragmatically ignorable ways'

Pragmatic Halos

(Lasersohn, 1999)

Questions/Issues:

- No account for which numerical expressions interpreted imprecisely.

  (17) Sue arrived at three o'clock — felicitous for arrival at 3:01

  (18) Sue arrived at three-oh-one — not felicitous for arrival at 3:00

  - 'different in only pragmatically ignorable ways' not symmetric?

- No account for which pairs form acceptable WDs.

Pragmatic Halos

(Lasersohn, 1999)

Questions/Issues:

- No account for precise interpretation of round numbers in complex expressions:

(19) Mabel owns fifty sheep.

- Potentially felicitous if she owns 48.

(20) Mabel owns more than fifty sheep.

- Infelicitous if she owns 49 (though $49 > 48$).

Same problem with *between* sentences.

Philosophical issue

Much of what speakers say using numerical expressions must be analyzed as false!

Scale Granularity

(Krifka, 2007, 2009)

Core idea: Results of measurement can be reported w.r.t. scales that differ in their granularity level (size minimal unit).

Example: distance measurement

```
…-39m-40m-41m-42m-43m-44m-45m-46m-47m-48m-49m-50m-51m-52m-…
…-----40m----------------45m----------------50m---------…
…-----40m-----------------------------------50m---------…
```

- Common granularities: powers of 10 and results of halving/doubling (exception: specific domains such as time measurement).

Approx. interpretation = interpretation w.r.t. coarse scale.

Pragmatic principles favor most likely interpretation.

- For simple numerical expressions, this is the approximate one.

Approximators function by (re)setting granularity level (Sauerland and Stateva, 2007).

Scale Granularity

(Krifka, 2007, 2009)

What it gets right:

- Account for imprecise interpretation and which numerical expressions allow them
  - Those occurring on coarser-grained scales
- Account for restrictions on pairs producing WDs
  - Two sequential values on a scale of some standard granularity level.

Scale Granularity

(Krifka, 2007, 2009)

Questions/Issues:

- No full account for (im)precision in complex expressions:

(21) Mabel owns more than 50 sheep.
  - Predicted: 50 interpreted precisely ✔

(22) Mabel owns between 50 and 60 sheep.
  - Predicted: 50 and 60 interpreted approximately ✘

- No account of role of scale granularity in pragmatic inferencing (Cummins et al., 2012)
  - E.g. (21) implicates that Mabel doesn't own more than 60/75/100 sheep.

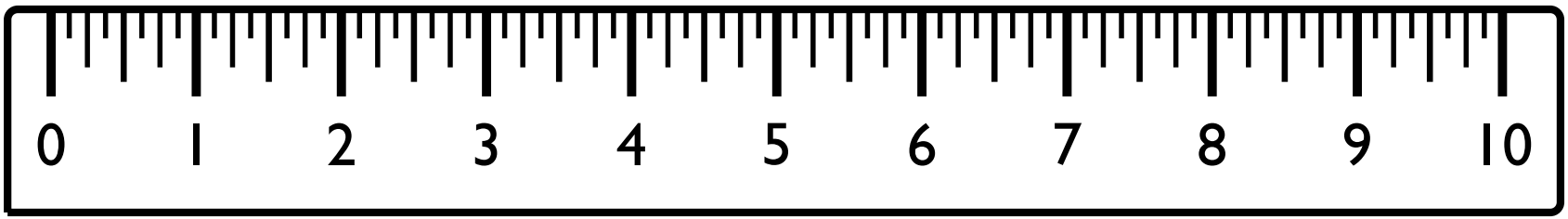
Philosophical issue

An expression such as *fifty* has an interpretation according to which it denotes a range!

Outline

Ruler Metaphor

The ruler: a continuous scale on which discrete hierarchical structure is imposed, allowing expression of measurements at varying precision levels



This suggests how we might conceptualize scale granularity as it pertains to natural language measurement expressions.

Three Components

1. Precise denotations for number words
2. Granularity as sets of alternatives
3. Truth relativized to granularity level

1. Precise denotations for number words

We take number words to denote quantifiers over degrees (type $\langle dt, t \rangle$), which can be lowered to a term meaning (type d).
(Kennedy, 2015).

(23) a. $[\![\text{sixty}_{\langle dt,t \rangle}]\!] = \lambda I_{\langle dt,t \rangle}.max(I) = 60$

b. $[\![\text{sixty}_{d}]\!] = 60$

This contrasts with previous granularity-based analyses in which measure expressions have interpretations on which they denote scalar ranges.

- NB: An 'exact' interpretation of the number word is crucial.

2. Granularity as sets of alternatives

Granularity may now be represented in terms of sets of alternatives to a measure expression. We start with some standard unit $gran$, which defines a standard sequence S_{gran}.

For example:

(24) a. $gran = 10$; $S_{10} = \{10, 20, 30, \dots\}$
b. $gran = 20$; $S_{20} = \{20, 40, 60, \dots\}$

- Per above, typical choices for $gran$ are powers of 10 and the results of halving and doubling these.

2. Granularity as sets of alternatives

For a measure expression α and granularity level $gran$, a set of alternatives to α can then be defined as follows:

(25) $$ALT_{gran}(\alpha) = \{\alpha' : [\![\alpha']\!] \in S_{gran}\}$$

For example:

(26) a. $ALT_{10}(\textit{sixty})$ ={…, *fifty, sixty, seventy*, …}

b. $ALT_{20}(\textit{sixty})$ ={…, *forty, sixty, eighty*, …}

The members of ALT_{gran} can be likened to the markings on the ruler: they represent the (only) choices the speaker has to report a measurement at granularity level $gran$.

3. Truth relative to granularity level

The truth / falsity of a sentence containing a measure expression is relativized to the granularity level at which that expression is interpreted.

- For each measure expression, granularity level $gran$ contextually determined via an assignment function g.
- Truth at a granularity level defined in terms of the scalar distance that the actual measure would need to be displaced in order to achieve truth under a perfectly precise ($gran = 0$) interpretation.

3. Truth relative to granularity level

Formal definition

For a sentence ϕ containing a measure expression α, $[\![\phi]\!]^{g} = 1$ iff there is no $\alpha' \in ALT_{gran}(\alpha)$ such that $[\![\phi[\alpha'/\alpha]]\!]^{g[gran=0]} = 1$ would require a smaller scalar displacement of the actual measure than $[\![\phi]\!]^{g[gran=0]} = 1$.

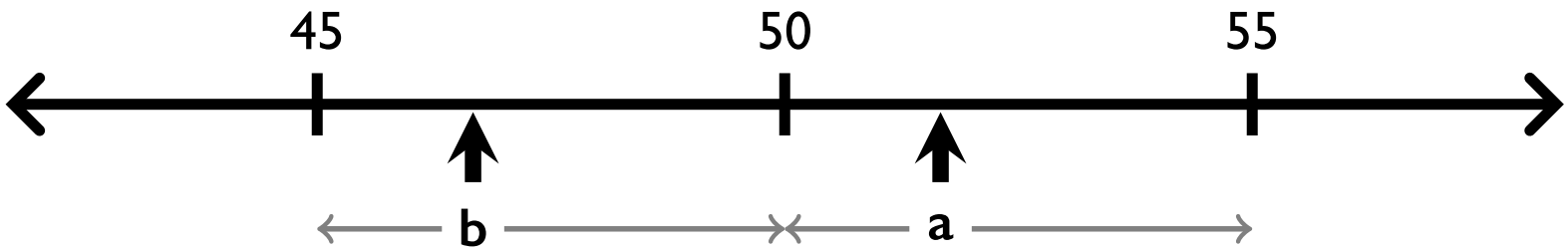
- More simply: a proposition containing a measure expression is evaluated as true iff there is no better choice of expression at the relevant granularity level.

3. Truth relative to granularity level

(27) There were fifty people at the meeting.
$max(\{d$: there were d people at the meeting$\}) = 50$

Assume $gran = 5$. We then consider the following at $gran = 0$:

(28)
a. $[\![$There were forty-five people at the meeting$]\!]^{g[gran=0]}$
b. $[\![$There were fifty people at the meeting$]\!]^{g[gran=0]}$
c. $[\![$There were fifty-five people at the meeting$]\!]^{g[gran=0]}$



Situation a: $[\![(27)]\!]^g = 1$ – (28b) requires smallest displacement

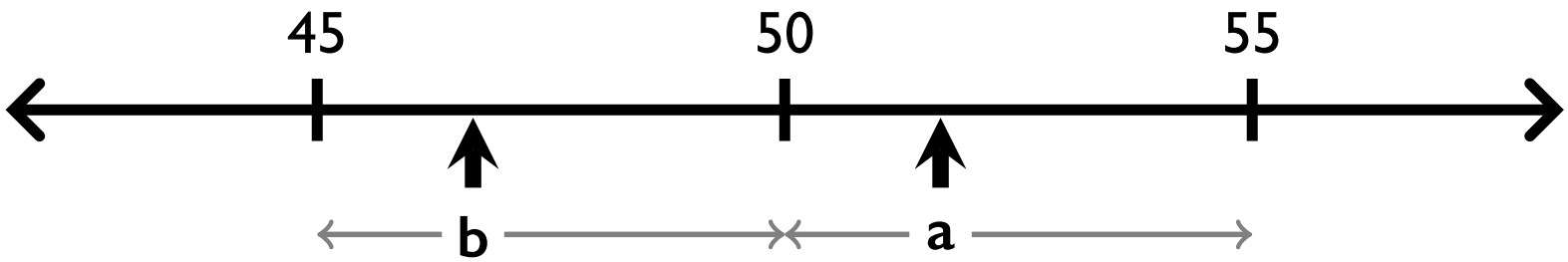
Situation b: $[\![(27)]\!]^g = 0$ – smaller displacement makes (28a) true

3. Truth relative to granularity level

(29) There were more than fifty people at the meeting.

$max(\{d$: there were d people at the meeting$\}) > 50$

(30) a. $[\![$There were more than forty-five people …$]\!]^{g[gran=0]}$
b. $[\![$There were more than fifty people …$]\!]^{g[gran=0]}$
c. $[\![$There were more than fifty-five people …$]\!]^{g[gran=0]}$



Situation a (and all other cases > 50): $[\![(29)]\!]^g = 1$ – because (30b) already true with no displacement of actual measure.

Situation b (and all other cases ≤ 50) $[\![(29)]\!]^g = 0$ – because there is always an alternative true with no displacement (here (30a)).

- Precise interpretation regardless of $gran$

Summary

The alternative theory of granularity yields …

- approximate interpretations for point-denoting measure expressions;
- precise interpretations for measure expressions occurring as constituents of range-denoting expressions.

Number words have precise denotations – but sentences containing them may nonetheless be evaluated as true if the actual value deviates from precise denotation.

Alternatives also available as input into implicature calculation.
(Cummins et al., 2012; Solt, 2014)

Outline

Granularity and range-based approximation

The above analysis extends readily to Weak Disjunctions, and accounts for the differences versus *between* constructions.

Weak Disjunction: Disjunction of two point-denoting expressions

***between* construction:** Range-denoting expression

Weak Disjunction

(31) There were forty or fifty people at the meeting.

We take *or* to be interpreted as set union:

(32) $[\![\text{forty or fifty}]\!] = [\![\text{forty}]\!] \cup [\![\text{fifty}]\!]$
$= \lambda I_{\langle dt,t \rangle}.max(I) = 40 \text{ or } 50$

Thus (31) receives the following interpretation:

(33) $max(\{d : \text{there were } d \text{ people at the meeting}\}) = 40 \text{ or } 50$

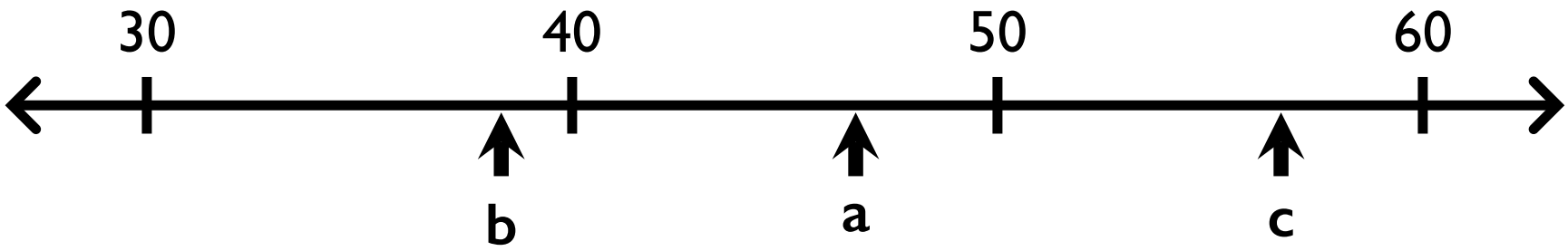
We further take the disjunction to overtly set $gran$ to the value of the gap (here: $gran = 10$)

- Gap must be acceptable choice for $gran$
- Two values must have interpretations in S_{gran}

Weak Disjunction

(34) There were forty or fifty people at the meeting.

- Truthfully assertable for all values for which *forty* or *fifty* is at least as good a description as any other member of $ALT_{10}(\textit{forty})(=ALT_{10}(\textit{fifty}))$
  - all values in between 40 and 50 (e.g. **a**)
  - Values outside of this range that are closer to 40 or 50 than any other value at $gran = 10$ (e.g. **b** but not **c**)



End result: *forty or fifty* effectively conveys an approximate range.

between sentences

(35) There were between forty and fifty people at the meeting.

We take *between* to directly introduce a range:

(36) $[\![\text{between}]\!] = \lambda n \lambda m \lambda I . n \leq max(I) \leq m$

(37) $[\![\text{between forty and fifty}]\!] = \lambda I . 40 \leq max(I) \leq 50$

This yields the following as the interpretation for (34):

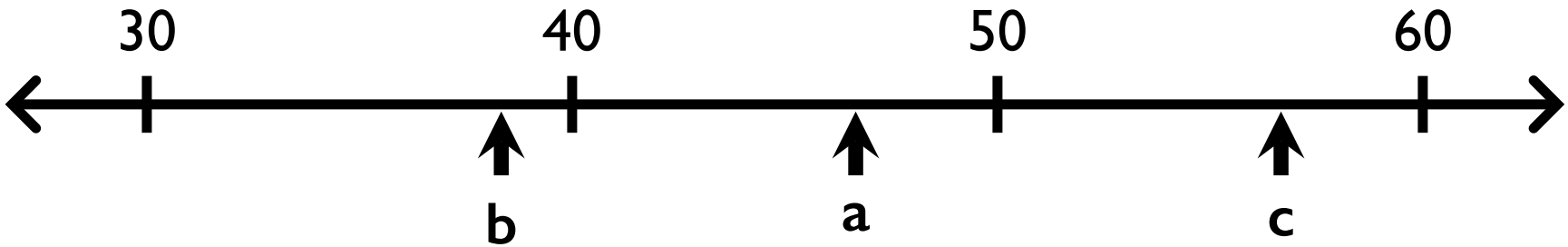
(38) $40 \leq max(\{d : \text{there were } d \text{ people at the meeting}\}) \leq 50$

- No restrictions on choice of m and n

between sentences

(39) There were between forty and fifty people at the meeting.

- Regardless of choice of $gran$, (39) evaluated as true for any value between 40 and 50 (e.g. **a**), since it is already true for such values at $gran = 0$.
- Conversely, it is evaluated as false for all values outside of this range (e.g. **b,c**), since there are alternatives that would make it true at $gran = 0$ (for **b** e.g. *between 30 and 40*)



End result: *between forty and fifty* conveys a range with sharp boundaries.

Outline

Conclusions

- Weak Disjunction provides evidence for a granularity-based account of numerical imprecision.
- An approach in which granularity is construed in terms of sets of alternatives yields a straightforward semantic analysis of Weak Disjunction, and addresses shortcomings of previous accounts:
  - Versus Pragmatic Halos, explains restrictions on which numerals can be used approximately (alone or in WDs).
  - Versus previous granularity-based analyses, derives the necessarily precise interpretation of number words in complex numerical expressions.

Open questions

- *to* constructions

(40) There were forty to fifty people at the meeting.

- Composition with overt approximators

(41) There were about fifty people at the meeting.

(42) There were about forty or fifty people at the meeting.

(43) #There were about between forty and fifty people at the meeting.

…but

(44) ?There were roughly between forty and fifty people at the meeting.

THANK YOU

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