On measurement and quantification: The case of *most* and *more than half*

Stephanie Solt
Zentrum für Allgemeine Sprachwissenschaft
Schützenstraße 18
10117 Berlin, Germany

stephanie.solt@gmail.com

+49 176 96974654

Revised version, resubmitted to *Language*

November 22, 2013

Acknowledgments

Portions of this material have been presented at the 4th Workshop on OT and Interpretation, the CUNY Syntax Supper, Modeling Interaction, Dialog, Social Choice, and Vagueness (MIDiSoVa), the 84th Annual Meeting of the LSA, and Logical Models of Reasoning with Vague Information (LoMoReVI), and I would like to thank these audiences for their comments and suggestions. For very helpful discussions and comments on earlier versions of this paper, I am especially grateful to Anton Benz, Justin Halberda, Larry Horn, Manfred Krifka, Bill McClure, Rick Nouwen, Uli Sauerland and Jakub Szymanik. I would also like to express my gratitude to two anonymous *Language* referees and to editor Greg Carlson, whose input has been of immense help in preparing this version. Finally, thanks to Nicole Gotzner and Camilo Rodriguez Rondero for assistance with the corpus analysis. Work on this paper was supported by the European Science Foundation (ESF) and the Deutsche Forschungsgemeinschaft (DFG) under grants 925/4 and 1157/1-1.

On measurement and quantification: The case of *most* and *more than half*

Abstract

The quantifiers *most* and *more than half* pose a challenge to formal semantic analysis. On the one hand, their meanings seem essentially the same, prompting accounts that treat them as logically equivalent. On the other hand, their behavior is known to diverge in a number of interesting ways. This paper draws attention to some previously unnoticed contrasts in distribution and interpretation between the two, and develops a novel semantic analysis of them, based on principles of measurement theory. *Most* and *more than half* have logical forms that are superficially equivalent (per Hackl 2009), but which place different requirements on the structure of the underlying measurement scale: *more than half* requires a ratio scale, while *most* can be interpreted relative to an ordinal scale or one with a semi-ordered structure. The latter scale type is motivated by findings from psychophysics, and by psychological models of humans' approximate numerical abilities. A corpus analysis is presented which confirms the predictions of the present account. These findings add to other recent work demonstrating the relevance of measurement theoretic concepts to natural language meaning.

1 Introduction

1.1 Modes of comparison

Suppose I have two objects – rocks, let's say – and I want to know whether the first is heavier than the second. There are (at least) two ways that I might go about finding this out. To take one approach, I could place the first rock on a scale and write down its measured weight, do the same with the second rock, and then compare the two values I have recorded (e.g. 452 grams vs. 319 grams). Alternately, I could place the two rocks on the two pans of a balance scale, and observe which side hangs lower, without in any way representing or recording the weight of either rock individually.

Now suppose that the question is whether the first rock weighs more than half as much as the second. In this case, the first procedure can still be applied: I weigh the first rock and record that value, weigh the second rock and divide the resulting value by two, and then compare the two values. But in this case, the second procedure will not work: "half as much as the second rock weighs" is not an object that can be placed on a balance.

There is a fundamental difference between the two modes of comparison illustrated above. The first assigns each entity a numerical measure which can be compared to the measures of other individuals, and which may serve as input to mathematical operations (in the example above, division by two). The second, on the other hand, merely establishes a relation of 'greater than' between two entities. Given some set of entities, we could via a series of pairwise comparisons establish a rank ordering of its members. This ordering could in turn be given a numerical representation in an order-preserving way, for example by assigning the number 1 to the highest ranked entity, 2 to

the next highest ranked, 3 to the third highest ranked, and so forth. But the numbers assigned in this way would in an important sense be arbitrary. By contrast, in the case of the first procedure, the only arbitrariness is in the choice of the unit of measurement; once this is fixed, so too is the value assigned to each individual.

These observations are by no means new. There is an entire field of study, namely measurement theory, devoted to understanding how the properties of and relationships between entities can be represented numerically (Kranz et al. 1971). A basic finding is that not all types of relationships can be given a numerical representation that supports comparisons such as that between the ratios of measures. Results from cognitive psychology support a somewhat similar distinction in how humans perform comparisons: in particular, comparison of the sizes of two sets may proceed via a precise symbolic representation of their cardinality, or via a more basic analog representation of their magnitude (Dehaene 1997).

The main theme of the present paper is that distinctions of this sort are relevant to language as well. Specifically, certain expressions of measure assume the possibility, at least in principle, of applying one or the other of these procedures to their assessment. This distinction has consequences for both the distribution and the interpretation of these items.

The domain of inquiry within which I will explore this topic is quantification, with specific focus on the proportional quantifiers *most* and *more than half*, as in the following:

(1) a. <u>Most</u> Americans have broadband internet access.
    b. <u>More than half</u> of Americans have broadband internet access.

(2) a. <u>Most</u> of the electricity used in the United States is produced at power plants.
    b. <u>More than half</u> of the electricity used in the United States is produced at power plants.

The specific claim that I will argue for is that *more than half* assumes the first of the two modes of comparison discussed above, while *most* favors something closer to the second.

More broadly, the results from this domain will provide evidence for the relevance of measurement theoretic concepts to natural language meaning. Specifically, measurement scales vary in the strength of the ordering relations that they are based on, and this has linguistic consequences. Beyond this, this case study will point to a meaningful connection between the semantics of quantificational expressions and the cognitive representation of quantity and measure.

1.2 On *most* and *more than half*

The pair *most* and *more than half* presents a challenge from the perspective of formal semantic analysis. On the one hand, they are on the surface quite similar in meaning. For example, both (1a) and (1b) would seem to be true in the case that the following holds:

(3) # of Americans who have broadband > # of Americans who don't have broadband

This apparent equivalence is recognized in both classic works on quantification (e.g. Westerstahl 1985; Benthem 1986; Keenan and Stavi 1986; Higginbotham 1995; Chierchia 1998a) as well as elementary semantics textbooks (e.g. Chierchia and McConnell-Ginet 2000), all of which assign *most* a logical representation that renders it equivalent to *more than half*.

On the other hand, it is now well known that the behavior of *most* and *more than half* diverges in a number of non-trivial ways. Most obviously, speakers commonly have the intuition that *most* and *more than half* differ in their lower bounds: while a simple majority is sufficient to establish the truth of an examples such as (1b) with *more than half*, a greater proportion is required for the corresponding *most* example (e.g. (1a)) (see e.g. Huddleston and Pullum 2002; Horn 2005). This is illustrated nicely by examples such as the following:

(4) a. ??Most of the American population is female.
b. More than half of the American population is female.

In the situation where the American population has a very slight female skew (in 2011, the numbers were 50.8% female vs. 49.2% male), (4b) is a true statement, while (4a) is at the very least infelicitous. Speakers do not entirely agree as to the nature of the infelicity: some judge (4a) as outright false, while others feel it to be true but inappropriate or misleading. But there is no disagreement that there is a sharp contrast here.

Contrasts in interpretation between *most* and *more than half* have also figured prominently in the debate on the semantics/pragmatics interface. *More than half* is unarguably defined by its lower bound. On the neo-Gricean view championed in particular by Horn (2005), *most* is likewise semantically lower bounded, with the upper bound ('not all') derived pragmatically via scalar implicature. An alternate view is offered by Ariel (2004, 2005), who proposes that the lexical meaning of *most* provides an upper as well as a lower bound, denoting a proper subset of a set that is larger than any other subset in a partitioning.

From another perspective, Szabolcsi (1997), following Sutton (1993), points out surprising contrasts of the following form:[1]

(5) a. *The professors met most of the boys each.
b. The professors met more than half of the boys each.

(6) a. *There will be most of the boys in the yard.
b. There will be more than half of the boys in the yard.

Finally, Hackl (2009) demonstrates experimentally that despite their apparent equivalence, *most* and *more than half* exhibit differences in online sentence processing. Hackl's findings will be discussed in more depth below.

Yet there are further differences between *most* and *more than half* that have not, to my knowledge, been previously discussed. Consider the following pairs, involving examples of the use of *most* drawn from the Corpus of Contemporary American English (Davies 2008-)[2], and the corresponding sentences formed by replacing *most* with *more than half*. In each case there is a noticeable contrast: while the (a) sentence is entirely felicitous, the (b) sentence is either odd or has a quite different feel to its interpretation:

[1] Szabolcsi in fact discusses *more than 50%* rather than *more than half*, but my intuition is that *more than half* behaves essentially the same in this respect.

[2] Details on this corpus are provided in Section 3.

(7) a. Most sadness diminishes with the passage of time . . . (Public Interest, Winter 2005)
b. ??More than half of sadness diminishes with the passage of time . . .

(8) a. Most pastel hues have a calming effect. (Prevention, December 2003)
b. ??More than half of pastel hues have a calming effect.

(9) a. Most teens want to fit in with their peers. (CNN YourHealth, 31/8/2002)
b. ??More than half of teens want to fit in with their peers.

(10) a. She noted lights on in most of the houses she passed . . .
(The Silenced: A Novel, Brett Battles. New York: Dell, 2001)
b. She noted lights on in more than half of the houses she passed . . .

Thus while (7a) is entirely felicitous, (7b) is peculiar, an effect which seems to come about because it implies that sadness can somehow be quantitatively measured. Likewise, in contrast to the unobjectionable (8a), (8b) has the odd implication that we have some exhaustive list of pastel hues, and have gone down it to determine for each whether or not it has a calming effect. (9) presents a different sort of contrast: the (a) sentence is entirely natural, having a generic-like interpretation, similar to what would obtain with a bare plural and a quantificational adverb (e.g. *Teens usually/mostly/typically want to fit in with their peers.*) The (b) sentence on the other hand is odd, and in particular the generic flavor is lost entirely. Rather, to the extent that it is acceptable at all, it has what might be termed a 'survey results' interpretation, seeming to report on some sort of survey of teens. Finally, in (10) the contrast is not so much in acceptability, but rather more subtly in interpretation and possible context of use: (10a) could be used to report the individual's impressions, while (10b) seems to require that she has counted the total number of houses, and the number in which the lights are on.

As far as I am aware, there has been no attempt in the semantics literature to address patterns such as these. Nor is it clear that any of the few existing accounts of this pair could provide a natural explanation of these facts. To take one possibility, Horn (2005) suggests that the greater length and complexity of *more than half* relative to *most* could be expected to constrain its use, in that marked expressions are typically restricted to marked contexts. But while this might explain some interpretive effects (and I will suggest below that something like this has a role to play), it is hard to see how length or syntactic complexity alone could account for the sharp contrasts in acceptability exemplified in (7)-(9).

In this paper, I will attempt to offer a comprehensive answer the puzzle that is posed by these two quantifiers: how is it that *most* and *more than half* are on the one hand seemingly equivalent, while on the other hand diverging in their behavior in the various ways discussed above. The central claim will be that what lies at the heart of the matter is the difference between the two sorts of measurement procedures discussed above.

The structure of the paper is the following: Section 2 lays out the core of the proposal, and shows how it accounts for data such as (7)-(10). Some broader predictions for the distribution and interpretation of the two quantifiers are outlined. Section 3 presents a corpus analysis in which these predictions are tested, and some further differences between *most* and *more than half*

are identified. Section 4 offers a more formal account of the relationship between noun phrase semantics and measurement, and compares the present account to an alternate approach, that of Szabolcsi (1997). Section 5 concludes, and briefly discusses possible extensions.

2 Proposal

2.1 Lexical Semantics

I take as my starting point a proposal by Hackl (2009) that *most* and *more than half* have distinct logical forms that respect their internal composition. *Most* should be analyzed as the superlative form of *many* (an idea that goes back to Bresnan 1973, and much earlier work), while *more than half* should be analyzed as a comparative, incorporating the notion of half, namely division by two. Building on this, I propose the following logical forms for the two quantifiers, which are slightly more general versions of Hackl's:

(11) a. $[\![\text{most}]\!](A)(B) = 1$ iff $\mu_S(A \cap B) \succ \mu_S(A - B)$
b. $[\![\text{more than half}]\!](A)(B) = 1$ iff $\mu_S(A \cap B) \succ \mu_S(A)/2$

Here A and B range over sets[3], and μ_S is a measure function, that is, a function that maps entities to degrees on the scale *S*. For both quantifiers, the specific scale is left unspecified. In the case of count nouns, the scale can be (though is not always) the counting numbers, in which case μ_S corresponds to a cardinality operator on set sizes. In the mass domain, a scale tracking some other dimension is involved; for example, in (2), the scale might be electrical energy as measured in kilowatt hours.[4] Below we will see that the underspecification of the scale has further consequences.

As pointed out by Hackl, (11a) and (11b) are, at least on the surface, logically equivalent. Again taking the sentences in (1) as an example, if the number of Americans who have broadband internet access is greater than the number who do not (per (11a)), then it is also the case that the number who have broadband is greater than half the total number of Americans (per (11b)), and vice versa. Nonetheless, Hackl shows experimentally that the difference in form between (11a) and (11b) impacts sentence processing, an effect he attributes to their giving rise to two distinct verification procedures. Sentences with *most* are preferentially verified via a 'vote-counting' procedure: for each *A* that is *B*, determine whether there is at least one *A* that is not *B*; if not, the sentence is true. *More than half*, by contrast, triggers a procedure involving totaling the number of *A* that are *B*, and comparing that number to half the total number of *A*. Hackl shows that in a task which favors the vote-counting procedure, subjects are quicker when the quantifier presented is *most* than when it is *more than half*.

I would like to suggest that this difference in processing represents only the tip of the iceberg, so to speak, in the consequences from the difference in logical form. The central claim I will argue for here is that while both the logical forms in (11) are based on a measure function μ_S, they differ

[3]We will have reason to refine this somewhat below.

[4]In Hackl's formulation, the lexical entries of *most* and *more than half* are defined in terms of a set cardinality operator, which limits their application to occurrence with count nouns. The formulations in (11) are thus more general.

in the underlying structure of the scale that serves as the range of this function, a difference that corresponds to that between the two weighing procedures described at the start of this paper. The divergent behavior of *most* and *more than half* discussed in the previous section will be shown to derive from this.

To develop this idea further, it is necessary to look in more depth at what scales are, and how they differ in their structures. I turn to this now.

2.2 Scale structure and measurement theory

Following work by Bartsch and Vennemann (1973); Cresswell (1977); Bierwisch (1989); Kennedy (1997); Heim (2000); Hackl (2000) and others, I adopt a degree-based semantic framework, in which the ontology is extended to include degrees as a basic type (type *d*). Degrees are organized into scales. As a first approximation (to be refined below), a scale *S* can be conceptualized as a triplet of the following form:

(12) $S = \langle D, \succ, DIM \rangle$, where

- D is a set of degrees,
- $\succ$ is an ordering relation on that set, and
- DIM is a dimension of measurement.

Here a dimension *DIM* is a property that an entity can have more or less of, or equivalently, a property on which two entities can be compared. Examples of dimensions that are relevant to the study of quantification include weight, volume, area and of course number of elements as applied to sets.

On the definition of a scale embodied in (12), a single dimension *DIM* can potentially be associated with multiple scales, which differ in the composition of the set of degrees *D* and/or the properties of the ordering relation $\succ$. A framework in which this possible variation can be characterized is provided by the field of measurement theory, the branch of applied mathematics concerned with the numerical representation of properties of and relationships between entities (see Kranz et al. 1971; Roberts 1979 for basic introductions to measurement theory, and Krifka 1989; Klein 1991; Nerbonne 1995; Bale 2008; van Rooij 2011b; Lassiter 2011; Sassoon 2010 for linguistic applications).

Since Stevens (1957), it is common in measurement theory to distinguish several levels of measurement, varying from weaker to stronger. An **ordinal** scale represents a simple rank ordering, with no notion of distance between scale points. A simple example of such a scale is the rank ordering of runners finishing a race (first, second, third, etc.). An **interval** scale adds a fixed unit of measure, such that it is now meaningful to talk about the distance between scale points. The classic example of an interval scale is temperature measured in Celsius or Fahrenheit. Finally, a **ratio** scale adds a non-arbitrary zero point. Examples of physical measures based on ratio scales include height in centimeters and weight in grams.

Scales that are higher in this hierarchy support a wider range of comparative statements, and in that sense can be considered stronger or more informative. Ordinal scales support comparisons

of measures (e.g. *runner A finished the race before runner B*), but nothing further. Interval scales allow these as well as statements of magnitude of difference (e.g. *today is 3 degrees warmer than yesterday*). Finally, ratio scales also allow comparisons of ratios of measures (e.g. *this rock is twice as heavy as that rock*).

As is well known, an ordinal scale can be constructed from a simple qualitative ordering of individuals (see e.g. Kranz et al. 1971; Cresswell 1977; Klein 1991; Bale 2008). As one way to do this, we begin with some set of entities *A* and a binary comparison relation *R* between them that satisfies the properties of a strict weak order.[5] Examples of such relations include "weighs more than" (applied to physical objects), "finished the race before" (applied to the participating runners) and "has more elements than" (applied to some set of sets). The equivalence classes under this relation become the degrees of the scale (where *a* and *b* are equivalent under *R* iff for all *c*, *R(a,c)* iff *R(b,c)*, and *R(c,a)* iff *R(c,b)*). A relation between degrees is then derived from the relation between individuals as follows: for equivalence classes (i.e. degrees) $\overline{a}$ and $\overline{b}$ containing individuals *a* and *b*, respectively, $\overline{a} \succ \overline{b}$ iff *R(a,b)*. Derived in this way, the $\succ$ relation satisfies the properties of a strict linear order[6], and as such the resulting scale consists of a linearly ordered set of degrees; this is another way to characterize an ordinal scale. The second of the two weighing procedures discussed at the start of this paper – the one based on the balance – would serve as the basis for creating a scale of this sort. As noted earlier, such a scale can be given a numerical representation by associating its degrees with numbers in an order-preserving way, but the numbers assigned in this way have no meaning beyond their representation of the original "greater than" relation *R*.

By contrast, a qualitative ordering is not sufficient to serve as the basis for a ratio scale. For this, it is necessary also to define a concatenation operation $\oplus$ on elements of the domain of the relation *R*, and a corresponding operation of addition on degrees of the scale, such that the following holds:

(13) $\mu_S(a \oplus b) = \mu_S(a) + \mu_S(b)$

To take the dimension of weight as an example, the concatenation of two objects corresponds to placing them together; the weight of two objects placed together is the sum of their individual weights. A scale where this property holds can be given a numerical representation by selecting a standard object to form the basis for a unit of measure; in the case of weight, for example, this might be an object of exactly 1 gram. In contrast to the case with an ordinal scale, the numerical measures assigned in this way convey something beyond position in an ordering; for example, an

[5] A strict weak order is a binary relation (A, R) that is:

(1) **Irreflexive**: for all $a \in A$, it is not the case that aRa

(2) **Asymmetric**: $a, b \in A$, if aRb then it is not the case that bRa

(3) **Transitive**: for all $a, b, c \in A$, aRb and $bRc \Rightarrow aRc$

(4) **Transitive w.r.t. incomparability**: for all $a, b, c \in A$, if a is incomparable with b and b is incomparable with c, then a is incomparable with c

[6] A strict linear order is a binary relation $A, \succ$ that is asymmetric, transitive and complete:

(1) **Complete**: for all $a \neq b \in A$, $a \succ b$ or $b \succ a$

object of weight 100 grams can meaningfully be said to be twice as heavy as one of weight 50 grams. The first of the two weighing procedures described in Section 1 is based on a scale of this nature.

To relate this discussion to the earlier definition of a scale in a degree-based semantic framework (12), we can now distinguish two separate types of scales underlying linguistic meaning:[7]

(14) **Ordinal Scale:**
$S = \langle D, \succ, DIM \rangle$, where

- $\succ$ is a strict linear order on D

(15) **Ratio Scale:**
$S = \langle D, \oplus, \succ, +, DIM \rangle$, where

- $\succ$ is a strict linear order on D
- $\mu_S(a \oplus b) = \mu_S(a) + \mu_S(b)$

A number of authors have demonstrated that concepts from measurement theory are relevant to natural language (Krifka 1989; Klein 1991; Nerbonne 1995; van Rooij 2011b, among others). Recently, Sassoon (2007, 2010) has argued that contrasts in the occurrence of measure phrases (e.g. *3 feet tall* vs. **50 degrees warm*) and modifiers such as *twice* (e.g. *twice as tall* vs. ?*twice as short*) can be related to the level of measurement assumed by the adjective: positive adjectives such as *tall* are associated with ratio scales, while those such as *short* and *warm* invoke only interval scales. Lassiter (2011) similarly shows that differences in level of measurement can account for differences in the behavior of epistemic versus deontic modals. In what follows I argue that these distinctions are relevant to quantification as well.

2.3 Back to quantification

With the preceding discussion in mind, let us return to the quantifiers *most* and *more than half*. It should now be apparent that there is in fact a crucial difference between the two logical forms in (11), namely that they place different requirements on the structure of the underlying scales that support their assessment. That for *most* in (11a) needs only an ordinal scale, in that it is based simply on a 'greater than' relation between measures. But that for *more than half* in (11b) requires a stronger scale that allows statements about ratios of measures, that is, a ratio scale.

On the surface, this may seem like a moot point, in that the typical dimensions involved in quantification can be measured at the ratio level. This is particularly the case for the dimension of number as applied to set sizes: measuring set size via counting is in Steven's framework an example of ratio-level measurement.[8]

[7] Here I leave aside interval scales, which are not relevant to the analysis of quantification, though they may play a role in other linguistic domains.

[8] Some authors (e.g. Roberts 1979) in fact characterize counting as involving a level of measurement higher than ratio, namely absolute level, in that unlike for example the measurement of weight, there is no choice of measurement unit. The point remains that set size can be assessed at least at the ratio level.

But a dimension where ratio level measurement is possible can also be measured at a less informative level. Take again the example of a race. Runners' finishing times can be measured in minutes and seconds (ratio level), allowing us to say things like *runner A took twice as long to finish as runner B*. But as noted above we can also simply associate each runner with his finish order. Relative to this less informative level of measurement, a statement like this no longer makes sense; we can only make statements such as *runner A finished after runner B*. The rock-weighing example discussed at the start of the paper similarly illustrates that weights of objects could likewise be measured at either the ratio or ordinal level. By the same token, the number of students in each of a set of university classes could be measured by counting (ratio level); but we could also simply rank the classes by number of students and represent this ordering by assigning them the numbers 1, 2, 3, . . . , or any other increasing sequence of numbers (ordinal):

(16) $\mu_{\#-ratio}(Class\ A) = 137$
$\mu_{\#-ratio}(Class\ B) = 74$
$\mu_{\#-ratio}(Class\ C) = 42$
$\mu_{\#-ratio}(Class\ D) = 11$

(17) $\mu_{\#-ordinal}(Class\ A) = 1$
$\mu_{\#-ordinal}(Class\ B) = 2$
$\mu_{\#-ordinal}(Class\ C) = 3$
$\mu_{\#-ordinal}(Class\ D) = 4$

The claim I would like to make is that there are cases where weaker than ratio level measurement is the only option, and it is in precisely these cases where quantification with *most* is possible, while quantification with *more than half* is not. The previously discussed pairs in (7)-(10) illustrate different sorts of situations where this is the case. Let us examine each of these individually.

Mass nouns are a good starting point, as the crucial point is particularly easy to appreciate. There are certain mass nouns, particularly abstract ones, that correspond to dimensions for which there is no available ratio level measurement scale. A case in point is sadness in (7).

(7) <u>Most/??more than half</u> of sadness diminishes over time.

There is no standardized unit for measuring amounts or occurrences of sadness, nor is it easy to imagine how such a measure could be developed (cf. Cresswell 1977 for discussion of a similar point relative to dimensions like beauty and strength). Recall that a fixed unit is one of the characteristics of a ratio scale. Yet we nonetheless seem able to rank manifestations of sadness relative to one another, as illustrated by the following example:

(18) Sometimes the aftermath of the holidays can bring <u>more sadness</u> than the holidays themselves. (http://www.mcginnis-chambers.com/content.php?sid=4143&ssid=9679)

That we can impose such orderings indicates that instances of sadness can be measured at the ordinal level, which is sufficient to support the logical form for *most*, but not that for *more than half*.

Some other corpus examples with *most* that make a similar point are in (19). How would we measure credit, interest or editorial opinion? Again there are no standard units. Contrast this with

the sort of mass nouns that do occur with *more than half*, as illustrated in (20). Quantities of oil can be measured in gallons or liters, money in dollars and territory in acres; all of these represent examples of ratio scales.

(19) a. Most of the credit goes to a key provision that allows private litigants to sue in the government's name and share in any recovery. (ABA Journal, October 2006)

b. Most of the interest in the space program today comes from an older generation that remembers the romantic times of a now older space era. (USA Today, 17/25/2011; quoting Richard Berendzen)

c. . . . most editorial opinion was against supporting the UNrrA leader. (Africa Today, 1st/2nd Quarters 1992)

(20) a. China imported more than half the oil it consumed last year . . . (Chicago Sun Times, 11/25/2011)

b. The stadium will cost $ 255 million, and more than half of the money will come from the current ownership group. (Atlanta Journal Constitution, 12/24/1995)

c. In contrast to Turkey's relative abundance more than half of Syrian territory gets less than 250 millimeters of rainfall per year. (Journal of International Affairs, Summer 1995)

A similar phenomenon is observed in the case of noun phrases that, while syntactically count, denote domains whose members are vague and cannot be individuated in a non-arbitrary way. Consider again *pastel hues* in (8).

(8) Most/??more than half of pastel hues have a calming effect.

What counts as a pastel hue? Where does one end and the next begin? Is lavender, for example, a single hue? Or do pale lavender and dark lavender each count as separate hues? Absent a fixed set of atoms, it is not possible to assign a precise cardinality to this set, or to any of its subsets. But it still may be possible to compare the numerosities of two subsets (e.g. those that do and do not have a calming effect), perhaps by generalizing over different possible individuations. This again could be represented on an ordinal scale, supporting the use of *most*.

The examples in (21) are further illustrations of the phenomenon. For example, what constitutes a single issue around accessibility in the workplace? How should the entire space of issues of this sort be chopped up into discrete entities? Similarly, what are the tasks whose difficulty is discussed in (21b). It is hard to see how an exhaustive list could be compiled in any non-arbitrary way. Observe that again typical instances involve abstract nouns such as *issues, tasks, things in life*. But we also see this effect with concrete noun phrases where atomicity fails to hold, as in (21d): a part of a part of the country is also a part of the country. In all of these cases, my intuition is that replacing *most* with *more than half* would result in infelicity, unless (and this is crucial) we are able to assume some pre-established list of issues, tasks, parts of the country, etc. over which to quantify.

(21) a. With the technology that is available today, most issues around accessibility in the workplace are solvable. (San Francisco Chronicle, 7/17/1999)

b. Successful people know that most tasks are much more difficult than they seem. (Town and Country, July 1998)

c. As with most things in life, quality costs money. (Accent on Living, Spring 1995)

d. In most parts of the country, a westerly wind predominates ... (Outdoor Life, September 1997)

A different sort of issue is presented by (9), where the quantifier combines with a bare plural noun phrase:

(9) Most/??more than half of teens want to fit in with their peers.

Teens – like other sorts of humans – are eminently countable sort of entities, but the plural noun *teens* here does not seem to refer to any particular set of teens. Rather, it is teens in general, or perhaps teens as a kind, whose behavior is characterized. It is in this context that replacing *most* with *more than half* results in infelicity. Another similar example is (22).

(22) Now most people don't know how a lock works, not even local farmers. (NPR Morning, 9/3/2001)

Contrast these examples with two corpus examples in which *more than half* occurs felicitously with the noun *teens*:

(23) a. The study found that more than half the teens surveyed admitted to dangerous driving habits. (Atlanta Journal Constitution, 5/13/2007)

b. More than half of teens surveyed said they are "not too careful or not at all careful" to protect their skin. (Todays Parent, July 2006)

The definite description *the teens surveyed* in (23a) denotes a particular set of teens, localized in space and time, namely those who took part in a certain study. A similar effect obtains in (23b), despite the absence of an overt definite form.

While the plural noun in (9) has an interpretation seemingly similar to that of bare plurals in generic sentences (e.g. *Teens (usually/mostly) want to fit in with their peers*), that in (23b) is closer to specific plural indefinites, whose interpretations have a referential flavor (Fodor and Sag 1982; Reinhart 1997). As an example, on the most salient reading of (24a), it is a specific set of three relatives whose collective deaths will make me rich. Other relevant parallels include the anomalous plurals that can take wide scope over operators, as in Carlson's (1977) (24b), which has a reading where *parts of this machine* scopes over negation, and Condoravdi's (1974) functional reading, as in (24c), where *students* picks out the totality of relevant students. In each of these cases, the plural noun phrase identifies a particular spatiotemporally bounded set of individuals.

(24) a. If three relatives of mine die, I'll inherit a million dollars.

b. John didn't see parts of that machine.

c. In 1985 there was a ghost haunting the campus. Students were aware of the danger.

The correct semantic treatment of examples such as these remains open to debate, and the terminology itself is not entirely standard. As such, I will in what follows refer to the interpretation of the plural *teens surveyed* in (23b) as the 'particular' interpretation, in contrast to the 'kind' reading of unmodified *teens* in (9). In Section 4 I will return to give a more formal characterization of the difference in the denotations of the two sorts of noun phrases, and the consequences of this for measurement. For the present, I will confine myself to stating the intuition that the extensions of kind noun phrases like *teens* in (9) are simply too underspecified to allow precise counting of members, which is necessary to satisfy the ratio level presupposition of *more than half*. That is, ratio level measurement such as precise counting is possible only in the case of definite noun phrases or indefinites with a particular interpretation, which introduce into the logical form a particular set or entity.

A final factor influencing level of measurement involves not the structure of the domain of quantification but rather the knowledge state of the speaker. In some situations, the speaker might know it to be the case that one set or entity is larger on the relevant dimension than another, without knowing the exact measure of either one. To some extent this is relevant to all of the cases discussed above; for example, how likely is it that the speaker knows the exact number of teens who want to fit in with their peers? But a particularly clear example is provided by (10).

(10) She noted lights on in <u>most/more than half</u> of the houses she passed . . .

Here, the individual in question may have come to know that there were more houses with their lights on than those with their lights off, without ever counting the number of each set. For example, she might have noted that for every house passed with the lights off, there were several with the lights on (this is essentially Hackl's vote counting procedure). Abusing the terminology slightly, we might call this 'ordinal level knowledge', i.e. knowledge as to position on an ordinal level scale. This is sufficient to support the use of *most* in an example such as (10); but use of the *more than half* version requires a higher level of knowledge, specifically measure relative to a ratio-level scale, such as would be obtained by actually counting.

The general theme running through the discussion of all of these examples is that a relation of 'greater than' between two sets or entities can be semantically represented, without it being the case that the precise cardinality or measure of either is represented. In the context of degree semantics this is a rather unusual idea, but there is a very close parallel from the field of cognitive psychology, specifically the branch concerned with number cognition. In fact, findings from that perspective suggest that such representations are the default option. I turn to this now.

2.4 Tolerant comparison and semi-ordered scales

Underlying the above-described procedure for the derivation of ordinal scales is the assumption that the ordering relation between individuals is fully transitive. But there is ample evidence that real life comparison does not work this way. Specifically, while the 'greater than' relation may be transitive, 'not greater than' often is not.

Research in psychophysics has shown that across a wide range of perceptual properties, humans' ability to differentiate two stimuli – for example, to determine which of two weights is

heavier – is subject to a threshold of discriminability known as the 'just noticeable difference' (jnd), which can typically be expressed as a ratio of the objective measures of the stimuli being compared (Stevens 1975). Two stimuli whose objective measures fall within the relevant threshold are not reliably distinguishable. Luce (1956) points out a similar pattern in preference between options; for example, one might prefer a cup of coffee with one spoonful of sugar to one with four spoonfuls of sugar, but certainly have no preference between a cup with one spoonful plus n grains of sugar and one with one spoonful plus $n + 1$ grains. In both cases the process of comparison exhibits what might be termed tolerance, in that it is insensitive to small differences between entities. It is easy to see that a consequence of this is the intransitivity of perceived sameness or indifference.

A particularly well-studied instance of tolerant comparison involves the perception of the numerosity of sets of objects. A large body of research supports the existence of two separate cognitive systems for the representation and processing of quantity (see especially Dehaene 1997; Gallistel and Gelman 2000; Feigenson et al. 2004). The first and most familiar, which could be termed the 'precise number system', allows the representation of exact number, and is involved in calculations such as addition, subtraction, multiplication and division using symbolic notation. When you solve a mathematical problem such as $327 \times 809 =?$, for example, you are using this precise number system. The representation and processing of exact number is closely linked to linguistic ability; these capabilities are present primarily in verbal humans, and may be lost in patients with aphasia.

But in addition to this capacity to represent precise quantity, humans also possess a second system which has been termed the 'approximate number system' (ANS), which allows us to represent and reason with quantities approximately. This system is essentially analog in nature, with (approximate) quantities thought to be encoded as patterns of activation on the equivalent of a 'mental number line'. These representations can serve as the basis for comparison of quantities, and for simple approximate arithmetic such as addition and subtraction. In the preceding section I proposed that a 'greater than' relation between two entities could be represented semantically without the precise measure of either being represented. Work on the ANS shows that this has a cognitive counterpart, in that the numerosities of two sets can be represented and compared mentally without their precise number being calculated or stored.

The hallmark of the ANS is its ratio dependence. When two quantities (say, two arrays of dots) are compared in a task where time constraints preclude counting, subjects' response times decrease and their accuracy improves as the numerical distance between the two quantities increases (distance effect). Similarly, for a given numerical distance between two quantities, response times decrease and accuracy improves as the magnitude of the numbers themselves decreases (size effect). Both of these effects are accounted for by the Weber-Fechner law, which states that the differentiability of two stimuli is a function of the ratio of their measures. Simply stated, two pairs of values with comparable ratios (e.g. 120 vs. 60 and 12 vs. 6) are equally differentiable, and two values whose ratio is sufficiently close to one are not differentiable via the ANS, or perhaps more accurately, only differentiable in a noisy and stochastic way. For adults, consistently reliable differentiation typically requires a ratio of 8:7 or greater between set sizes. The ratio dependence of the ANS has been captured via psychological models in which the representations it generates are

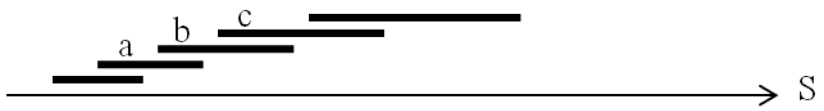
modeled by Gaussian curves with either linearly increasing center values and linearly increasing standard deviations, or logarithmically increasing center values and constant standard deviations (Feigenson et al. 2004). In either case, the differentiability of two values is a function of the degree of overlap of their curves.

The ANS is developmentally and evolutionarily more basic than the ability to represent precise numerosity. It is operational not just in adults who possess a system of number words, but also preverbal infants, who have been shown to exhibit approximate numerical abilities (Xu and Spelke 2000). Likewise, members of societies without complex number systems have been found to perform successfully on tasks involving approximate quantity comparison and arithmetic, but not those involving precise operations (Pica et al. 2004). The approximate system may be preserved in aphasics who have lost the ability to perform exact mathematical operations (Dehaene and Cohen 1991). Non-human animals, too, show similar approximate numerical abilities (Dehaene et al. 1998). Furthermore, in verbal/numerate adults, the role of the approximate number system is not superseded by the possession of learned, precise numerical abilities. Even on tasks involving quantities presented symbolically, such as judging which of two values represented in Arabic numerals is larger, size and distance effects are still observed, indicating that even in this case the ANS is engaged (Buckey and Gilman 1974).

To return to the semantics of gradation and quantity, scales have been argued to arise out of our ability to make comparisons (see e.g. Bierwisch 1989). But the above-described threshold-sensitive patterns of differentiability and preference are not faithfully represented by a scale in which the ordering on degrees is complete, i.e. in which for any two distinct degrees a and b, either $a \succ b$ or $b \succ a$. Yet while completeness follows from the assumption that scales are derived from strict weak orders on individuals, and is taken for granted in most current degree-based work in semantics, it is not inherent to the definition of a scale in (12). Cresswell (1977), in what is often considered to be the foundational work in degree semantics, leaves open the possibility that scales may be based on a weaker ordering relation, and it is this idea that I pursue here.

There are various possibilities for how a tolerant, ratio-dependent comparison operation can be represented in a degree-based semantic framework.[9] One fairly straightforward option, based loosely on the above-described psychological models of the approximate number system, is to formalize degrees not as points but rather intervals whose length is a function of their magnitude, with the 'greater than' relation obtaining between degrees (i.e. intervals) if and only if one exceeds the other without overlap. This is represented visually below, where for the three degrees a, b, c, $a \sim b$ and $\sim c$ but $c \succ a$.

(25)



A more formal definition is given in (26):

[9] In a related approach, van Rooij (2011a) and Cobreros et al. (2012) develop a logical system based on the notion of tolerance which they apply to the analysis of vague predicates such as *tall*; see also Burnett (2012) for a further extension. These works embody similar insights to those discussed here, but as they are couched in a very different framework I do not attempt to discuss them in detail.

(26) **Semi-ordered scale:**
$S = \langle D, \succ, DIM \rangle$, where

- every $d \in D$ is an interval of the form $d' \pm n \bullet d'$ for some precise measure d' and some constant n
- for any $d_1, d_2 \in D$, $d_1 \succ d_2$ iff d_1 as a whole exceeds d_2 as a whole

Crucially, though, the above characterization of degrees as intervals around a precise point is intended as a descriptive statement rather than an account of their derivation. A scale with the structure in (26) is not derived from one based on precise measurement, but rather represents comparison that proceeds in the absence of precise measurement.

Intuitively, the scale structure represented in (25) and (26) captures the tolerant nature of many real life sorts of comparisons, including those made via the ANS. Formally, the relation $\succ$ meets the criteria of a semi-order, a type of ordering introduced by Luce (1956) to account for indifference in preference (per above).[10]

The reason that a scale with this structure is relevant to the present topic is that it can shed light on the interpretation of *most*, a connection already made by Pietroski et al. (2009). Recall the observation that *most* is typically judged most felicitous for proportions considerably greater than 50%. This can be accounted for as interpretation of the logical form in (11a) relative to a scale with the structure in (26). If the cardinality of $A \cap B$ exceeds that of $A - B$ by some magnitude-dependent threshold value, which is what is captured by the formulation in (26), then it is also necessarily the case that the proportion of As that are B exceeds 50% by some corresponding threshold. For example, if (11a) is interpreted relative to a semi-ordered scale in which the parameter *n* is set at 0.1, then *most* will obtain for percentages of 55% and higher.

Pietroski et al. (2009) provide evidence that the ANS plays a role in the online processing of sentences with *most*. Through a series of experiments, they demonstrate that verification of a sentence of the form *Most of the dots are yellow* exhibits size and distance effects that can be characterized by the Weber-Fechner law. The connection is made tighter by Halberda et al. (2008), who show that among young children, understanding of *most* is independent of knowledge of large numbers. Specifically, children who do not yet know the meaning of number words larger than *two* may still be able to evaluate a sentence such as *Most of the crayons are yellow*, but only if the ratio between the numbers of yellow and non-yellow crayons is sufficiently large, suggesting that the children are drawing on their approximate numerical capabilities.

My claims above amount to taking this a step further. Not only is the ANS involved in the online processing of *most* sentences, but something related finds its way into the semantics of

[10] Per (Luce 1956), a structure $\langle A, \succ \rangle$ is a **semi-order** iff $\forall a, b, c, d \in A$:

(1) $\neg(a \succ a)$

(2) $((a \succ b) \wedge (c \succ d)) \rightarrow ((a \succ d) \vee (c \succ b))$

(3) $((a \succ b) \wedge (b \succ c)) \rightarrow ((a \succ d) \vee (d \succ c))$

Note however that for the present purposes it is not sufficient to define a semi-ordered scale in terms of a semi-order on degrees, as such a structure does not necessarily capture the ratio-dependent property of tolerant comparison; hence the more complicated formulation in (26). See van Rooij (2011a) for further linguistic applications of semi-orders.

most, in the form of a measurement scale that embodies the ratio-dependent property of tolerant comparison.

Furthermore, the observed tendency for *most* to be restricted to cases where there is a significant difference between set sizes suggests that interpretation relative to a semi-ordered scale is not only a possibility, but actually the default option. This is less far fetched that it might at first seem. Recall the above-cited evidence that approximate comparison of numerosities is more basic than precise comparison, and our approximate numerical abilities are activated by default even in tasks that involve precise representation. A semi-ordered scale thus models our most primitive representation of quantity. The strong tendency for the use of *most* to be restricted to situations where truth obtains relative to a semi-ordered scale might thus be aligned to what Horn (1984) terms R-based implicature, where a more general predicate is pragmatically restricted or narrowed to stereotypical instances. Such implicatures derive from Horn's R Principle: "say no more than you must". Examples of R-based implicatures discussed by Horn include the strengthening of ability modals (such that *John was able to solve the problem* R-implicates that he in fact solved it), the restriction of lexical causatives such as *kill* to cases of direct causation, and in the lexical domain, the inference from *John had a drink* that the beverage in question was alcoholic. When it comes to comparison of quantities, the psychological primacy of approximate comparison makes interpretation relative to a semi-ordered scale structure a very plausible candidate for the stereotypical instance.

A case can furthermore be made that a scale of this structure does not support the logical form for *more than half*. Such a scale is weaker than ratio level, having in particular no standard unit. More basically, it does not give content to the value 2, which encodes a precise measurement. As such, the formula in (11b) is undefined relative to such a scale. Interestingly, this claim too has a cognitive counterpart: psychological research has shown that representations generated by the ANS can support approximate addition and subtraction, but – crucially – not multiplication or division of values.

There have been previous proposals to account for the tendency to interpret *most* as 'significantly more than half', either by building this meaning into the semantics itself (Peterson 1979; Westerstahl 1985) or by invoking pragmatic principles (Horn 2005). To my knowledge, the present analysis is the only one to relate this aspect of *most*'s behavior to other seemingly unrelated patterns in its distribution and interpretation, all of which can be traced back to a common underlying factor, the structure of the measurement scales that give it content.

2.5 Summary and predictions

The central idea developed in this section is that the puzzle posed by *most* and *more than half* can be explained in terms of scale structure. Their logical forms are superficially equivalent, hence the initial intuition that their meanings are essentially identical. But they place different requirements on the structure of the scales that underlie their interpretation, a difference that has a range of consequences. The lexical entry for *more than half* in (11b) assumes a ratio scale, in that it expresses the ratio of two measures; that is, *more than half* presupposes measurement at the ratio level. There is nothing terribly surprising about this, in that counting, as well as typical measures of mass, volume, etc., represent classic examples of ratio level measurement. What is important is

that in the case of *most* in (11a), a ratio scale is not required. Rather, a simple ordering of entities will suffice, one that can be represented via an ordinal scale. That is, the logical form of *most* need only assume an underlying qualitative ordering, without building in notions of distances or ratios between scale points. And even a weaker scale structure, one in which the degrees are only semi-ordered relative to one another, is also sufficient.

This proposal has been seen to yield insight into a range of examples discussed up to this point. But if this is the right characterization of the facts, we expect differences in the distribution and interpretation of *most* and *more than half* to be detectable more broadly. In particular, we predict the following:

1. The distribution of *most* will be broader than that of *more than half*, in that the latter but not the former will be restricted to contexts where ratio level measurement is possible. Specifically, *more than half* will occur exclusively with noun phrases that ...
   - ...are definite in form, or if indefinite have a 'particular' rather than 'kind' interpretation;
   - ...have extensions whose structures allow ratio-level measurement. For count nouns, this reduces to having a non-arbitrary set of atoms; for mass nouns, the corresponding dimension will have a standard unit of measure.
2. *More than half*, whose interpretation is based on counting or other ratio-level measurement, will tend to be used by speakers/writers who have such measurement data available to them. This will not be observed to the same extent with *most*.
3. As a consequence of the preference towards interpretation relative to a semi-ordered scale, *most* will tend to be used to convey proportions significantly greater than 50%, and correspondingly will be used for higher proportions than *more than half*, for which interpretation relative to such a scale is not a possibility.

In the next section, a corpus study is presented in which these predictions are tested.

3 Corpus analysis

3.1 Data source and sampling

Data for the analysis were drawn from the Corpus of Contemporary American English (COCA) (Davies 2008-), a 450+ million word corpus of American English, including approximately 20 million words per year for the years 1990-2012. For each year included, the corpus is evenly divided between five genres: spoken language, fiction, popular magazines, newspapers, and academic texts. Searches can be carried out by word, phrase, lemma and part of speech, the latter made possible via automated tagging according to the UCREL CLAWS7 tagset. For each entry, a context consisting of the 10-15 word preceding and following the searched term is available for downloading; an expanded context is available online.

In total, there are 502,255 occurrences of *most* and 5,730 occurrences of *more than half* in COCA. However, not all of these involve the quantificational use that represents the focus of this paper. A significant proportion of *most* tokens involve its use in adjectival and adverbial superlatives (27a). Also represented are the so-called relative superlative *the most* (27b) (which could be paraphrased as 'the largest number of'), as well as fixed expressions such as *at most* (27c) and *for the most part* (27d), and a usage with a meaning of 'almost' (27e). Similarly, in the case of *more than half* we find in addition to the quantificational usage its use as an adverbial modifier (28a) and in measure expressions (28b).

(27) a. Gill's compilation of apprenticeship records is one of the best and most accessible sources for research into eighteenth-century Virginia craftsmen. (Magazine Antiques, May 1998)

b. The squad with the most points at the end of the week wins a trophy (Denver Post, 9/23/2001)

c. Common black paint absorbs, at most, 90 percent of the light that hits it. (Popular Mechanics, February 2012)

d. For the most part, it was folks from the other side of the tracks who were hardest hit. (NPR Weekly, 3/7/1998)

e. Most anything that gets broken can be fixed, and insurance covers 99.9 percent (Motor Boating, October 2009)

(28) a. Net private capital inflows to developing countries plummeted by more than half in 1996-98. (Foreign Affairs, September/October 2001)

b. That particular model is about fifteen centimeters long and a bit more than half a centimeter thick. (Analog Science Fiction & Fact, October 2007)

To address this, samples of the quantificational use of each expression were extracted for further analysis as follows. For *more than half*, COCA's sampling feature was used to extract 200 random tokens from each of the periods 1990-1994, 1995-1999, 2000-2004 and 2005-2009, and 100 tokens from the (shorter) period 2010-2012, resulting in a raw sample of 900 tokens. This raw sample was manually cleaned by removing non-quantificational tokens, resulting in a final sample of 684 tokens. For *most*, the search was first restricted to tokens tagged as determiners (CLAWS7 tag DAT) or as ambiguous between a determiner and another part of speech, and to exclude tokens preceded by *the* (to rule out the relative superlative use); this yielded 235,035 occurrences in total. From here, a raw sample of 900 tokens was extracted as described above, and then further manually cleaned, yielding a final sample of 762 tokens. Note that this procedure for *most* leaves open the possibility that some quantificational tokens were missed because they were incorrectly tagged with a tag other than DAT. To assess the magnitude of this issue, a sample of tokens receiving other tags was extracted and manually searched for instances of the quantificational use. Based on this analysis, an estimate was derived that fewer than 1% of quantificational tokens were falsely excluded via the extraction procedure described above. This was judged to be acceptable.

In the following, a set of analyses of these data are presented, which test the predictions outlined in the preceding section.

3.2 Distribution of *most* and *more than half*

The first of the predictions discussed above was that the distribution of *most* will be broader than that of *more than half*. By virtue of its logical form, we predict *more than half* to be restricted to contexts where ratio level measurement is possible. By contrast, *most*, whose logical form places less stringent requirements on the structure of the underlying scale, is expected to surface in a broader range of contexts. This requirement was broken down into two component predictions.

Definite/particular reference. The first requirement for the necessary level of measurement is that the noun phrase that the quantifier composes with denote a particular group or entity, bounded in space and time, rather than members of the kind in general. We saw in the preceding section that this requirement is met by definite noun phrases[11], as well as bare plurals with a 'particular' rather than 'kind' reading. As such, we predict that *more than half* – but not *most* – will be restricted to occurring with such noun phrases.

To assess this, each token in the two corpus samples was first coded for the structure of the noun phrase occurring with the quantifier. The results of this analysis are shown in Table 1.[12] Examples of each structure are given in (29) and (30).

(29) a. Most experts believe CFC-12 should be available for the next few years. . . . (Consumer Research, July 1997) **bare-plural**

b. An unproblematic local-nonlocal dichotomy is at variance with most current scholarship on globalization . . . (Geographical Review, July 1996) **bare-mass**

c. Most of the gun crimes in this country are caused by unregistered or stolen guns . . . (CNN Dr. Drew, 3/15/2012) **definite**

d. I moved into the neighborhood and drank at the local ginmills, most of them long gone now. (A Long Line for Dead Men, Lawrence Block. New York: Morrow, 1994) **pronoun**

e. When the Weigands ran their early findings by fellow archaeologists for confirmation, most agreed that they were onto something big. (Archeology, Nov/Dec 2006) **null**

(30) a. More than half of Missouri homeowners who live in the New Madrid area have earthquake insurance. (Archeology, January/February 2006) **bare-plural**

b. In 1996, for the first time in history, more than half of U.S. transportation oil came from overseas . . . (Environment, September 2000) **bare-mass**

c. More than half of the doctoral degrees in engineering awarded by American universities each year go to foreigners. (Associated Press, 1/6/2007) **definite**

[11] Here and in what follows, I use 'noun phrase' to refer to the entirety of the nominal expression following *most/more than half*, without making any commitment to its particular syntactic status (e.g. NP vs. DP).

[12] This analysis collapses together examples with and without the partitive *of* e.g. *more than half the students* vs. *more than half of the students*. The distribution of *of* is somewhat idiosyncratic, and differs between *most* and *more than half* (e.g. *most Americans* but *more than half of Americans*), which tends to obscure the main conclusions from the comparison. As will be discussed further below, I follow Matthewson (2001) in assuming that *of* here lacks semantic content.

Table 1: Noun phrase structure

	Most		More than half	
	#	%	#	%
(of) bare plural/mass noun	421	55	63	8
(of) definite description	218	29	442	58
(of) '*all* + noun phrase'	0	0	41	5
(of) pronoun	74	10	60	8
null	49	6	69	9
other	0	0	9	1
Total	762	100	684	100

d. In states such as Massachusetts, more than half of all drivers are in the assigned-risk pool. (Consumers Research Magazine, November 1991) ***all*+ noun phrase**

e. Today his company has 155 members, more than half of whom have been in Denver for at least 10 years. (Denver Post, 9/28/2003) **pronoun**

f. The study questioned pharmacists-in-charge at the state's 1,590 drugstores, and more than half answered. (Associated Press, 2/15/2000) **null**

As seen here, the greatest proportion (58%) of *more than half* tokens are followed by a definite description. An additional 8% of tokens are accounted for by pronouns, which have a definite interpretation, and 9% involve null-headed noun phrases, which also can be interpreted definitely (for example, the null nominal in (30f) corresponds to 'the phrmacists-in-charge at the state's 1,590 drugstores'). Only 8% of cases involve a bare plural or mass noun. By contrast, *most* is followed by a bare plural or mass noun in over half (55%) of cases, while only 29% of tokens involve a definite description. A chi-square test shows the difference in distribution between the two quantifiers to be significant (χ^2 = 392.6, df=5, p<0.0001). Thus as predicted, *more than half* skews towards occurring with definite noun phrases in a way that *most* does not.

An interesting sort of example in which *more than half* occurs involves noun phrases containing *all* followed by a bare plural or definite description, as in (30d). According to my intuitions, the corresponding examples without *all* remain grammatical, though less natural, and there is no obvious change in meaning. I return to this in the next section, where I discuss implications for the formal semantics of the relevant examples.

We are left to consider those cases where the quantifier occurs with a bare plural or mass expression, focusing on the question of whether this noun phrase can be interpreted as referring to a particular well-defined set or entity, rather than members of the kind in general. The most reliable indicator of this in corpus data is the presence of modification, as it is known that modification, and in particular post-nominal modification, facilitates such an interpretation (e.g. Fodor and Sag 1982; Matthewson 2001). Compare for example the previously discussed *teens* versus *teens surveyed*; the unmodified plural noun seems to range over teens in general, while the modified expression picks out a particular well-delineated set of teens. A similar contrast is seen between *experts* in (29a) and *Missouri homeowners who live in the New Madrid area* in (30a); only the latter can be

interpreted as denoting a well-defined set of individuals, while the former leaves unspecified which particular experts are being considered. Thus as a further step in the analysis, all tokens featuring the quantifier followed by a bare mass or plural noun phrase (421 for *most*, 63 for *more than half*) were coded for the presence of modification, and particularly post-nominal modification. Here again a difference between quantifiers was identified: 81% of *more than half* tokens but only 41% of *most* tokens were modified in some way (χ^2 = 30.4, df=1, p<0.0001). Furthermore, post-nominal modification was present in 29% of *more than half* examples compared to 14% for *most* (χ^2 = 9.2, df=1, p<0.01). Note that the tendency for *more than half* to occur with modified noun phrases is also observed in the case of noun phrases of the form '*all* + bare plural/mass' (25/37 tokens modified - 68%; 13/37 tokens with post-nominal modification - 35%).

In summary, the corpus data as a whole supports the claim that *more than half* is restricted to occurring with definite noun phrases, or with bare mass and plural terms that can be interpreted as referring to a particular group or entity. This restriction is not observed for *most*, which demonstrates a broader distribution.

Countability and measurability. Having a well-delineated extension is itself not sufficient for measurement at the level required to support the interpretation of *more than half*; in addition, the elements of that extension must lend themselves to the appropriate sort of measurement.

In the case of count nouns, what is required is a domain set whose members can, at least in principle, be individuated (separated from one another in a non-arbitrary way) and enumerated (put on an exhaustive list and counted). Earlier we saw several examples of plural noun phrases where these conditions were not fulfilled, e.g. *pastel hues*, *tasks* and *issues*. The intuition is that in all of these cases, quantification with *more than half* would be infelicitous. While it is difficult to operationally define 'individuation and enumeration', observe that most of the problematic cases involved abstract nouns. This leads to the overall prediction that *more than half* will occur less commonly than *most* with abstract nouns.

The corpus samples selected for analysis contained 362 tokens of *more than half* with a plural noun phrase, and 517 such tokens for *most*.[13] These were broken down according to the denotation of the noun phrase; results are shown in Table 2. As seen here, the prediction is to some extent confirmed. Both quantifiers skew towards quantifying over sets of humans and other concrete objects, but the skew is somewhat greater in the case of *more than half*. Conversely, while both quantifiers occur with nouns denoting more readily countable sorts of abstract entities such as organizations and institutions (e.g. countries, companies, teams) and events (e.g. deaths, visits, elections), *most* occurs at a higher level than *more than half* with other sorts of abstract nouns. Again the difference in distribution is significant (χ^2 = 19.4, df=5, p<0.01). The tendency for *more than half* to skew towards nouns with concrete and particularly human denotations is reinforced by looking at its occurrence with collective nouns such as *population* and *staff*, which are singular in form but denote sets of individuals. *More than half* occurs 40 times with such nouns, with 38 of these denoting groups of people; by contrast, *most* occurs only 7 times with a collective noun, with only 1 of these denoting a group of people.

[13]This corresponds to the structures *(of) bare*, *(of) definite description*, *(of) all + noun phrase* and *other* in Table 1, and excludes tokens involving pronouns or null-headed noun phrases, even if these had a count plural as antecedent.

Table 2: Noun phrase denotation - count nouns

	Most		More than half	
	#	%	#	%
Concrete:				
Humans	241	47	182	50
Other concrete nouns	67	13	61	17
Abstract:				
Organizations/institutions	49	9	32	9
Events	18	3	25	7
Kinds	15	3	3	1
Other abstract nouns	127	25	59	16
Total	517	100	362	100

A more direct though also somewhat subjective way to test the predicted difference in the sort of plural nouns the two quantifiers compose with is to determine for each given plural noun phrase whether its extension has well-defined atoms. An atom is an element of the extension of a noun phrase which has no proper subparts which are also in its extension. *Teens*, for example, has atoms that correspond to elements in the extension of the singular noun *teen*: if *x* is a teen, then no proper part of *x* is also a teen. Most count plurals behave in this way, but there are exceptions. For example, *parts of the country* – a plural noun phrase that occurs in the corpus sample with *most* – is not atomic in this sense: as noted earlier, a proper subpart of a part of the country is also a part of the country. Since precise counting is not possible in the absence of a non-arbitrary set of atoms, we expect such 'non-atomic' count plurals to occur with *most* but not *more than half.*

To assess this quantitatively, each plural noun phrase occurring with *most* and *more than half* was coded for (non-)atomicity. To avoid subjectivity as much as possible, the identity of the quantifier was masked, and coding was first carried out by a student assistant who was naive to the purpose of the analysis, with results then reviewed by the researcher. The test used for (non-)atomicity was felicity in the frame 'A part of an NP is an NP'.

In analyzing the results, we first excluded those tokens (9 for *most*, 24 for *more than half*) in which the noun phrase was syntactically plural, but quantification involved a dimension other than number. Examples include *more than half of Medicaid costs* (dimension is expense, measured in dollars) and *most of your vitamin A needs* (weight, measured e.g. in milligrams); such examples are not relevant to the question of atomicity. Of the remaining 338 *more than half* tokens, 5 (1.5%) included a noun phrase coded as non-atomic. By comparison, 28 out of 508 *most* tokens (5.5%) were coded as non-atomic. All noun phrases classified as non-atomic are listed in the Appendix. As seen here, these include examples such as those previously discussed, as well as 'subkind' readings of ordinary count nouns (e.g. *most vegetables* with the reading of 'most kinds of vegetables'); these are also non-atomic, in that there is no *a priori* established level of the taxonomy at which subkinds should be counted. The difference between quantifiers on this measure is significant (χ^2 = 8.8, df=1, $p<0.01$); thus while atomicity is, not surprisingly, the rule for both quantifiers, *most* is more likely than *more than half* to occur with non-atomic plural nouns.

Let us turn now to mass nouns. Here the dimension involved in quantification is not number or counting, but rather something else. We predict that *more than half* will occur exclusively with nouns corresponding to dimensions that can be measured at the ratio level. As discussed above, ratio-level measurement tends to go hand-in-hand with the existence of a standard unit of measure. We have already seen several *most* examples, e.g. *sadness* and *opinion*, where this requirement is not met. Conversely, the felicitous examples of mass nouns with *more than half* have featured dimensions that do have such measures, such as oil, quantities of which can be measured in gallons or liters.

To investigate whether this pattern holds more generally, all instances of mass nouns in the corpus samples (51 for *most*, 51 for *more than half*) were coded for the existence of a numerical measurement system based on a standard unit of measure. To avoid subjectivity, the identity of the quantifier was again masked, and coding was carried out independently by the researcher and a student assistant, and results were compared. Agreement between coders was 86%. Only those tokens which were coded by both coders as having no standard unit of measure were counted as such for the purposes of analysis.[14] In total, 24 of the mass nouns occuring with *most* satisfied the criterion of having a unit of measure (45%). By contrast, fully 50 (98%) of the *more than half* tokens did, the only exception being *legal education* (seen in example (31b) below). The full list of noun phrases coded as lacking standard measures is given in the appendix. The difference between quantifiers is significant (χ^2 = 33.3, df=1, $p<0.0001$).

Thus as predicted, the corpus analysis supports that *more than half* but not *most* is restricted to quantifying over domains that are countable, or for which there is a standard unit of measure.

The overall prediction discussed in this subsection was that the distribution of *most* would be broader than *more than half*. We have seen here that this prediction is supported by natural language data. *Most* occurs frequently with bare mass/plural nouns as well as definite noun phrases, and with expressions with a kind as well as particular interpretation, and furthermore is not limited to noun phrases whose denotations are countable or measurable at the ratio level. The same is not the case for *more than half*. These findings are consistent with the measurement theoretic analysis developed here.

3.3 Availability of supporting data

Above, it was suggested that the knowledge state of the speaker may play a role in determining which quantifier may be felicitously used. In that the semantics of *more than half* presupposes counting or other sort of ratio level measurement, we predict that it will most commonly be used by speakers/writers who are in the state of 'ratio level' knowledge, i.e. who have supporting ratio level measurement data available to them. Such a tendency is not predicted in the case of *most*, or at least not to the same extent, as its assertion can be based on weaker 'ordinal level' knowledge.

[14]The less than perfect agreement between coders can be attributed to the fact that it is not always clear whether or not a dimension has a standard unit. For example, there is no completely standard unit for measuring quantities of scholarship (a noun occurring with *most*), but we could imagine creating one, perhaps based on number of publications and their length; such a system may in fact exist somewhere. The process of counting only those cases where both coders agreed on the lack of a unit was intended to address this.

Table 3: Distribution by corpus genre

	Corpus	Most		More than half	
	%	#	%	#	%
Spoken	21%	124	16%	85	12%
Fiction	19%	75	10%	26	4%
Magazine	21%	246	32%	202	30%
Newspaper	20%	139	18%	219	32%
Academic	20%	178	23%	152	22%
Total	100%	762	100%	684	100%

These predictions cannot be tested directly via a corpus-based methodology, since for any given corpus token of an expression, it cannot be definitively determined what sort of evidence the speaker/writer had on which to base his/her assertion. However, some more indirect evidence can be brought to bear on the question.

Occurrence by corpus genre. At the coarsest level, the above prediction leads us to expect that the occurrence of *more than half* will skew to more information-oriented genres such as newspaper and magazine articles and academic papers, where numerical data is typically reported, and away from less informative genres such as fiction, and that this will be observed to a greater extent than for *most*. This was assessed by breaking down the samples of *most* and *more than half* according to the five genres represented in COCA. Results of this analysis are presented in Table 3. As seen here, the prediction is confirmed. *More than half* is overrepresented in the COCA genres newspaper (32% of tokens in the sample), magazine (30%) and to a lesser extent academic (22%), and is in particular underrepresented in fiction (4%). Recall that the full corpus is split equally between these five genres, so that this pattern in fact represents a skew in the usage of *more than half*, and not in the source material itself. *Most* is also underrepresented in fiction (10% of sample tokens) but overall shows a less extreme skew than does *more than half*. A chi-squared test shows the difference in distribution between the two quantifiers to be significant (χ^2 = 51.76, df=4, $p < 0.0001$).

Numerical data and sources. A more direct way to approach this question is to examine individual tokens of *most* and *more than half* for evidence of the availability of supporting data sourced via counting or other quantitative measurement. The examples below point to what might be signs of this. While the sentences in (31) could be taken to report generalizations or the authors' impressions, those in (32) mention specific data sources as well as numerical values.

(31) a. The pattern of life in the country and <u>most</u> suburbs usually involves long hours in the automobile each week . . . (E: The Environmental Magazine, 18(5), September/October 2007)

b. <u>More than half</u> of legal education is vocabulary training that actually gets in the way of effective communication. (ABA Journal, October 1995)

(32) a. Most of the 62 countries omitted from the UN list of 175 are small, underdeveloped countries . . . (CATO Journal, Fall 1998)

b. U.S. News's analysis of the 1992 clinic data reveals that more than half the 5,400 IVF babies came as part of a multiple birth. (U.S. News & World Report, Dec 5 1994)

c. Longo (Longo &; Sherman, 2007) reported that more than half of the 47 students studied (53%) experienced horizontal violence during their educational experience. (Creative Nursing, 17, 2011)

To assess quantitatively whether *most* and *more than half* differ in this respect, each token in the two analysis samples was first coded for the presence of a numeral or number word the immediately preceding or following context (i.e. in the context available for download from COCA). To avoid the need to make potentially arbitrary decisions as to which numerical expressions were relevant, all numerals/number words were counted with the exception of those in names, and those serving as chapter or line numbers. In total, 364 of the 684 *more than half* tokens (53%) co-occured with a numeral/number word, compared to 209 of the 762 *most* tokens (27%); this difference is statistically significant (χ^2 = 100.2, df=1, p<0.0001). Secondly, each token was coded for the presence in the immediate context of a word indicating a data source; the words searched were *analysis, data, poll, report, respondent, statistic, study, survey,* and their inflected forms. While the overall occurrence of these words is low in both samples, a difference between quantifiers is again found in the predicted directon (*most*: 40/762 - 5%; *more than half*: 80/684 - 11%; χ^2 = 19.7, df=1, p $<$ 0.0001). Both these tests support the conclusion that *more than half* – to a greater extent than *most* – is used when supported by numerical data from a particular source. This is further and more direct evidence for a difference in the knowledge state underlying the use of the two quantifiers, one that derives from the distinction in the required scale structure imposed by their logical forms.

3.4 Proportional ranges of *most* and *more than half*

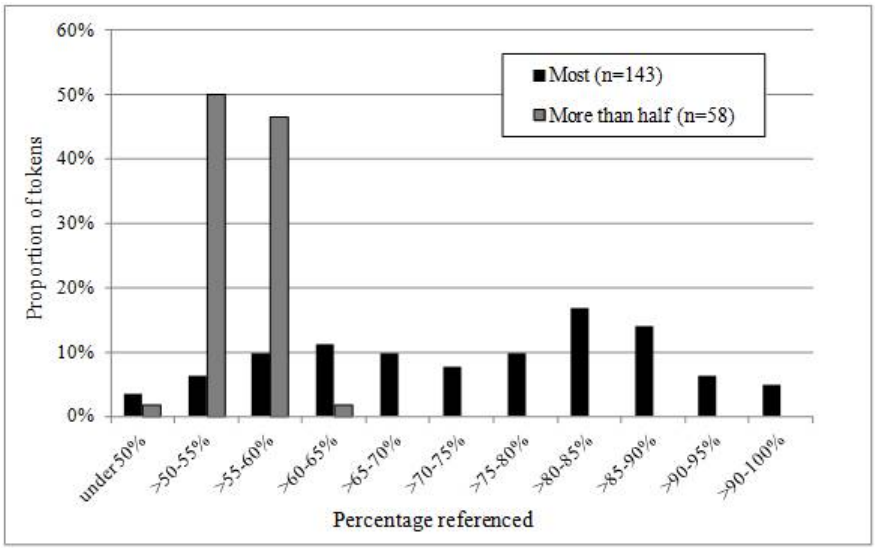
The final prediction was that *most* will tend to be used for higher proportions than *more than half*, and in particular will be used infrequently with proportions very close to 50%, a consequence of a default preference for its interpretation relative to a semi-ordered scale.

On the surface, corpus data seems ill-suited to testing this prediction. In the majority of corpus examples involving the use of a quantifier, it is not possible to determine what the actual proportion is. However, there is one particular type of example, found quite commonly in the reporting of survey data, where a quantifier is used in conjunction with an exact percent. Some examples of this were seen above; some further ones are the following:

(33) a. The poll also found that most Saudis, 85 percent, are very interested in seeing political reform. (CNN LiveSat, 6/19/2004)

b. Most (73%) of the visits were in the outpatient care setting . . . (American Journal of Public Health, 2012)

Figure 1: Percentages associated with *most* and *more than half*



c. And while more than half of us grill year-round (57 percent), summertime is overwhelmingly charcoal time. (Denver Post, 5/24/2000)

d. More than half of all students (54.1%) mentioned confusion about giftedness at least once. (Roeper Review, September 1992)

This usage gives us precisely the sort of data we need to investigate the proportions which are described by the two quantifiers in question. As the two samples used for the previously discussed analyses did not yield sufficient examples of this sort for quantitative analysis, searches were conducted on the entire COCA corpus for occurrences of each quantifier followed within five words by the word 'percent'. This does not pick up examples such as (33b,d) featuring a percent % sign rather than the word spelled out. This is unavoidable, as the COCA search interface does not allow searching by punctuation symbols; but there is no reason to think that the overall pattern would have been different had these sorts of examples been included. After cleaning of irrelevant hits (i.e. other uses of 'percent'), this yielded samples of 143 tokens for *most*, and 58 for *more than half*. These were classified according to the value of the numerical percentage.

Figure 1 shows the results of this analysis. As seen here, there is a dramatic difference in the range of proportions for which the two quantifiers are used. While *most* occurs for proportions from just over 50% to nearly 100%, the usage of *more than half* is restricted almost entirely to values less than or equal to 60%.[15]

On the one hand, these results confirm the stated prediction. On average *most* is used for higher proportions than *more than half*, and in particular only a small fraction of *most* tokens occur with proportions very close to 50% (just 6% involved a value $\leq$ 55%). This is consistent with the

[15]Five tokens of *most* and one of *more than half* occurred with a percent value below 50%. In the case of *most*, these appear to represent the relative superlative usage, which is more commonly spelled out as *the most*. The *more than half* example appears to describe more than half of a previously mentioned percentage, though this is not clear from the limited context available via COCA.

reported intuition that *most* is infelicitous for such values, and with the claim that it is by default interpreted relative to a scale with a tolerant or semi-ordered structure.

On the other hand, the most dramatic difference between the two quantifiers is one not predicted by the previously discussed aspects of scale structure, namely the almost complete absence of *more than half* for proportions greater than 60%.

I propose that this pattern derives from competition between *more than half* and other parallel expressions of proportion, an analysis that follows Gricean lines (Grice 1975; Horn 2005). If we are in a situation that supports an utterance of *more than half* – namely one in which the relevant sets or entities are measurable at the ratio level – then there are a range of alternative expressions that, depending on the percentage to be conveyed, could also have been used. Examples of such alternatives include *a quarter*, *two thirds*, *8-in-10*, and so forth. By standard (neo-)Gricean reasoning, the use of *more than half* will be pragmatically restricted to cases where no stronger alternative is available. In particular, since *more than two thirds* and *more than three quarters* represent equally lexicalized but stronger alternatives to *more than half*, the felicitous use of *more than half* will be limited to cases where the proportion in question is below this, that is, to the range from half to roughly two thirds. Conversely, the hearer can infer from the speaker's choice of *more than half* that no stronger alternative is available, and thus that the percentage falls within this range. The conclusion, then, is that the interpretation of *more than half* receives an upper bound via scalar implicature.[16]

An alternative possible explanation is that the interpretation of *more than half* is pragmatically upper bounded via competition not with numerical expressions such as *two thirds*, but rather with *most*. What argues against this is that a similar pattern is observed for other expressions of proportion beyond *more than half*. For example, all of the examples below seem to imply that the actual proportion is close to the value referenced in the quantifier. To the extent that corpus data can be used to investigate this, a similar conclusion is supported; for example, all of the percent values occurring with *less than half* in COCA fall in the range between 39% and 49%. In these cases however there is no simple quantifier comparable to *most* that the numerical expression could compete with; but the pattern can be explained as competition between the expression used and another numerical expression (e.g. *fewer than half* competes with *fewer than one third*, *more than a quarter* with *more than a third*, and so forth).

(34) a. More than a quarter of us grill year round.
b. More than two thirds of us grill year round.
c. Fewer than half of us grill year round.

Yet another possibility is suggested by a reviewer, namely that it is the mention of a particular threshold (half) that causes *more than half* to be restricted to proportions close to that value. A similar idea is put forth by Horn (2005). This is a valid point, but I think it is not so different from the proposal I have outlined above. *More than half* is largely restricted to values close to half

[16]It has been claimed that scalar implicatures do not arise with modified numerical expressions. However, Cummins et al. (2012) show that once scale granularity is taken into consideration, such implicatures are in fact observed for expressions such as *more than 100*.

because for values further away from this threshold, there are quite simply better ways to describe them. The same could be said about other proportional comparatives such as those in (34).

In contrast to *more than half*, *most* is not restricted to values close to 50%, but rather is used even for proportions approaching 'all'. A potential reason is that it does not compete with expressions such as *more than two thirds*, and as such upper bounded implicatures relative to these expressions are not generated. As support, in contexts in which *more than half* is infelicitous, so too are other numerical expressions of proportion (e.g. *??More than two thirds of teens want to fit in with their peers*). These expressions, like *more than half*, presuppose ratio level measurement, and are thus plausibly not alternatives to *most*. Instead, the possible competitors for *most* seem to be other quantifiers that express relationships between sets, and that are commonly organized on a Horn scale (Horn 1989, 2005), notably *all*.

3.5 Summary

In total, the corpus analysis supports the scale-based proposal developed in Section 2. Starting from the perspective of a degree-based semantic framework, and enriching this with insights from measurement theory and from the psychology of number and measure, we are able to develop a general account of both the similarities and differences between *most* and *more than half*.

In the following section I investigate in greater depth the interaction between noun phrase semantics and measurement, and along the way contrast the present account to an alternative, the account of Szabolcsi (1997).

4 Noun phrase denotations and measurement

In Section 2, I discussed some distributional differences in the sorts of noun phrases that *most* and *more than half* occur with. In this section I will seek to make this more formal. I focus here in particular on the count noun case, though at the end I will offer a few observations regarding mass nouns. In developing the account, I draw on proposals by Carlson (1977); Link (1983); Chierchia (1998a,b); Matthewson (2001) and others which, while not entirely standard, or standardly combined in the manner I propose, provide a useful framework for expressing the core insights.

Let us start by taking the domain of individuals to have the structure of a complete join semilattice, composed of atomic individuals as well as plural individuals formed as their individual sums (Link 1983; Chierchia 1998a; Landman 2004). If *a* and *b* are atoms, *ab* is their sum, to be interpreted as '*a* and *b* taken together'. Elements of the lattice are ordered via the 'part of' relation $\sqsubseteq$; for example, $a \sqsubseteq abc$ and $ab \sqsubseteq abc$. Singular count nouns have atomic individuals in their extensions; plural nouns have extensions based on closure under sum formation of the extension of the corresponding singular . As an (overly) simple example, if *dog* in a given world denotes the set $\{a, b, c, d\}$, *dogs* denotes $\{a, b, c, d, ab, ac, ad, bc, bd, cd, abc, abd, acd, bcd, abcd\}$.[17].

The next step is somewhat more controversial. Following Matthewson (2001), I propose that quantifiers take as their first argument an individual rather than a set. That is, the semantic type of

[17]There is some debate as whether atomic individuals are included in the denotation of plural nouns; see Chierchia (1998a); Krifka (2004) for discussion. This is not crucial for the present purposes

expressions such as *most* and *more than half* is not $\langle\langle et\rangle, \langle\langle et\rangle, t\rangle\rangle$, as is typically assumed, but rather $\langle e, \langle\langle et\rangle, t\rangle\rangle$. Matthewson's motivation for this proposal is the desire to develop a unified cross-linguistic approach to quantification by demonstrating that an analysis developed for the Salish language St'át'imcets (which more transparently requires it) can also be extended to English. She shows, however, that this analysis also offers a better explanation of some aspects of the English data than does the traditional view. Inherent in this analysis is the conclusion that partitive *of* is semantically vacuous.

Consider now the following examples, which on this account represent three options for how the noun phrase can provide an argument of type *e* to saturate the first argument of the quantifier:

(35) a. Most/more than half (of) <u>the teens surveyed</u> admitted to driving dangerously.
b. Most/more than half (of) <u>teens surveyed</u> admitted to driving dangerously.
c. Most/??more than half (of) <u>teens</u> want to fit in with their peers.

In (35a,b), the noun phrase *(the) teens surveyed* denotes an ordinary plural individual, the maximal plurality of teens surveyed. Exactly how this plural individual is arrived at in semantic composition depends on one's preferred view of definiteness and specificity. A standard approach to the definite description in (35a) is to take the definite article to denote a maximality operator, as in (36a) (Chierchia 1998a, a.o.). In the case of the plural *teens surveyed* in (36b), which has what I earlier termed a particular interpretation, we might follow Reinhart (1997) and others in taking the derivation to involve a contextually determined choice function applied to the extension of the noun phrase (36b).

(36) a. $[\![the\ teens\ surveyed_e]\!] = \iota\lambda x.teens(x) \wedge surveyed(x)$,
where $\iota P = max(P)$ if it exists; otherwise undefined
b. $[\![teens\ surveyed_e]\!] = f(\lambda x.teens(x) \wedge surveyed(x))$

What is required for a bare plural to have the reading derived via (36b) is that its linguistic content coupled with the context of use be sufficient to uniquely determine the function *f*. The previously mentioned tendency for this reading to be most available in the presence of modification can be attributed to this requirement. Recall also from the preceding section that *more than half* sometimes occurs with a noun phrase beginning with *all*, with little apparent difference in meaning relative to the version without *all* (e.g. *more than half of (all) teens surveyed*). A number of authors (Partee 1995; Lasersohn 1999; Brisson 2003) propose that *all* is not in fact a quantifier, but instead simply adds a meaning of exhaustiveness to the expressions it combines with. As noted for example by Lasersohn, *the teens surveyed* might allow exceptions, but *all the teens surveyed* does not. On this use *all* composes with entity-denoting expressions, which itself supports the above claim that noun phrases like *teens surveyed* in (35b) denote individuals. I thus propose that the function of *all* in the *more than half* cases is to enforce a maximal or exceptionless reading of the following referential expression, and in doing so to remove any possible ambiguity as to which plurality is intended.

In (35c), by contrast, the noun phrase *teens* does not denote an ordinary plural individual. Rather, Matthewson proposes that such noun phrases occurring with *most* denote kinds, just as do bare plurals in generic sentences such as *Teens want to fit in with their peers*. As evidence for this,

she notes that the distribution of unmodified plural noun phrases with *most* parallels that of bare plurals on their generic reading. For example, in episodic contexts where a bare plural would have an existential rather than generic reading (e.g. *I talked to linguists*), *most* is awkward (?*I talked to most linguists*).[18]

To compose via intersection with a sentential predicate, the individual-denoting expressions in (35) that serve as the first argument for the quantifier must shift to the type of sets. Here, an important difference arises between the two sorts of individuals discussed here, which has bearing on the possibilities for measurement. The ordinary plural individuals involved in (35a,b) stand in one-to-one correspondence with sets of individuals, namely the set of entities that are in the 'part of' relation to the plurality. In this case, the previously introduced logical forms for *most* and *more than half* can easily be restated in the following form:

(37) a. $[\![\text{most}]\!](x_e)(X_{\langle et\rangle})$ = 1 iff $\mu_S(A \cap X) \succ \mu_S(A - X)$
b. $[\![\text{more than half}]\!](x_e)(X_{\langle et\rangle})$ = 1 iff $\mu_S(A \cap X) \succ \mu_S(A)/2$
where $A = \{y : y \sqsubseteq x\}$

So long as the sets derived in this way have well-defined atoms, μ_S can simply reduce to counting of their atomic members. In the case of *more than half*, this is the only option. Put differently, for *more than half*, μ_S must 'look at' the individual atoms of the plurality, as this is required for precise counting. This fails only in the case where the atoms of the plurality can not be individuated in a non-arbitrary way. For *most*, there are other and perhaps even simpler possibilities. For example, the maximum members of each of the two subsets $(A \cap X)$ and $(A - X)$ might simply be compared in their entireties to determine which is greater in numerosity, without going down to the level of counting atoms. To borrow from the example discussed at the start of the paper, this could be likened to placing the two maximum pluralities on the two pans of a metaphorical balance scale, and determining whether the 'greater than' relation obtains between their sizes, without actually determining the precise cardinality of either. From a cognitive perspective, it could related to an analog judgment of set sizes made via the Approximate Number System. Either way, the results of such a holistic comparison could be recorded on an ordinal scale or, if the comparison procedure is characterized by tolerance, one with a semi-ordered structure.

The kind individuals in examples like (35c) are different in an important way: they do not stand in any such simple relationship to sets of ordinary individuals. Kinds are typically conceptualized as atomic individuals, or as functions from worlds to pluralities (Carlson 1977; Chierchia 1998b). In neither of these guises does a kind have subparts of the sort that characterize the plural individuals in (35a,b).

The relationship of kinds to the ordinary individuals that instantiate them is the topic of some discussion in the literature on generics. It is well known that statements about kinds, such as *Teens want to fit in with their peers*, need not apply to all members of the kind, but rather allow exceptions. However, formally characterizing what exceptions are allowed is challenging. Even when an overt quantifier is present (*every/most/many teen(s)*), the intention is typically not to quantify over all members of the kind; rather, the domain of quantification is restricted. Abstracting away from the

[18] I differ from Matthewson in finding some such examples at least marginally acceptable with sufficient contextual support. These cases would on my account be analyzed as in (36b).

complexities of the topic, the shift from kind to set might be captured as follows, where C is a covert variable that encodes a domain restriction (von Fintel 1994).

(38) $teens_{kind} \rightarrow \lambda x.teens(x) \wedge x \in C$

There is thus a level of underspecification in the kind-denotation case that is not present in the plural individual case. The latter sort of individuals identify well-defined sets whose members can be counted. Kind individuals by contrast correspond to sets that are fuzzy or vague in their delineation, a consequence of the presence of the variable C. We have already noted the possibility of vagueness in what constitutes an atom; here the vagueness is not in the identity of the atoms themselves, but rather in which ones are included in the set. I propose that this again rules out the possibility of assigning a precise cardinality sets of this sort or their subsets; that is, μ_S cannot be interpreted as an operation of exact counting. But again, a weaker level of measurement may be possible. In particular, the underspecification here may not stand in the way of determining a 'greater than' relation between set sizes. In other words, ordinal level measurement may still be possible. The contrast between *more than half* and *most* follows.

The above discussion has focused on count plurals, but similar considerations apply also in the mass domain. Here, measurement does not proceed by counting atoms; a measure function μ_S applied to a set operates by mapping the maximum member of that set to the scale S. But just as in the count plural case, there are different choices for μ_S, and in particular different possible options for the structure of S. Here, what determines what function can apply to a given set – that is, what determines possible level of measurement – is how well that maximum element is delineated, and whether the dimension in question can be associated with a standard unit, and thus measured at the ratio level.

Having discussed one way of giving a formal compositional analysis to *most* and *more than half* with various classes of noun phrases, it is an appropriate point to contrast the present analysis to another formal account of these expressions, that of Szabolcsi (1997).

Szabolcsi's analysis is based on a close connection between syntactic structure and semantic interpretation. Different quantificational determiner phrases (DPs) occur in distinct syntactic positions (either overtly or at logical form), and this corresponds to a representational or procedural difference in their interpretation. DPs based on *most* – like others that occur in the HRefP position high in the syntactic tree – contribute a plural individual to the interpretation of the sentence, corresponding to the elements of some minimal witness set of the quantifier; this individual serves as the logical subject of predication.[19] By contrast, DPs based on *more than half* – like others in the lower PredOp position – perform a counting operation on the property denoted by the remainder of the sentence, similar to the procedure more typically assumed for generalized quantifiers. Differences of this sort are argued to account for a range of divergences between individual quantifiers, relating among others to scope-taking behavior, anaphora, and availability of collective vs. distributive interpretations. While she does not provide a full analysis, Szabolcsi also proposes that this difference underlies the contrasts in (5) and (6) (e.g. *The professors met *most/more than half of the boys each*), in that *more than half* patterns in this respect with other 'counters' such as cardinal numerals, which are also analyzed as occurring in PredOp.

[19] A witness set of a generalized quantifier is an element of that quantifier which is also a subset of the extension of the noun phrase it is based on.

Let us try to relate this to the present account. First of all, the analysis I have given to *most* could, if desired, be restated in terms of plural individual discourse referents, as follows:

(39) *Most A*: *x*,
where *x* is a plural individual corresponding to some $X \subseteq A$ such that
$\mu_S(X) > \mu_S(A - X)$

Thus the semantics I have proposed is not inherently incompatible with Szabolcsi's representational/procedural framework.

But crucially, positing a distinction between individual-introducing and generalized quantifier-like interpretations is not in itself sufficient to explain the differences in distribution and interpretation that have been the focus of the present paper. Suppose we had two quantifiers Q_1 and Q_2 that were identical in their meaning except that the first worked by contributing a witness set to the interpretation of the sentence while the second operated by counting set members. It is not immediately obvious how this alone would result in differences in the sorts of noun phrase denotations they composed with, the range of proportions they were felicitously used for, or the knowledge state required for their assertion.

On the other hand, the present account, inspired by principles of measurement theory, offers the potential to explain why *most* and *more than half* pattern the way they do in Szabolcsi's system. Szabolcsi uses divergences between the two (or, more accurately, between *most* and *more than 50%*) as evidence that the difference between the two modes of operation she proposes is not purely denotational in nature: two denotationally equivalent quantifiers do not necessarily behave identically from a procedural perspective. She does not however attempt to explain why things are the way they are, as opposed, say, to reversed, i.e. with *more than half* as individual-introducing and *most* as a counter. The present account give a clue to why this is. At the core, the difference I propose between *most* and *more than half* relates to the nature of the mathematical formulas by which their meanings are encoded. That for *most* is based entirely on a 'greater than' relation. That for *more than half* involves, in addition to this, an operation of division by two. We have seen that this corresponds to a difference in the structure of the scale that serves as the range of the measure function; *more than half* forces interpretation relative to a ratio scale, while *most* does not. It is now less puzzling why *more than half* patterns with counters: its denotation is necessarily based on exact counting (or, as we must always add, some other sort of ratio level measurment). *Most*'s denotation, by contrast, does not assume any sort of precise counting or measurement beyond a basic and potentially tolerant operation of comparing magnitudes; it is likewise not surprising that it therefore patterns with other quantificational expressions whose denotations are not based on counting.

I do not attempt to address the question of whether a representational/procedural framework of the sort Szabolcsi proposes is necessary, as this is motivated primarily by a different set of facts than those considered here, relating in particular to scope-taking behavior. I also will not try to account for what it is about 'counters' like *more than half* that is responsible for the contrasts exemplified in (5) and (6). One might hypothesize that the aspect of *more than half*'s semantics that forces it to 'look at' individual atoms of a plurality might play a role here as well; but pursuing this intuition in depth would require a lengthy excursus into the semantics of distributivity, and of existential constructions. I hope however that this discussion has served to demonstrate that

even if one chooses to adopt a procedural framework, the core insight embodied in the present analysis still has explanatory value. Looking for a denotational explanation for what Szabolcsi terms differences in mode of operation is not a hopeless matter after all.

5 Conclusions and connections

There is no doubt that speakers sometimes talk about degrees, and correspondingly no doubt that degrees must somehow be semantically represented. Most current semantic theories that deal with measure, gradability and comparison in some way assume degrees and scales as part of the ontology. It is furthermore well recognized that scales vary in their structures, and that this has linguistic effects. In particular, Rotstein and Winter (2004) and Kennedy and McNally (2005) demonstrate that the distinction between scales that do and do not have endpoints has consequences for the interpretation of gradable adjectives in their positive forms, as well as for the distribution of degree modifiers. But beyond this, there is little consensus as to the structure of scales, and in particular no established view on the full range of parameters on which scales can vary in their structures.

The present work points to another dimension along which scales vary, one that corresponds to distinctions that are well known from measurement theory, and to findings from the psychology of number and perception. Measurement can be carried out by various procedures, and these differences can be represented via scales that differ in the level and nature of the information they encode. Perhaps surprisingly, this is true even of what is arguably the most basic sort of measurement, namely assessing the numerosity of sets of entities. Scales are not simply linearly ordered sets of points. Some have more structure than this, namely a standard unit of measure. Others have less structure than this, in that the degrees that constitute them are not totally ordered with respect to one another. The comparison of *most* and *more than half* shows that this aspect of scale structure – which might be called 'ordering strength' – also has linguistic consequences. *More than half* (and by extension other numerical expressions of proportion) requires a ratio level scale, and is thus restricted to use in cases where entities can be mapped to a scale of this sort. *Most*, by contrast, can be interpreted with respect to a simple qualitative ordering of entities, one which would be reflected by an ordinal or semi-ordered scale. As demonstrated by the corpus analyses reported here, this distinction results in quite dramatic differences in the distribution of the two quantifiers, as well as in the interpretations they may receive.

If the structures of scales – even those tracking the same dimension – can vary in this way, we should expect to see this reflected in other domains of natural language as well. While it is beyond the scope of the present paper to consider this issue more broadly, there is evidence that this is in fact the case. It was noted above that authors such as Sassoon (2010) have argued that the ratio/interval distinction is relevant to areas such as the semantics of gradable adjectives. That between totally ordered and semi-ordered scales also may play a role elsewhere. There is another well-known contrast that parallels that between *most* and *more than half*, and an approach very similar to that developed here has been applied towards analyzing it. The topic has to do with the difference in interpretation between explicit comparatives such as (40a) and implicit comparatives such as (40b), which was first pointed out by Kennedy (2007):

(40) a. Fred is taller than Barney.
b. Fred is tall compared to Barney.

While (40a) can be felicitously used to describe a situation where Fred's height is just slightly greater than Barney's (say, 1 cm), the felicitous use of (40b) requires there to be a significant difference in the two individuals' heights. This corresponds closely to the distinction between *more than half* and *most*, the latter requiring for felicity a 'significant' difference in set sizes. In an approach that closely parallels the present one, Fults (2011) proposes that the interpretation of explicit comparatives involves an 'ordinary' linearly ordered scale, while that of implicit comparatives is based an 'analog magnitude scale' parallel in structure to the output of the approximate number system. van Rooij (2011a) develops a similar insight in a non-degree-based framework, holding that the difference between (40a) and (40b) corresponds to that between strict weak orders and semi-orders; again the similarity to the present analysis is clear.

That factors relating to the type of ordering relation invoked have explanatory value in this domain as well suggests the broader relevance of the measurement theoretic account developed here.

Appendix: Nouns occurring with *most* and *more than half*

A. Plural noun phrases with non-atomic extensions

Most: areas outside Kabul, belongings, bones and rubbish, chemical reactions, conversations, crops [subkind] (2x), fossils, garden pests [subkind], hair types, low-fat ice creams [subkind], major habitat types, mollusks [subkind], nascent social movements, orchids [subkind], overexposed areas in an image, parts of the country, parts of the economy, parts of India's rural countryside, pastel hues, periods, places in the world, possessions, radio waves, species and gear selectivities, things in life, urban areas throughout the country, vegetables [subkind]

More than half: hunting injuries, kinds of molecules, neighborhoods surveyed, processed foods on the European market [subkind], tumors

B. Mass noun phrases without standardized measurement units

Most: action, American architecture, business (2x), care, children's art media, commuting, credit (2x), current scholarship on globalization, dinner-party conversation, editorial opinion, enjoyment, evidence, hardware, interest, privitization, reading, real shopping, research on low vision, sadness, scholarship, sculpting of the planetary nebula, training, ultimate damage, work (2x)

More than half: legal education

References

Ariel, Mira. 2004. *Most*. *Language* 80:658–706.

Ariel, Mira. 2005. A 'just that' lexical meaning for *most*. In *Where semantics meets pragmatics*, ed. Klaus von Heusinger and Ken Turner, 49–91. Amsterdam: Elsevier.

Bale, Alan Clinton. 2008. A universal scale of comparison. *Linguistics and Philosophy* 31:1–55.

Bartsch, Renate, and Theo Vennemann. 1973. *Semantic structures: A study in the relation between syntax and semantics*. Frankfurt: Athaenum Verlag.

Benthem, Johan F.A.K. van. 1986. *Essays in logical semantics*. Dordrecht: Reidel.

Bierwisch, Manfred. 1989. The semantics of gradation. In *Dimensional adjectives*, ed. Manfred Bierwisch and Ewald Lang, 71–261. Berlin: Springer Verlag.

Bresnan, Joan. 1973. Syntax of the comparative clause construction in English. *Linguistic Inquiry* 4:275–343.

Brisson, Christine. 2003. Plurals, 'all', and the nonuniformity of collective predication. *Linguistic Inquiry* 26:129–184.

Buckey, P.B., and C. B. Gilman. 1974. Comparison of digits and dot patterns. *Journal of Experimental Psychology* 103:1131–1136.

Burnett, Heather. 2012. The grammar of tolerance: On vagueness, context-sensitivity and the origin of scale structure. Doctoral Dissertation, University of California Los Angeles.

Carlson, Gregory. 1977. Reference to kinds in English. Ph.D. dissertation, University of Massachusetts.

Chierchia, Gennaro. 1998a. Plurality of mass nouns and the notion of 'semantic parameter'. In *Events and grammar*, ed. Susan Rothstein, 53–103. Dordrecht: Kluwer.

Chierchia, Gennaro. 1998b. Reference to kinds across languages. *Natural Language Semantics* 6:339–405.

Chierchia, Gennaro, and Sally McConnell-Ginet. 2000. *Meaning and grammar*. Cambridge, MA: MIT Press, 2nd edition.

Cobreros, Pablo, Paul Égré, David Ripley, and Robert van Rooij. 2012. Tolerant, classical, strict. *Journal of Philosophical Logic* 41:347–385.

Condoravdi, Cleo. 1994. Descriptions in context. Ph.D dissertation, Yale University.

Cresswell, Max J. 1977. The semantics of degree. In *Montague grammar*, ed. Barbara Partee, 261–292. New York: Academic Press.

Cummins, Chris, Uli Sauerland, and Stephanie Solt. 2012. Granularity and scalar implicature in numerical expressions. *Linguistics and Philosophy* 35:135–169.

Davies, Mark. 2008-. The Corpus of Contemporary American English (COCA): 450 million words, 1990-present. Available online at http://corpus.byu.edu/coca/.

Dehaene, Stanislas. 1997. *The number sense: How the mind creates mathematics*. Oxford: Oxford University Press.

Dehaene, Stanislas, and Laurent Cohen. 1991. Two mental calculation systems: a case study of severe acalculia with preserved approximation. *Neuropsychologia* 11:1045–1074.

Dehaene, Stanislas, Ghislaine Dehaene-Lambertz, and Laurent Cohen. 1998. Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences* 21:355–361.

Feigenson, Lisa, Stanislas Dehaene, and Elizabeth Spelke. 2004. Core systems of number. *Trends in Cognitive Science* 8:307–314.

von Fintel, Kai. 1994. Restrictions on quantifier domains. Ph.D. dissertation, University of Massachusetts, Amherst.

Fodor, Janet Dean, and Ivan A. Sag. 1982. Referential and quantificational indefinites. *Linguistics and Philosophy* 5:355–398.

Fults, Scott. 2011. Vagueness and scales. In *Vagueness and language use*, ed. Paul Égré and Nathan Klinedinst, 25–50. New York: Palgrave Macmillan.

Gallistel, C.R., and Rochel Gelman. 2000. Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Science* 4:59–65.

Grice, H. Paul. 1975. Logic and conversation. In *Syntax and semantics 3: Speech acts*, ed. Peter Cole and Jerry L. Morgan, 41–58. New York: Academic Press.

Hackl, Martin. 2000. Comparative quantifiers. Ph.D. dissertation, Massachusetts Institute of Technology.

Hackl, Martin. 2009. On the grammar and processing of proportional quantifiers: *Most* versus *more than half*. *Natural Language Semantics* 17:63–98.

Halberda, Justin, Len Taing, and Jeffrey Lidz. 2008. The development of "most" comprehension and its potential dependence on counting-ability in preschoolers. *Language Learning and Development* 4:99–121.

Heim, Irene. 2000. Degree operators and scope. In *Proceedings of the 9th Semantics and Linguistic Theory Conference*, ed. Brendan Jackson and Tanya Matthews. CLC Publications.

Higginbotham, James. 1995. Mass and count quantifiers. In *Quantification in natural languages*, ed. Emmon Bach, Eloise Jelinek, Angelika Kratzer, and Barbara H. Partee, 383–419. New York: Springer.

Horn, Laurence R. 1984. Toward a new taxonomy for pragmatic inference: Q-based and R-based implicature. In *Meaning, form and use in context: Linguistic applications*, ed. Deborah Shiffrin, 11–89. Washington, D.C.: Georgetown University Press.

Horn, Laurence R. 1989. *A natural history of negation*. Chicago: University of Chicago Press.

Horn, Laurence R. 2005. The border wars: A neo-Gricean perspective. In *Where semantics meets pragmatics*, ed. Klaus von Heusinger and Ken Turner. Amsterdam: Elsevier.

Huddleston, Rodney, and Geoffrey K. Pullum. 2002. *The Cambridge grammar of the English Language*. Cambridge: Cambridge University Press.

Keenan, Edward L., and Johnathan Stavi. 1986. A semantic characterization of natural language determiners. *Linguistics and Philosophy* 9:253–326.

Kennedy, Christopher. 1997. Projecting the adjective: The syntax and semantics of gradability and comparison. Ph.D. dissertation, University of California at Santa Cruz.

Kennedy, Christopher. 2007. Vagueness and grammar: The semantics of relative and absolute gradable predicates. *Linguistics and Philosophy* 30:1–45.

Kennedy, Christopher, and Louise McNally. 2005. Scale structure and the semantic typology of gradable predicates. *Language* 81:1–37.

Klein, Ewan. 1991. Comparatives. In *Semantics: An international handbook of contemporary research*, ed. Arnim von Stechow and Dieter Wunderlich, 673–691. Berlin: de Gruyter.

Kranz, David H, R. Duncan Luce, Patrick Suppes, and Amos Tversky. 1971. *Additive and polynomial representations*, volume I of *Foundations of Measurement*. New York: Academic Press.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In *Semantics and contextual expression*, ed. Renate Bartsch, Johan van Benthem, and Peter von Emde Boas, 75–115. Dordrecht: Foris.

Krifka, Manfred. 2004. Bare NPs: Kind referring, indefinites, both, or neither. *Empirical Issues in Syntax and Semantics* 5:111–132.

Landman, Fred. 2004. *Indefinites and the type of sets*. London: Blackwell.

Lasersohn, Peter. 1999. Pragmatic halos. *Language* 75:522–551.

Lassiter, Dan. 2011. Measurement and modality: the scalar basis of modal semantics. Ph.D. dissertation, New York University.

Link, Godehard. 1983. The logical analysis of plurals and mass terms. In *Meaning, use, and interpretation of language*, ed. Rainer Buerle, Christoph Schwarze, and Arnim von Stechow, 302–323. Berlin: Mouton de Gruyter.

Luce, R. Duncan. 1956. Semiorders and a theory of utility discrimination. *Econometrica* 24:178–191.

Matthewson, LIsa. 2001. Quantification and the nature of cross-linguistic variation. *Natural Language Semantics* 9:145–189.

Nerbonne, John. 1995. Nominal comparatives and generalized quantifiers. *Journal of Logic, Language and Information* 4:273–300.

Partee, Barbara H. 1995. *Quantification in natural languages*, chapter Quantificational structures and compositionality, 541–602. Kluwer.

Peterson, Philip L. 1979. On the logic of "few", "many", and "most". *Notre Dame Journal of Formal Logic* 20:155–179.

Pica, Pierre, Cathy Lerner, Veronique Izard, and Stanislas Dehaene. 2004. Exact and approximate arithmetic in an Amazonian indigene group. *Cognition* 306:499–503.

Pietroski, Paul, Jeffrey Lidz, Tim Hunter, and Justin Halberda. 2009. The meaning of 'most': semantics, numerosity and psychology. *Mind and Language* 24:554–585.

Reinhart, Tanya. 1997. Quantifier scope: How labor is divided between QR and choice functions. *Linguistics and Philosophy* 20:335–397.

Roberts, Fred S. 1979. *Measurement theory with applications to decisionmaking, utility, and the social sciences*. Cambridge: Cambridge University Press.

van Rooij, Robert. 2011a. Implicit versus explicit comparatives. In *Vagueness and language use*, ed. Paul Égré and Nathan Klinedinst, 51–72. New York: Palgrave Macmillan.

van Rooij, Robert. 2011b. Measurement, and interadjective comparisons. *Journal of Semantics* 28:335–358.

Rotstein, Carmen, and Yoad Winter. 2004. Total adjectives vs. partial adjectives: Scale structure and higher order modifiers. *Natural Language Semantics* 12:259–288.

Sassoon, Galit. 2007. Vagueness, gradability and typicality, a comprehensive semantic analysis. Ph.D. dissertation, Tel Aviv University.

Sassoon, Galit. 2010. Measurement theory in linguistics. *Synthese* 174:151–180.

Stevens, Stanley Smith. 1957. On the psychophysical law. *Psychological Review* 65:153–181.

Stevens, Stanley Smith. 1975. *Psychophysics: Introduction to its perceptual, neural and social prospects*. New York: Wiley.

Sutton, Melody. 1993. Binomial *Each*. Master's thesis, UCLA.

Szabolcsi, Anna. 1997. Strategies for scope taking. In *Ways of scope taking*, ed. Anna Szabolcsi. Dordrecht: Kluwer.

Westerstahl, Dag. 1985. Logical constants in quantifier languages. *Linguistics and Philosophy* 8:387–413.

Xu, Fei, and Elizabeth S. Spelke. 2000. Large number discrimination in 6-month-old infants. *Cognition* 74:b1–b11.