Q-adjectives and the semantics of quantity

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Abstract

The adjectives of quantity (Q-adjectives) many, few, much and little stand out from other quantity expressions on account of their syntactic flexibility, occurring in positions that could be called quantificational (many students attended), predicative (John’s friends were many), attributive (the many students), differential (much more than a liter) and adverbial (slept too much). This broad distribution poses a challenge for the two leading theories of this class, which treat them as either quantifying determiners or predicates over individuals. This paper develops an analysis of Q-adjectives as gradable predicates of sets of degrees or (equivalently) gradable quantifiers over degrees. It is shown that this proposal allows a unified analysis of these items across the positions in which they occur, while also overcoming several issues facing competing accounts, among others the divergences between Q-adjectives and ‘ordinary’ adjectives, the operator-like behavior of few and little, and the use of much as a dummy element. Overall the findings point to the central role of degrees in the semantics of quantity.

1 Introduction

1.1 The puzzle of Q-adjectives

What semantic means does English (or any language) have for expressing quantity? This paper approaches this question from the perspective of the words many, few, much and little – a class that I will call ‘adjectives of quantity’, or Q-adjectives for short. The goal of the paper is to provide a unified semantic analysis of this class, and in doing so to shed some light on the semantics of quantity more generally.

Q-adjectives pose some interesting questions relating to their vagueness and context sensitivity, or even ambiguity. But arguably the distinguishing feature of the class – and the one most in need of explanation – is their broad syntactic distribution, which sets them apart from other expressions of quantity.

To start with many and few, they occur in positions that could be called quantificational (1a), predicative (1b), attributive (1c) and differential (1d):
Many/few students attended the lecture.  
John’s friends are many/few.  
The many/few students who attended enjoyed the lecture.  
Many more/few more/many fewer than 100 students attended the lecture.

Much and little similarly occur in quantificational (2a) attributive (2c) and differential (2d) positions, though not predicatively (2b), at least not in constructions parallel to (1b).

(2) a. Much/little water is left in the bucket.   
b. *The water in the bucket was much/little.  
c. The little/*much water in the bucket  
d. Much/little more than a liter of water is left in the bucket.

Much/little also have a differential use in adjectival comparatives (3a) and excessives (3b), and occur as adverbial modifiers of verbs (4a,b) and certain adjectives (4c):

(3) a. Fred isn’t very tall – but he’s much taller than his father /  
in fact, he’s little taller than his father.  

(4) a. I much prefer wine to beer.  
b. I slept little.  
c. much loved/little known; much/little alike/different

Finally, much occurs in a context that has come to be known as much support (Corver 1997), where an adjective has been pronominalized with so:

(5) Fred is generous; in fact, he is too much so / so much so that it worries me.

Despite considerable work on the semantics of Q-adjectives, there is as yet no fully general account that is able to handle all the data in (1)-(5). Nor is it obvious that such a general account is possible. On the surface, the examples above seem to involve interpretations at several different semantic types. In their quantificational uses Q-adjectives look like ordinary quantifiers (cf. ‘every/most/no student(s) attended the lecture’). In their attributive and predicative uses they look instead like ordinary adjectives (cf. ‘John’s friends are nice’; ‘the diligent students who attended’). The adjectival parallel is strengthened by the oft-noted fact that Q-adjectives are like gradable adjectives in having comparative and superlative forms, and in combining with the same degree modifiers:

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1 A complication in the data is that on some uses, bare much and to a lesser extent many are awkward in positive sentences, to the point that some authors have considered them negative polarity items (see especially Israel 1996). I take the unavailability of much in (2c) to be attributable to this. I will as much as possible sidestep this issue, as it is largely tangential to the central questions investigated here.

2 Only those of few are morphologically transparent. But authors back to Jespersen (1970/1914) and Bresnan (1973) have taken more to be the spell-out of many+er and much+er, most to be the spell-out of many+est and much+est, and less and least to be the spell-outs of little+er and little+est, respectively.
Finally, in their adverbial and differential uses Q-adjectives seem to be degree modifiers of some sort. Must we conclude that this class is simply ambiguous?

While the flexibility illustrated above is particular to Q-adjectives, number words and measure phrases occur in some of these same positions:

(7) a. Twenty students attended the lecture.  
   b. The twenty students who attended enjoyed the lecture.  
   c. Twenty more students than we expected attended at the lecture.

(8) a. We bought five pounds of rice.  
   b. The five pounds of rice we bought is on the shelf.  
   c. We bought five pounds more rice than we needed.

This suggests that a solution to this problem will have relevance beyond the small class of Q-adjectives themselves.

1.2 Proposal in a nutshell

In what follows I will argue that, despite appearances, Q-adjectives are not ambiguous. The central claim of the paper is that across all of their uses, Q-adjectives denote gradable predicates of sets of degrees, or equivalently, gradable quantifiers over degrees. To put it in somewhat informal terms (ignoring for now the matter of gradability), the proposal is that the lexical entries for Q-adjectives are the following:

(9) Proposal: Q-adjectives as predicates of sets of degrees:
   For a set of degrees $D$ in a context $c$:
   a. many/much is true of $D$ iff $\text{max}(D)$ is greater than the relevant standard in $c$
   b. few/little is true of $D$ iff $\text{max}(D)$ is less than the relevant standard in $c$

This is not an entirely new idea: Similar proposals have been made for certain uses of Q-adjectives by Klein (1982), Schwarzschild (2006) and Heim (2006), and Rett (2006, 2008) extends this kind of approach to several of the sorts of examples discussed above. My intention here is to take a much broader view than in previous work, and demonstrate that it is by taking this as the basic lexical semantics for Q-adjectives that we can achieve a fully general and compositional analysis encompassing all of the uses exemplified in (1)-(5). What allows Q-adjectives to surface in all of these varied contexts is that in each case, the logical form provides a set of degrees of which a member of this class can be predicated. Once again being rather informal, the relevant sets of degrees are the following:

(10) a. Quantificational: Many students attended the lecture.  
   \{d : \# students attending $\geq d$\}
   b. Predicative: John’s friends are many.  
   \{d : \# John’s friends $\geq d$\}
The prediction is thus that Q-adjectives will occur in all positions where an argument of this type is available. Evidence for a very similar generalization is provided by Doetjes (1997), who demonstrates on the basis of extensive cross-linguistic data that the distribution of much and similar items can be characterized in terms of their sensitivity to the presence of a scalar theta position. Some exceptions to the predicted distribution are discussed below.

The bulk of this paper is devoted to developing a compositional implementation of this idea, and demonstrating that it overcomes a number of issues facing previous approaches to Q-adjectives. The structure of the paper is the following: Section 2 summarizes the two leading semantic approaches to Q-adjectives, and outlines a set of problems they face in accounting for the relevant data. Section 3 presents the core of the present proposal, focusing on the quantificational use as the base case. It will be seen that the Q-adjective in this case is located in the specifier position of a DP-internal functional head Meas, whose role is to introduce a degree argument; from here, it undergoes QR to a position of sentential scope, where it takes as argument the set of degrees formed by lambda abstraction over the degree trace in its base position. Section 4 extends the analysis to the differential and adverbial uses, and to the phenomenon of much support, which can be accommodated via the same compositional mechanisms, while Section 5 investigates the adjective-like cases, where some further ancillary proposals are required. Finally, Section 6 takes on a more detailed comparison of the present approach to one based on type ambiguity, and Section 7 wraps up with conclusions, and some brief remarks on possible extensions to number words and measure phrases.

2 Accounting for the diversity

2.1 Previous approaches

There are two now fairly standard semantic treatments of Q-adjectives. I summarize these here, focusing in particular on many and few, which have been the subject of the most extensive research to date.

Q-adjectives as quantifiers  If there can be said to be an orthodox view on the semantics of many and few, it is that they are quantifiers (cf. introductory semantics textbooks such as Heim and Kratzer 1998; Chierchia and McConnell-Ginet 2000). The quantificational analysis in its most standard form is represented by the Generalized Quantifier Theory of Barwise and Cooper (1981), according to which many and few are
‘quantifying determiners’ that express relationships between two sets (type \(\langle et, \langle et, t \rangle \rangle\)),
specifying that the cardinality of their intersection exceeds \(\text{many}\) or falls short of \(\text{few}\) some standard determined by the context \(c\):

\[
\begin{align*}
\text{(11)} & \quad a. \ [\text{many}_{\langle et, \langle et, t \rangle \rangle}]^c = \lambda P \lambda Q. |P \cap Q| > n_c, \text{ for some large } n_c \\
& \quad b. \ [\text{few}_{\langle et, \langle et, t \rangle \rangle}]^c = \lambda P \lambda Q. |P \cap Q| < m_c, \text{ for some small } m_c
\end{align*}
\]

Quantificational analyses have been proposed by Westerstahl (1985); Lappin (1988, 2000); Partee (1989); Diesing (1992); Kamp and Reyle (1993); Higginbotham (1995); Herburger (1997); Chierchia (1998a), among others. Hackl (2000) offers an update to the quantificational approach according to which \(\text{many}\) is analyzed as a parameterized determiner whose first argument is a degree argument that is saturated or bound by degree morphology. Hackl’s analysis has the benefit of establishing a compositional relationship between bare \(\text{many}\) and \(\text{few}\) and their modified forms, but is nonetheless fundamentally a quantificational one.

Q-adjectives as cardinality predicates A second well-established view on the semantics of \(\text{many}\) and \(\text{few}\) holds that they have adjectival semantics, being like ‘ordinary’ adjectives in denoting predicates or modifiers. In what might be considered a precursor of this view, Link (1983) begins his seminal paper on mass terms and plurals with an anecdote about German magazine publisher Rudolph Augstein, who when asked what quality he most appreciated in his friends, replied “that they are few.” Link remarks:

Clearly, this is not a property of any one of Augstein’s friends; yet, even apart from the esprit it was designed to display the answer has a straightforward interpretation. The phrase [‘that they are few’] predicates something collectively of a group of objects, here: Augstein’s friends. (1983; p. 302)

Link does not return to this example to present a formal semantics for \(\text{few}\) as a predicate of groups or pluralities, but proposals of this nature have been made by Mil- sark (1974, 1977); Klein (1981); Hoeksema (1983); Partee (1989); Kennedy and McNally (2005b). On this approach, \(\text{many}\) and \(\text{few}\) have the semantics of cardinality predicates: predicates that hold true of a group/plurality if its cardinality exceeds or falls short of the appropriate contextual standard:

\[
\begin{align*}
\text{(12)} & \quad a. \ [\text{many}_{\langle et \rangle}]^c = \lambda x. |x| > n_c, \text{ for some large } n_c \\
& \quad b. \ [\text{few}_{\langle et \rangle}]^c = \lambda x. |x| < m_c, \text{ for some small } m_c
\end{align*}
\]

A related approach (e.g. Hackl 2009) takes \(\text{many}\) and \(\text{few}\) to be noun modifiers, i.e. functions from sets to sets. The adjectival analysis of Q-adjectives represented here also aligns to a much broader tradition in which cardinal numerals are analyzed as cardinality predicates or noun modifiers (Hoeksema 1983; Partee 1987; Krifka 1999; Landman 2004; Ionin and Matushansky 2006).

On their own, each of the two approaches sketched out above handles some but not all of the uses of Q-adjectives. The quantificational analysis, not surprisingly, can account for their occurrence as apparent quantifiers (per (13a)), but has less to say about their other uses. Conversely, the cardinality predicate analysis is suited to the predicative and attributive uses (per (13b)), but less so to the quantificational case.
(13) a. Many students attended the lecture.
   \[\text{[many}(_{et},(et,0))\text{]}^c(\text{[students]}^c)(\text{[attended the lecture]}^c)\]
b. John’s friends are many.
   \[\text{[many}(_{et})\text{]}^c(\text{[John’s friends]}^c)\]

To the extent that proponents of these two analyses have addressed the multiplicity represented in (1)-(5), the typical approach is to invoke some sort of type shifting operations (see especially Partee 1989; De Swart 2001). For example, taking the predicative entry for \textit{many} in (12a) as basic, and assuming an existential-based predicate-to-quantifier shift along the lines of Partee’s (1987) \textit{A} operation (14), we derive the truth conditions for quantificational \textit{many} in (15), which is equivalent to what would be obtained via the quantifier entry (11a):

(14) \[A(P_{(et)}) = \lambda Q_{(et)}. \exists x [P(x) \land Q(x)]\]

(15) a. \[A([\text{many students}]^c) = A([\text{many}]^c \cap [\text{students}]^c)\]
   \[= \lambda Q_{(et)}. \exists x [^\ast \text{student}(x) \land |x| > n_c \land Q(x)]\]
b. \[\text{[Many students attended the lecture]}^c = 1 \iff \exists x[^\ast \text{student}(x) \land |x| > n_c \land \text{attended}(x, \text{lecture})]\]

This sort of approach to Q-adjjectives falls under the well-established tradition of non-quantificational analyses of indefinite noun phrases, and could be implemented in a variety of ways other than type shifting, e.g. via global existential closure (Heim 1985; Diesing 1992), existential closure over choice functions (Reinhart 1997), or composition with a null determiner with existential semantics (Krifka 1999). Alternately, quantificational and predicative noun phrase interpretations could be related via a quantifier-to-predicate shift such as Partee’s \textit{BE} (De Swart 2001; McNally 1998), or a type shift could be defined between predicative and quantificational entries of Q-adjjectives themselves (Klein 1981).

In that such an approach seems to capture a range of data on the basis of a single underlying lexical entry for the Q-adjective coupled with more general and independently motivated semantic operations, it is appealing from the point of view of parsimony.

If this were the end of the story – and I think it is often assumed that it is – there would not be much to write about here. But in fact, the ‘standard’ approach described here does not account for the full range of facts in (1)-(5). In the following, I describe four issues for the line of analysis described above.

2.2 Issues for previous approaches

**Issue 1: Incorrect results under type shifting**  
Above it was shown that a predicate-to-quantifier shift based on an existential operator produces an intuitively correct result in the case of quantificational \textit{many}. But the same is not true for quantificational \textit{few}. Applying \textit{A} to a predicative NP based on \textit{few} in (12b) produces the following:

(16) \[A([\text{few students}]^c) = \lambda Q_{(et)}. \exists x [^\ast \text{student}(x) \land |x| < m_c \land Q(x)]\]

(17) \[\text{[Few students attended the lecture]}^c = 1 \iff \exists x[^\ast \text{student}(x) \land |x| < m_c \land \text{attended}(x, \text{lecture})]\]
(17) simply asserts the existence of a group of students numbering less than \( m_c \) who attended the lecture, and crucially does not exclude the possibility that some larger group (numbering \( m_c \) or more), a superset of the smaller group, likewise attended. That is, the operation that works in the case of \textit{many} yields incorrect results in the case of \textit{few}.

The issue in (17) exemplifies what has come to be known as van Benthem’s problem (Benthem 1986; see also Herburger 1997; McNally 1998; Hackl 2000; De Swart 2001; Landman 2004): The application of an existential operator to a predicative expression including an upper-bounded cardinality predicate (e.g. \textit{few}) incorrectly produces a lower-bounded ‘at least’ reading. To be sure, this does not mean that it is impossible to formulate a type shift that will produce the correct results in the case of \textit{few} as well as \textit{many}. Some authors (De Swart 2001; Landman 2004) have proposed systems in which an existential-based shift is augmented with a further operation intended to produce the correct results for upper-bounded predicates such as \textit{few}. But in introducing additional or more complex type shifts, such accounts also introduce the challenge of constraining their application appropriately. Minimally, if we assume that some sort of existential-based shifting operation is part of the grammar, we need to account for why it seemingly cannot apply to a predicative noun phrase containing \textit{few}. In Section 6 I return to the complications involved in making this work.

**Issue 2: Non-quantificational/non-adjectival uses**  The discussion above has focused on the quantifier-like and adjective-like uses of Q-adjectives, but there are other occurrences of these items that cannot be analyzed as either. A case in point is the differential use, some further examples of which are given below\(^3\):

\[(18) \quad \text{a. There were 100 seats in the lecture hall, but unfortunately many more than 100 students showed up for the lecture.} \]
\[\text{b. The hall has 500 seats, but few more than 100 students were in attendance.} \]
\[\text{c. The whole class of 100 was supposed to attend the lecture, but many fewer than 100 students actually showed up.} \]

The Q-adjectives in (18) do not seem to be quantifying determiners, since the logical form does not provide two appropriate sets of individuals that could serve as their arguments. In the ‘more than’ comparatives in (18a,b), \textit{many} and \textit{few} might be analyzed as cardinality predicates, predicated of that set of students in excess of the first 100 who attended the lecture. But in the corresponding ‘fewer than’ comparative in (18c), there is no equivalent group of students to whom the property of ‘many-ness’ can be ascribed.

\(^3\)Some speakers find examples of this form marginal. Differential occurrences of Q-adjectives are however readily found in natural language data. For example:

\[\text{(i) 5.8 million Afghan children are enrolled in school, and 35% are girls, many more than at any point in Afghanistan’s history.} \]
\[\text{(ii) Few more than 400 Sumatran tigers survive in the wild.} \]
\[\text{(iii) The American economy created 103,000 jobs, many fewer than expected.} \]
This point becomes even clearer if we extend our focus to *much* and *little* in their role as modifiers of adjectival comparatives:

(19)  a. John is much taller than Fred.
     b. John is much younger than Fred.

On other uses, *much/little* might be treated as predicates over portions of matter (similarly to the cardinality predicate approach to *many/few*), but that is implausible here. We would not, for example, wish to say that *much* in (19a) is predicated of that portion of John’s anatomy that extends above Fred’s maximal height. And even if we convinced ourselves that this were reasonable, in (19b) there no portion of matter of any sort to serve as the argument for the Q-adjective. Similar issues come up when we consider the adverbial uses of *much* and *little*. In fact, when the full range of data in (1)-(5) is considered, it becomes clear that the ‘quantifier/cardinality predicate + type shift’ analysis in the form described above accounts for only a small proportion of it. There are, to be certain, semantic analyses of these other uses of Q-adjectives (e.g. Klein 1982 on the differential use and Kennedy and McNally 2005a; Heim 2006 on adverbial cases), but there have been few attempts to relate these to quantificational/predicative/attributive Q-adjectives, a notable exception being Rett (2008) (to be discussed further below).

**Issue 3: Non-adjective-like behavior** The strongest argument for the cardinality predicate analysis of Q-adjectives (or one in which a predicative interpretation can be derived) is that they occur in canonical adjectival positions, namely as predicates in copular sentences (1b) and as attributive modifiers (1c), (2c). But as several authors have noted, the behavior of Q-adjectives diverges in various ways from that of ‘ordinary’ adjectives, including in some cases the adjective *numerous*, whose meaning is otherwise very similar to that of *many*. For example, attributive Q-adjectives are more restricted in the determiners they follow (20). Hackl (2000) points out that while *many* and *few* occur predicatively in full clauses, they do not do so in small clause complements of *consider*, in contrast to both adjectives such as *tall* and *numerous* (21).

(20)  a. The/those/his/*most/*some many students
     b. The/those/his/most/some intelligent students

(21)  a. *I consider the guests many/few
     b. I consider the guests tall/numerous

Conversely, Kayne (2005) notes that *many* and *few* allow a subsequent ‘unpronounced NP’, while ordinary adjectives (and *numerous*) do not:

(22)  a. Many linguists like phonology, but many don’t.
     b. *Good linguists like phonology, but bad don’t.
     c. *Numerous linguists like phonology, but numerous don’t.

Some further divergences will be discussed in Section 5 below. An analysis of Q-adjectives that takes them to have the same semantic type as ordinary gradable adjectives such as *tall* is faced with explaining their differences in behavior vs. the latter class.
**Issue 4: Scope splitting** The final issue is not so much a failing of the standard account, but rather a separate set of data that it does not explain. It has been noted that quantificational *few/little* and their inflected forms allow readings in which their semantic content seems to be split across a modal expression or intensional verb. For example, (23a), from Hackl (2000), has the reading paraphrased in (23b), where negation scopes over *need*, which in turn scopes over the existential component.

(23)  
   a. To get tenure at MIT, you need to publish fewer than 4 papers.  
   b. ‘It does not need to be the case that there are 4 or more papers that you publish’

Something comparable is possible with bare *few*. The following sentence (based on a similar example for adverbial *little* discussed by Heim 2006) is ambiguous between two readings, which are brought out by the two continuations that follow; the second exhibits the same sort of splitting seen in (23):

(24)  
   a. . . . and still graduate with honors.  
   ‘can take a ¬large number of advanced classes’
   b. . . . because I need to take so many introductory courses.  
   ‘¬ can take a large number of advanced classes’

Note that the nominal *advanced classes* in (24b) is interpreted *de dicto* rather than *de re*: the claim is not that there is a small set of specific advanced classes I can take, but rather that however many such individual classes there are, the total number of them I can take is small. As such, this example cannot involve the noun phrase *few advanced classes* as a whole taking scope over the modal.

The pattern exemplified here is similar to the phenomenon of scope splitting with negative indefinites such as *nobody*, which has prompted analyses that decompose them into an indefinite and a wide-scope negation operator (Penka 2012). The data in (24) in particular suggest that *few* also requires a decompositional treatment that separates the negative aspect of its content from the cardinality operator. This is not reflected in either the quantificational or the cardinality predicate theories, which treat this word as an unanalyzed whole.

In what follows, I will show that once Q-adjectives are analyzed as having degree-based semantics – and not as quantifiers or predicates over individuals – solutions to all of these issues fall out naturally.

### 3 Compositional analysis of Q-adjectives

#### 3.1 Formal preliminaries

I begin here by briefly summarizing the semantic framework I adopt in formalizing the analysis of Q-adjectives. To start, I assume an ontology that includes degrees as a basic type, type $d$ (Cresswell 1977 and ff.). Degrees are organized into scales; scale $S$ is a triple $\langle D, >, DIM \rangle$, where $D$ is a set of degrees, $>$ is an ordering relation on that set, and
DIM is a dimension of measurement. Dimensions include those such as length, height, duration and so forth; but number (cardinality) as well as measures of quantity in the mass domain (e.g. volume, weight) are also examples of dimensions.

Individuals (type $e$) are related to degrees via measure functions. The function $\mu_S$ maps an entity $x$ to the unique degree $d$ on the scale $S$ that represents the measure of $x$ with respect to the corresponding dimension DIM.

With regards to the syntax and semantics of gradability more specifically, I take as my starting point an analysis of gradable adjectives that is essentially that of von Stechow (1984) and Heim (2000), and which has been productively applied to a wide range of phenomena relating to degree and comparison (see Beck 2011 for a recent review). On this approach, gradable adjectives head adjective phrases (APs), and degree modifiers, which have the syntactic status of degree phrases (DegP), occur in the specifier position of the AP (25). Semantically, gradable adjectives denote relationships between individuals and degrees (26):

\[
\text{(25) \quad John is } [AP [DegP too] [A' tall]]
\]

\[
\text{(26) \quad [tall] = } \lambda d \lambda x. \mu_{\text{HEIGHT}}(x) \geq d
\]

Degree modifiers denote either degrees (plausibly the case for measure phrases and demonstratives such as that in that tall) or quantifiers over degrees (e.g. too, so, comparative and superlative morphology). In the latter case, the degree modifier cannot be interpreted in situ, in that the gradable adjective needs an expression of type $d$ to saturate its first argument position, but the degree expression is of type $\langle dt, t \rangle$. This type mismatch is resolved by quantifier raising (QR) of the degree modifier to a sentence adjoined position (27); the type $d$ argument of the adjective is saturated by the trace of the raised DegP, with subsequent lambda abstraction over this variable providing an expression of type $\langle dt \rangle$ to serve as the argument of the degree modifier (28):

\[
\text{(27) \quad [DegP too] }_1 [\text{John is } [AP \ t_1 [A' \ tall]]]
\]

\[
\text{(28) \quad [too]_(dt,t) (} \lambda d. \mu_{\text{HEIGHT}}(\text{John}) \geq d)
\]

In the case of the unmodified positive form of the adjective (as in John is tall), where there is no overt degree morphology to saturate the degree argument, it is typically assumed that a null degree morpheme POS (for ‘positive’) plays this role (see e.g. Cresswell 1977; Kennedy 2007). I follow this approach, adopting the definition of POS introduced by Stechow (2007), which is based on a ‘neutral range’ $N_S$ on the scale $S$, whose value is specified by an assignment function $g_c$ provided by the context $c$ (29), and which consists of the range of degrees on $S$ that would be considered neither large nor small with respect to $c$. In the case of height, for example, $N_{\text{HEIGHT}}$ would correspond to those heights that would in the given context count as neither tall nor short. Thus (30a) receives the interpretation in (30b), which corresponds to the situation depicted in (30c):

\[
\text{(29) \quad [POS]_{(dt,t)} }^w = \lambda I_{(dt)}. \forall d \in N_S [I(d)]
\]

\[
\text{(30) \quad a. \quad John is tall}
\]

\[
\text{b. \quad } \forall d \in N_{\text{HEIGHT}} [\mu_{\text{HEIGHT}}(\text{John}) \geq d]
\]
This formulation of the semantics of \textit{POS} has the benefit of allowing a unified treatment of both positive and negative polar expressions (e.g. tall/short, many/few), and has been chosen for that reason. But nothing in my proposal depends crucially on the specific semantics of \textit{POS} given in (29), or indeed on the choice of a \textit{POS}-based analysis over another approach to the positive form (e.g. type shifting, per Kennedy and McNally 2005b).

I further adopt a non-quantificational approach to indefinites, in which quantificational force arises as the result of the application of a covert existential operator (Heim 1982; Diesing 1992). For explicitness I assume the following rule, although again my proposal would be compatible with alternative formulations:

\begin{equation}
\text{Existential Closure (\exists C): Unbound variables are existentially bound at the IP level.}
\end{equation}

Finally, I assume the compositional rules of functional application (FA), predicate modification (PM) and predicate abstraction (PA) (Heim and Kratzer 1998), as well as a rule of Degree Argument Introduction (DAI), which will be introduced below.

### 3.2 The semantics and syntax of Q-adjectives

In Section 1 it was proposed that Q-adjectives, across all of their uses, denote gradable predicates of sets of degrees. I formalize this as follows:

\begin{equation}
\text{(32) Lexical entries of Q-adjectives:}
\begin{align*}
a. \quad & \text{[many/much}_{(d,\langle dt,t \rangle)} = \lambda d \lambda I_{(dt)}.I(d) \\
b. \quad & \text{[few/little}_{(d,\langle dt,t \rangle)} = \lambda d \lambda I_{(dt)}.\neg I(d)
\end{align*}
\end{equation}

These entries assume the negation theory of antonymy proposed in Heim (2004, 2006); Stechow (2007). In particular, \textit{few} and \textit{little} as defined in (32b) have the semantics that Heim (2006) ascribes to \textit{little} on its adverbial uses, and which I will argue in fact underlie all of the occurrences of these items. The corresponding entry for \textit{many/much} is the natural positive counterpart, which we will see has some interesting consequences.

On the definitions in (32), Q-adjectives are like gradable adjectives in that their first argument is of type \textit{d}. But they differ from ordinary gradable adjectives in that their second argument ranges over sets of degrees rather than individuals. In fact, once their first argument is saturated, they have the type of degree modifiers. This difference vs. adjectives of the \textit{tall} class does not affect their ability to compose with the same degree modifiers. As discussed above, degree modifiers either saturate the degree argument of the gradable expression or take as argument a set of degrees formed via lambda abstraction over a trace of type \textit{d}, and as such the full type of the gradable expression itself does not have an effect. However, we will see in what follows that the type difference between
Q-adjectives and ordinary gradable adjectives accounts for the unique behavior of the former class.

At this point, it is necessary to say something about what distinguishes many from much, and correspondingly few from little. Surely there is some differentiating factor, since the two pairs are not interchangeable (e.g. many/*much students; much/*many water). The difference is typically taken to correspond to that between mass and count nouns (Higginbotham 1995 a.o.); but since much and little occur also outside of the nominal domain (e.g. in adjectival differentials), it seems necessary to explain it in other terms. A plausible characterization of the facts is that the choice is sensitive to dimension of measurement, in that many/few are used when the dimension in question is number or cardinality, while much/little are used for other dimensions. This would limit many/few to occurrence with plural count nouns, whose denotations can be counted, while much/little occur with mass nouns and in the adjectival and verbal domains. I thus propose that many/few lexically encode cardinality as a dimension of measurement. Much/little I take to be unspecified for dimension of measurement, with the result that they may be used in a wider range of contexts; the exception is with canonical plural count nouns, where they are blocked by the more specific many/few.4

Turning to their syntax, the occurrence of Q-adjectives with degree modifiers indicates that they too, like ordinary adjectives, head maximal projections, which I will call quantifier phrases (QP). In the case of quantificational Q-adjectives, I following a number of authors who posit a functional projection between N and D that is the locus of numerical expressions and/or weak quantifiers more generally, and take the QP to occur in the specifier position of a functional head which I will call Meas, for ‘measure’ (the corresponding projections are Abney’s (1987) QP, Zamparelli’s (1995) PDP, Cheng & Sybesma’s (1999) NumP, Borer’s (2005) #P, and Svenonius’ (2008) UnitP). The relevant structure is thus the following:

\[ [DP [MeasP [QP [DegP too/that/very/POS/etc.] [q’ many]] Meas NP]] \]

I further propose that the functional head Meas plays a semantic role. The degree-based entries in (32) do not allow a Q-adjective to compose directly with an NP denoting a set of individuals. I therefore propose that composition between the Q-adjective and its apparent NP sister is established by Meas, which has the function of introducing a degree argument and linking it to an individual argument.

The idea that there is a phonologically null but semantically contentful element between the Q-adjective and the following NP is similar to a proposal by Kayne (2005),

\[ (1) \] These/*this mashed potatoes/refried beans/scrambled eggs are/*is cold.

\[ (2) \]

a. Q: How many scrambled eggs do you want?
   A: Three

b. Q: How much scrambled eggs do you want?
   A: A lot/a little/a scoopful/about half the amount you gave him

---

4Support for this view is provided by data relating to anomalous plurals such as refried beans, mashed potatoes and scrambled eggs, which syntactically show at least some of the characteristics of plural count nouns (1), but semantically denote noncountable substances or portions of matter. These items are at least marginally acceptable with both many and much, but with a difference in meaning; for example, (2a) asks for a number as an answer, while (2b) seems to want an answer relating to a mass dimension. Thus the choice of the Q-adjective constrains the available dimension of measurement.
who posits a null functional noun NUMBER or AMOUNT in this position. Schwarzschild (2006) similarly argues that the connection between quantity words and nominal expressions is mediated by a syntactic head Mon, which has the semantic function of introducing a dimension of measurement (Schwarzschild’s Mon spans the role of both NUMBER and AMOUNT in Kayne’s system).

Importantly for the purposes of the present paper, Kayne shows that a variety of divergences in the behavior of Q-adjectives and ordinary adjectives can be attributed to the presence of this null element. For example, it was noted in Section 2.2 that Q-adjectives, unlike ordinary adjectives, allow a subsequent unpronounced noun (per (22), repeated below). Kayne argues that it is NUMBER that licenses this, just as would be possible with overt number.

(22) a. Many linguists like phonology, but many don’t.
    b. *Good linguists like phonology, but bad don’t.

(34) A large number of linguists like phonology, but a large number don’t.

I propose that this line of reasoning be carried over to Meas, whose semantics I take to correspond closely to ‘number/amount’. Specifically, the interpretation I assign to Meas is:

(35) \[ [\text{Meas}]^{g_c} = \lambda x, \gamma d, \mu_S(x) \geq d \]

Here \( S \) is a variable over measurement scales, whose value is determined by the assignment function \( g_c \). As such, Meas encodes an underspecified measure function, with the choice of scales contextually determined.

I propose, however, that the choice of \( S \) is not totally unconstrained. Rather, I follow Schwarzschild in requiring the corresponding dimension to be monotonic on the part/whole relationship between entities, meaning that for any \( x, y \) such that \( x \) is a proper subpart of \( y \), the measure of \( x \) relative to \( \text{DIM} \) is strictly less than the measure of \( y \). Intuitively, dimensions that are monotonic in this sense are dimensions of quantity or amount. A consequence is that much gold, for example, can be a portion of gold that exceeds some contextual standard in terms of weight or volume (both monotonic), but not one that exceeds some standard in terms of purity (non-monotonic). Note that one particular dimension that satisfies this condition is cardinality (the cardinality of any proper subset of a set of entities is strictly less than that of the original set). As such, cardinality is one possible choice for the dimension introduced by Meas, and thus it is not necessary to assume a distinction between NUMBER and AMOUNT as in Kayne’s system.\(^5\)

When applied to an individual of type \( e \), Meas produces a set of degrees representing its number or amount (hence the parallel to Kayne’s null nouns). This possibility will be relevant in Section 5 below. Alternately, in the configuration in (33), I propose that Meas

---

\(^5\)Reviewers have pointed out that in the case of abstract mass nouns, monotonicity does not always seem to be satisfied. Relevant examples are I’ve never before felt so much happiness and too much heat will cause the pan to warp. These are tricky in that it is not entirely clear what the individual is that is being measured: What is a portion of heat, or happiness? I do not have much to say about these cases, though I tentatively hypothesize that they involve construing heat, happiness, etc. as substances whose amount can be measured.
composes with the noun phrase via the following rule of Degree Argument Introduction, which is intended as a variant of Kratzer’s (1996) rule of Variable Identification, with the modification that the argument targeted for composition in this way – the individual argument – is demoted to second position in the lambda prefix (cf. the RESTRICT operation of Chung and Ladusaw 2003).

(36) **Degree Argument Introduction (DAI):**
If \( \alpha \) is a branching node and \( \{ \beta, \gamma \} \) are the set of \( \alpha \)'s daughters, \( [\beta] = \lambda x. P(x), \)
\( [\gamma] = \lambda d. \lambda \ell. Q(d)(x), \) then \( [\alpha] = \lambda d. \lambda \ell. P(x) \land Q(x)(d) \)

3.3 **Quantificational Q-adjectives – the base case**

With the above pieces in place, we are in a position to provide a compositional analysis of Q-adjectives in their quantificational use, as in (1a), repeated below. In surface structure, the QP occurs in the specifier position of Meas. But with this structure, the Q-adjective cannot be interpreted *in situ*: At the level of the MeasP, a degree-denoting expression is needed as argument; but the QP does not have this type. As a result, the QP raises covertly to take sentential scope, leaving behind a trace of type \( d. \) POS likewise must QR for purposes of interpretability, leaving a trace of type \( d \) to saturate the first argument of the Q-adjective.

(1a) Many/few students attended the lecture. 

![Diagram](image)

The corresponding semantic derivation in the case of *many* is the following (where \( \Rightarrow \) indicates a non-syntactic operation of predicate abstraction or existential closure):

(38) \( \llbracket [\text{Meas' Meas students}]^{\ell} \rrbracket = (\llbracket \text{Meas} \rrbracket^{\ell})(\llbracket \text{students} \rrbracket^{\ell}) \)
\( = (\lambda x. \lambda d. \mu_S(x) \geq d)(\lambda x. \ast \text{student}(x)) \)
\( = \lambda d. \lambda x. \ast \text{student}(x) \land \mu_S(x) \geq d \)

by DAI
As described above, Meas introduces an underspecified measure function, featuring a many dimension of cardinality. Thus the derivation will proceed only if the situation where that number falls short of this range:

number of students who attended exceeds the neutral range, while that (39) describes the relevant differences in the case of few:

As depicted below, the final formula in (38) describes a situation where the maximum number of students who attended exceeds the neutral range, while that (39) describes the situation where that number falls short of this range:

As described above, Meas introduces an underspecified measure function, featuring a variable S ranging over measurement scales. However, many lexically encodes the dimension of cardinality. Thus the derivation will proceed only if S has been assigned to the scale corresponding to cardinality, the number line. This is indicate with a switch to the subscript # at the point where many enters the derivation.

The relevant differences in the case of few are the following:

(39) }[Q_P t_2 few]^{gc} = [few]^{gc}(d_2) = \lambda I. \neg I(d_2) by FA

As depicted below, the final formula in (38) describes a situation where the maximum number of students who attended exceeds the neutral range, while that (39) describes the situation where that number falls short of this range:
Seen slightly differently, the sets of degrees of which Q-adjectives are predicated can be viewed as scalar intervals, in the above examples intervals on the number line. The latter stages of the derivations in (38) and (39) can then be viewed as the creation and manipulation of a series of scalar intervals. In (38), the lowermost IP denotes (after existential closure and predicate abstraction) the interval from 0 to the number of students who attended the lecture, henceforth abbreviated ATTENDANCE (41a). Application of *many* and subsequent predicate abstraction produces the same interval (41b), and the application of *POS* specifies that the neutral range $N_\#$ is within this interval (41c).

(41) Many students attended the lecture.
   a. $[IP \ t_1 \ Meas \ students \ attended \ the \ lecture] \quad [0, ATTENDANCE]$
   b. $[IP \ t_2 \ many \ [IP \ t_1 \ Meas \ students \ ...]] \quad [0, ATTENDANCE]$
   c. $[IP \ POS_2 \ [IP \ t_2 \ many \ [IP \ t_1 \ Meas \ ...]]] \quad N_\# \subseteq [0, ATTENDANCE]$

In (39), the application of *few* maps the original interval to its complement, with *POS* specifying that $N_\#$ is a subset of this interval:

(42) Few students attended the lecture.
   a. $[IP \ t_1 \ Meas \ students \ attended \ the \ lecture] \quad [0, ATTENDANCE]$
   b. $[IP \ t_2 \ few \ [IP \ t_1 \ Meas \ students \ ...]] \quad (ATTENDANCE, \infty)$
   c. $[IP \ POS_2 \ [IP \ t_2 \ few \ [IP \ t_1 \ Meas \ ...]]] \quad N_\# \subseteq (ATTENDANCE, \infty)$

Note finally that while the discussion to this point has focused on *many* and *few*, the same account can be extended to their counterparts *much* and *little* in their quantificational uses. For example:

(43) a. Little rice is left.
    b. LF: $[POS_2 \ [t_2 \ little_1 \ [t_1 \ Meas \ rice \ is \ left]]]$
    c. $\forall d \in N_S \ [\neg \exists x \ [rice(x) \land \mu_S(x) \geq d \land left(x)]]$

Since *little* does not encode a particular dimension of measurement, the dimension must be determined via the context; here, likely dimensions would be weight (*little rice is left - only a pound!* or volume (*little rice is left - only two cups!*).

---

6I use standard interval notation: $[a,b]$ is the closed interval from $a$ to $b$, $(a,b)$ is the corresponding open interval, and so forth.
3.4 Observations and issues resolved

Before proceeding, let me point out some consequences of the analysis developed here as it relates to the goals of this paper, namely developing a unified account of Q-adjectives while avoiding the issues faced by previous approaches.

The present analysis is a decompositional one, in that much of the semantic content that other theories ascribe to Q-adjectives themselves is instead analyzed as coming from null functional elements and semantic operations. The standard of comparison for the positive form is provided by $\text{POS}$, not the Q-adjective. More significantly, quantification over individuals has been removed to the operation of Existential Closure, while the measure function aspect of meaning (i.e. in the case of $\text{many}/\text{few}$, the cardinality operator) is taken to be part of Meas. Q-adjectives themselves are left simply as gradable predicates of sets of degrees.

This may at this stage seem unnecessary and overly complicated. Furthermore, in the case of $\text{many}$ and $\text{much}$, the result of this decompositional approach is rather counterintuitive. As seen in (38) and especially (41), $\text{many}$ as defined here takes as argument a set of degrees and, after lambda abstraction over the trace in its specifier position, returns the same set of degrees. That is, once the other meaning components are removed, what is left in the case of $\text{many}$ is a degree operator that is essentially vacuous – an identity element on sets of degrees. The same point could be made about $\text{much}$. This seems at odds with the intuition that these words are contentful.

If we were to confine ourselves to Q-adjectives on their quantificational use, these criticisms would have validity. For examples of this sort, simpler and more intuitive options would be available, in which some or all of the removed content was built back into the semantics of the Q-adjective itself. But while this would suffice for the quantificational case, it would not give us what we need for the other uses of Q-adjectives. As will be seen below, it is precisely by stripping these various elements of meaning out of the lexical entry of the Q-adjective, and placing them elsewhere, that we achieve a semantics for Q-adjectives that can extend to all the uses discussed earlier. Even the surprising analysis of $\text{many}/\text{much}$ as essentially null elements will be seen to account for an additional and puzzling property of $\text{much}$. I discuss these consequences in the next section.

In the case of $\text{few}/\text{little}$, though, the benefits of the decompositional approach can be appreciated already at this stage. As seen in (39), $\text{few}$ cannot be interpreted in situ, since the logical form at that level of the derivation (the MeasP) does not provide a set of degrees to serve as its argument. As a result, it must QR, acquiring as an argument the set of degrees formed by lambda abstraction over the trace of type $d$ in its base position. But crucially, this can only occur after the stage at which Existential Closure applies to produce an expression of type $t$; if lambda abstraction were to occur at an earlier stage, the result would not be an expression of type $\langle dt \rangle$, and therefore could not serve as argument to the Q-adjective. The result is that the Q-adjective comes to outscope the existential operator. That is, the scope relationship is that in (44a), not that in (44b):

\begin{align}
\text{(44)} & \\
& \begin{aligned}
a. & \text{few > } \exists \\
b. & \exists > \text{few}
\end{aligned}
\end{align}

By this means, we are able overcome the first of the issues discussed in Section 2.2 above. Recall that a serious challenge faced by the non-quantificational cardinality predicate
analysis of Q-adjectives is that the application of an existential operator (e.g. via Existential Closure or Partee’s $A$ operation) to a cardinality predicate few incorrectly produces a lower-bounded ‘at least’ reading (van Benthem’s problem). The present account avoids this issue, while at the same time maintaining a non-quantificational approach which assumes no predicative-to-quantificational type shifts beyond the application of a covert existential operator. The correct interpretation falls out as a direct consequence of the lexical entry of the negative few and little, without the need to add anything further to the system.

Furthermore, the same mechanism can account for the last of the four issues discussed in Section 2.2, namely the availability of split scope readings with few and little, where the negative meaning component of the Q-adjective seems to take separate and higher scope from the remainder of its (apparent) content. Consider again the two readings of an example such as (24), repeated below. These can be analyzed as involving two different scope options for the Q-adjective relative to the modal operator:

(45) I can take few advanced classes . . .
   a. . . . and still receive my degree with honors.
      $\Diamond > \text{POS + few}_i > \exists d_i\text{-Meas advanced classes that I take}$
   b. . . . because I need to take so many introductory courses.
      $\text{POS + few}_i > \Diamond > \exists d_i\text{-Meas advanced classes that I take}$

On the split reading (45b), the degree operators $\text{POS}$ and few QR to a position above the possibility operator, while the existential operator and measure function introduced by Meas remain in the scope of the modal.

This is by no means a new claim, as it is precisely the analysis Heim (2006) gives to comparable ambiguities with adverbial little; in fact, examples of this sort serve as a primary motivation for the degree-operator analysis of little she advocates. The point to be made here is that the same mechanism that yields an intuitively correct analysis of quantificational few/little also gives an explanation for a separate set of facts, namely the possibility of ambiguities such as that in (45), and in particular the existence of the second, split reading.

4 Beyond quantifiers

The second of the issues discussed in Section 2.2 is that Q-adjectives have uses where they appear to neither quantify over nor be predicated of individuals, and which are therefore not handled by the two leading approaches to this class. A central advantage of the present analysis is that it can be extended to these uses as well. Furthermore, this is possible with the same lexical entries that were taken to underlie the quantificational use. In this section I discuss the relevant cases.

4.1 Differential Q-adjectives

The first case to be considered is the differential use of Q-adjectives. To repeat some earlier data:
Many more/few more/many fewer than 100 students attended the lecture.

Fred isn’t very tall – but he’s much taller than his father / in fact, he’s little taller than his father.

To analyze these, we require to start a theory of the syntax and semantics of the comparative. This is of course the subject of an extensive literature. I will attempt to avoid the complexities of this subject, and adopt here a fairly standard view based in particular on von Stechow (1984) and Heim (1985, 2000). On this account, the comparative morpheme -er – like other degree modifiers – heads a degree phrase DegP. The than phrase originates as the complement of -er, though may be right extraposed in the surface structure. This leaves the specifier position of the DegP for a differential expression. To illustrate with a simple adjectival example, (46a) has the underlying structure in (46b).

At LF, the DegP undergoes QR, as in (46c):

(46)   a. John is 3 cm taller than Fred.
       b. John is \[ AP [DegP 3 cm [Deg' -er [ than Fred ]] tall] \]
       c. [DegP 3 cm [Deg' -er [ than Fred]]] \[ John is [AP I1 tall] \]

With regards to the semantics, we require an entry for -er that allows its modification by a differential degree (e.g. 3 cm in the above example). A fairly standard option is that in (47), based somewhat loosely on von Stechow (1984) (see also Beck 2011). Here, -er takes three arguments corresponding to the than phrase (d), the differential degree (d′) and the degree predicate formed from the matrix clause out of which DegP has raised (I). For (46), this yields the interpretation in (48).

(47)   [-er] = \[ \lambda d \lambda d′ \lambda I(d).max(I) \geq d + d′ \]

(48)   \[ max(\lambda d.\mu_{HEIGHT}(John) \geq d) \geq \mu_{HEIGHT}(Fred) + 3 \text{ cm} \]

As stated, however, the entry in (47) is however not quite sufficient for the present purposes, in that it does not translate directly to negative antonyms. For example, if short is defined comparably to tall but with the \( \geq \) operator replaced by \( \leq \), (49) incorrectly receives the analysis in (49a), which among other issues has the problem that the maximum is undefined. Instead, we want something along the lines of (49b):

(49)   a. John is 3 cm shorter than Fred.
       b. \[ max(\lambda d.\mu_{HEIGHT}(John) \leq d) \geq \mu_{HEIGHT}(Fred) + 3 \text{ cm} \] ×
       c. \[ min(\lambda d.\mu_{HEIGHT}(John) \leq d) \leq \mu_{HEIGHT}(Fred) - 3 \text{ cm} \] ✔

As this is relevant for comparatives formed with fewer, I will instead make use of the following slightly modified formulation:

\[ \text{Fewer is defined as: } \mu_{HEIGHT}(John) \leq \mu_{HEIGHT}(Fred) - 3 \text{ cm} \]
[-er] = λdλd′λI(d)I(d)∧dif(supDIR(I),d) ≥ d′

where dif returns the positive difference between two degrees, and supDIR is a direction-sensitive supremum operator.

Here supDIR is a function that returns the supremum of a set of degrees I that is lower closed by 0 (the smallest degree d such that ∀d′ ∈ I, d ≥ d′), and the infimum of a set I that is upper bounded by ∞ (the largest degree d such that ∀d′ ∈ I, d ≤ d′). For example:

\[(51)\]
\[
a.\ supDIR([0, 20]) = sup([0, 20]) = 20 \\
b.\ supDIR((20, ∞)) = inf((20, ∞)) = 20
\]

supDIR is thus similar to the direction-sensitive maximum operator of Fox and Hackl (2006), which picks out the most informative degree in an interval, as well as to the maximalization operator assumed by Beck (2010) to derive points from intervals. I however use a (direction sensitive) supremum rather than maximum operator because some of the intervals that will be encountered are bounded but open on the relevant end (i.e. such that supDIR(I) ∉ I, as in (51b)).

The reader may verify that (50) produces results equivalent to (48) and (49c) for taller and shorter, respectively. In both cases, the differential degree specifies the minimum scalar distance between the degree introduced by the than clause and the relevant endpoint of the interval derived from the matrix clause. In the case of unmodified comparatives (e.g. John is taller/shorter than Fred), it is assumed that the differential degree slot is existentially closed, with the specification that the degree be greater than 0.

Let us apply this now to Q-adjectives on their differential use. (52) gives the LF of a relevant example. As in the quantificational case, the entire QP many fewer than 100 undergoes QR; from here, the DegP headed by -er also QRs, as does the QP many and finally the degree modifier POS:

\[(52)\] Many fewer than 100 students attended the lecture.

The denotation of the DegP headed by -er is the following:
For the sentence as a whole, the derivation proceeds as it did in the quantificational case, via the creation and manipulation of a series of scalar intervals formed by lambda abstraction over the traces of each stage of QR. The relevant intervals are:

\[
\text{IP}\,^3 = \lambda d_3.\neg \exists x \left[\text{student}(x) \land \mu#(x) \geq 100 \land \text{attended}(x,\text{lecture})\right] \land \\
\text{dif}(\text{ATTENDANCE}, 100) \geq d_3 \\
\text{IP}\,^5 = \underline{1} \text{ iff} \neg \exists x \left[\text{student}(x) \land \mu#(x) \geq 100 \land \text{attended}(x,\text{lecture})\right] \land \\
\forall d \in N_# \left[\text{dif}(\text{ATTENDANCE}, 100) \geq d\right] \\
N_# \subseteq [0,100 − \text{ATTENDANCE}] \\
\]

Here, the application of the DegP \( t_3 \)-er than 100 and subsequent lambda abstraction creates an interval that represents the scalar distance between the maximal number of attendees (which is less than 100) and 100. For example, if 60 students attended, the interval derived is \([0,40]\) (since the distance between 60 and 100 is 40). Applying many and then POS to this interval specifies that it extends past the contextually relevant standard of comparison \(N_#\), i.e. that the difference between the maximal number of attendees and 100 is large relative to the context. This is what we want.⁸

The adjectival differential use (3a) can be handled identically.

4.2 Adverbial Q-adjectives

Let us now briefly consider the adverbial use of Q-adjectives, as in the following examples from the original data set:

⁸The reader might note that the value assigned to the neutral range \(N_#\) must be different in the differential case vs. the corresponding quantificational case. For example, how many books I need to have to count as having ‘many books’ is different from how many more than 100 I need to have to have ‘many more than 100 books’. The latter, in particular, seems in part to depend on the degree specified by the \(\text{than}\) clause, i.e. 100. Developing an account of how the context determines the neutral range is beyond the scope of the present paper; though see Solt (2009, 2011) for some steps towards working this out.
(4)  a. I **much** prefer wine to beer.  
    b. I slept **little**.  
    c. **much** loved/little known; **much**/little alike/different

It should be clear that these can be accommodated very similarly to the differential case. We require simply that some element of the semantic representation introduces a set of degrees of which the Q-adjective can be predicated. Exactly this sort of analysis is applied by Heim (2006) to examples parallel to (4b). Following this approach, suppose the verb includes a degree argument:

(55) $[\text{sleep}] = \lambda d \lambda x. \text{sleep}(x)$ for duration $\geq d$

Then (4b) has the LF in (56a), and thus receives the interpretation in (56b):

(56) a. LF: [POS$_2$ $[t_2$ little$_1$ [I slept $t_1$]]]
   b. $\forall d \in N_{\text{DURATION}}, \neg [\text{i slept for duration}] \geq d$

A similar analysis can be extended to cases where the Q-adjective serves as an adjectival modifier (4c), an approach which is made plausible by the fact that it is almost exclusively deverbal adjectives that occur with **much**/**little**. The exception to this is the pair alike/different, which feature a ‘differential’ component to their meaning, and thus might be aligned instead to differentials.

### 4.3 Much support and more

In the previous section, a peculiarity in the present treatment of **many** and **much** was pointed out, namely that they behave essentially as identity elements. As seen in the derivation in (38), when we start with a set of degrees, apply **many**/**much**, and abstract over the trace of a DegP in its specifier position, we end up with the same set of degrees. This is in fact already evident from the lexical entries given for Q-adjectives: **many**/**much** simply relate an interval to the degrees within it. On the decompositional approach taken here, once other components of the apparent meaning of Q-adjectives are stripped away to other sentential elements and operations, what we are left with in the case of **many**/**much** is a gradable predicate of sets of degrees that is itself essentially contentless.

While this may seem to be an unwelcome consequence of the present analysis, it is in fact well known that **much** in particular can serve as a semantically contentless dummy element. The first example of this involves the previously discussed phenomenon of **much** support (Corver 1997): When a gradable adjective is pronominalized with **so**, it cannot combine directly with a degree modifier; instead, a ‘dummy’ **much** must be inserted:

(5) **Fred is generous; in fact, he is too much so / so much so** that it worries me.

The puzzle here is that if **so** is a pronominal copy of the adjective, **much** does not appear to make any semantic contribution at all.

A similar issue comes up in the context of analytic comparatives formed with **more**. (57a) and (57b) are parallel in interpretation, involving the comparative forms of smart and intelligent, respectively, implying that **more** and -er are semantically equivalent. But if **more** in (57b) is analyzed as **much** + -er, as has been assumed up to this point, we again seem to have an ‘extra’ **much** without semantic content.
(57) a. Mabel is smarter than Fred.
   b. Mabel is more intelligent than Fred.

Under the present analysis, the *much* in these contexts is not at all exceptional. Since *much* is essentially vacuous, it can function as a dummy element. Take a relevant example to have the surface structure in (58a) (where the AP has the QP *too much* in its specifier position) and the corresponding LF in (58b). Then the interpretation in (58c) is equivalent to one in which *much* is not present.

(58) He is too much so *generous*.
   a. SS: He is \[[AP \textit{QP \textit{DegP}} \textit{too} \textit{much}] \textit{generous}\]
   b. LF: \[[\textit{DegP} \textit{too}]_2 [[\textit{QP} \textit{t}_2 \textit{much}]_1 \textit{he is} \textit{[AP} \textit{t}_1 \textit{generous}]]\]
   c. \[[\textit{too}_2](\textit{t}_2 \textit{much})(\lambda d_1. \textit{he is} d_1 \textit{generous})\]
   \[[\textit{too}_1](\lambda d_1. \textit{he is} d_1 \textit{generous})\]

Comparatives with *more* can be handled identically:

(59) Mabel is more intelligent than Fred.
   a. LF: \[[\textit{DegP} \textit{-er than Fred}]_2 [[\textit{QP} \textit{t}_2 \textit{much}]_1 [\textit{IP} \textit{Mabel is} \textit{t}_1 \textit{intelligent}]]\]
   b. \[[\textit{-er than Fred}_2](\textit{t}_2 \textit{much})(\lambda d_1. \textit{Mabel is} d_1 \textit{intelligent}))\]
   \[[\textit{-er than Fred}_1](\lambda d_1. \textit{Mabel is} d_1 \textit{intelligent})\]

Thus under the approach to Q-adjectives developed here, the ‘dummy’ *much* of *much* support and *more* comparatives can be aligned in a straightforward way to *much* in its apparently contentful occurrences. This represents an advantage for the present theory over previous treatments of Q-adjectives, including that of Rett (2008), the existing work closest in spirit to the present one. Based on somewhat different data than considered here, Rett proposes that Q-adjectives denote functions that map a set of degrees $I$ to the singleton set containing the degree that represents the measure (i.e. the length) of $I$. Like the account developed here, this analysis allows a unified treatment of the quantificational and (adjectival) differential uses. But in that it attributes semantic content to *much*, it cannot be extended directly to *much* support. Instead, Rett (like Corver and other past authors) must posit a separate contentless *much* for this case (and something similar for the *many* of *how many?* questions). The present account does not require us to assume such a duality.

There is, however, an issue that arises in this connection, namely that the present analysis seems to overgenerate. Specifically, we incorrectly predict the grammaticality of examples of the form *much tall*, where *much* (in either bare or modified form) occurs as a modifier of a gradable adjective. Such examples would have the surface structure and LF in (60a,b), and could be interpreted in essentially the same way as the *much* support cases, per (60c).

(60) *John is too/so/as/⊘ much tall.*
   a. SS: John is \[[AP \textit{QP \textit{DegP} too/so/as/POS} \textit{much}] \textit{tall}]\]
   b. LF: \[[\textit{DegP} \textit{too/so/as/POS}]_2 [[\textit{QP} \textit{t}_2 \textit{much}]_1 \textit{[IP} \textit{John is} \textit{[AP} \textit{t}_1 \textit{tall}]]]\]
   c. \[[\textit{too/so/as/POS}_2](\textit{t}_2 \textit{much})(\lambda d_1. \textit{John is} d_1 \textit{tall}))\]
   \[[\textit{too/so/as/POS}_1](\lambda d_1. \textit{John is} d_1 \textit{tall})\]

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While I do not have a full explanation for this gap, there is evidence that a form of blocking plays a role. Note first that in the tall case, much is not needed, in that gradable adjectives can combine directly with degree morphology. That is, the following structure, featuring a DegP in the specifier position of the AP, is possible (cf. Section 3.1), and furthermore yields the same interpretation as the corresponding structure with much in (60a):

(61) John is \[AP [DegP too/so/as/POS] tall]\n
In contrast, in all of the grammatical much examples we have seen, the presence of much is obligatory. This is well known to be true for much support and more comparatives, where eliminating much results in ungrammaticality (*too/so/that so generous; *intelligenter), but the same point can be made about the quantificational, differential and adverbial uses, where again degree modifiers cannot appear without much (so/too/as *(much) water; so/too/as *(much) taller than Mary; I slept so/too/as *(much)). Taking the differential use as an example, in these cases the QP headed by much cannot be replaced by a simpler ‘bare’ DegP in the same position:

(62) a. John is \[AP [DegP [QP [DegP too/so/as/POS] much] -er than Mary] tall]\n    b. *John is \[AP [DegP [DegP too/so/as/POS] -er than Mary] tall]\n
This pattern can be summarized by saying that QPs headed by much occur in only those positions where bare DegPs cannot. This can be formulated in terms of the following blocking principle, which has the effect of ruling out *much tall.

(63) \[DegP \alpha \rightarrow * [QP [DegP \alpha] much]\nA QP headed by much is blocked in contexts where a DegP is allowed.

Here it might initially seem that the case where there is no overt degree morphology represents a counterexample, in that for example both (64a,b) are grammatical. If these are taken to have the underlying structures in (62a,b), respectively, in both cases including a DegP headed by the phonologically null \textit{POS}, this would contradict the claim that the availability of (62b) blocks (62a).

(64) a. John is much taller than Mary.
    b. John is taller than Mary.

But a case can be made that this is not in fact the correct analysis of (64b). If \textit{POS} (as defined in (29)) were present in (64b), its meaning would be paraphrasable as ‘John’s height exceeds Mary’s by a large degree (relative to the context)’. This is the correct paraphrase for (64a) – indicating \textit{POS} is present here – but it is not a possible interpretation for (64b). Rather, the latter means only that John’s height exceeds Mary’s by some positive degree \(d\); as noted in Section 4.1, I take this to arise via existential quantification over the differential degree slot of -er, rather than a (null) degree morpheme. Thus a DegP headed by \textit{POS}, like one with an overt degree morpheme, cannot occur in the configuration in (62b), allowing (62a) to surface.

What remains to be explained is the restricted distribution of degree morphemes themselves: Why do DegPs occurring as specifiers of APs and QPs, but not in other contexts? To this I do not have answer, though I refer the reader to Doetjes (1997) and Neeleman et al.
(2004), who investigate similar data, and starting from somewhat different syntactic and semantic assumptions than those adopted here develop an account based on selectional restrictions introduced by Deg heads. I have to leave it as an open question whether a similar account could be worked out within the present framework.

4.4 Summary

The second of the issues discussed in Section 2.2 related to uses of Q-adjectives where they neither quantify over nor are predicated of individuals. In this section we have considered several of these cases, and shown that they can be handled by the same semantics as applied to the quantificational case, namely where the Q-adjective is predicated of a set of degrees or scalar interval. Even the phenomenon of much support falls out as another case of this more general pattern; there is no need to posit a separate ‘dummy’ much. Thus the present theory has in this respect broader empirical coverage that either of the standard views discussed at the start of this paper.

At this point, there is however an alternate possibility that might be considered, one based on a generalization of the previously discussed cardinality predicate analysis of Q-adjectives. Specifically, we might take Q-adjectives to denote gradable predicates over entities of underspecified type, which introduce a generalized version of a measure function that assigns degrees not just to individuals, but also to events and scalar intervals (and perhaps other sorts of entities as well):

\[
\text{\[many/much\]}^{ge} = \lambda d \lambda \alpha. \mu_S(\alpha) \geq d
\]
where \(\alpha\) ranges over individuals, events and scalar intervals

Few/little might then be analyzed as composed of many/much + degree negation.\(^9\)

Such an approach would be similar to proposals by Wellwood et al. (2012); Nakanishi (2007) and others, which are based on a parallel between measurement of individuals and events. It could seemingly provide a unified analysis of the quantificational, differential and adverbial cases discussed so far, as well as the adjective-like uses, all of which would involve many/much providing a measure of some sort of entity.

In the next section, however, evidence will be presented that, despite appearances to the contrary, Q-adjectives are in fact never predicated directly of individuals (i.e., of entities of type \(e\)). Such a conclusion makes a generalized version of the predicate-over-individuals theory less attractive as a general solution to the semantics of Q-adjectives.

5 Adjective-like uses

The strongest motivation for adjectival analyses of Q-adjectives is that they occur in canonical adjectival positions, specifically as sentential predicates and in post-determiner position. To repeat our original examples:

\[\begin{align*}
(1) & \quad \text{b. John’s friends are many/few.} & \text{\underline{predicative}} \\
& \quad \text{c. The many/few students who attended enjoyed the lecture.} & \text{\underline{attributive}}
\end{align*}\]

\(^9\)I thank an anonymous reviewer for suggesting this possibility.
Under the cardinality-predicate approach, the analysis of such examples is straightforward. (1b) would involve many/few being predicated of the maximal plurality of John’s friends. In (1c), many/few would first compose intersecively with the nominal expression students who attended. Adopting a fairly standard approach to the definite article as a maximality operator, the DP in (1c) would come to denote the maximal set of students we invited, with the presupposition that this group is small in number.

\[(66)\]
\[\text{a. } \left[\text{NP few students who attended}\right] = \left[\text{few}\right] \cap \left[\text{students}\right] \]
\[= \lambda x. \text{student}(x) \land \text{attended}(x) \land |x| \leq m_c \]
\[\text{b. } \left[\text{DP the few students who attended}\right] = \]
\[= \iota \lambda x. \text{student}(x) \land \text{attended}(x) \land |x| \leq m_c, \]
\[\text{where } \iota P = \text{max}(P) \text{ if it exists; otherwise undefined} \]

This among other things captures the backgrounded status of the information introduced by the Q-adjective (for example, in the few students who attended didn’t enjoy the lecture, it is the enjoyment that is negated, not the small size of the group).

For the present theory, on the other hand, such examples seem problematic. In that Q-adjectives denote predicates of sets of degrees, they cannot be predicated directly of pluralities of individuals. Nor can they combine intersecively with predicates over individuals. In fact, using just the mechanisms introduced so far, Q-adjectives cannot be interpreted at the DP level at all, but rather must take sentential scope.

On the other hand, in Section 2.2 we saw evidence that Q-adjectives do not always behave like ordinary gradable adjectives (the third of the four issues discussed in that section). Some further divergences between the two classes will be seen below. The conclusion will be that the totality of the data is not consistent with the cardinality predicate approach, but rather supports the degree-based theory developed here.

5.1 Predicative Q-adjectives

Let us start by examining some patterns of acceptability. What has not to my knowledge been previously observed is that predicative many and few are allowed with a restricted class of subjects: bare plurals, definites, possessives, demonstratives and wh-pronouns (67a-g). They are disallowed with quantificational subjects, plural indefinites and conjoined referring expressions (67h-j).

\[(67)\]
\[\text{a. Clues to the suspect’s identity were many/few.} \]
\[\text{b. The advantages of the new treatment are many/few.} \]
\[\text{c. John’s good qualities are many/few.} \]
\[\text{d. His friends were many/few.} \]
\[\text{e. As for occurrences of side effects, those cases were many/few.} \]
\[\text{f. As for the advantages of the new treatment, those are many/few.} \]
\[\text{g. . . . the advantages of the new treatment, which are many/few} \]
\[\text{h. *All/most/both lawyers are many/few.} \]
\[\text{i. *Some lawyers are many/few.} \]
\[\text{j. *Fred, John and Frank are many/few.} \]
Such restrictions are of course not found with ordinary adjectives in predicative position:

(68)  a. Every/all/most/both/some lawyer(s) is/are greedy.
      b. Fred, John and Frank are greedy.

But a very similar pattern is observed in noun phrases of the form ‘the number of DP’:

(69)  a. The number of clues to the suspect’s identity was large/small.
      b. ?The number of the advantages of the new treatment is large/small.
      c. The number of John’s good qualities is large/small.
      d. The number of his friends was large/small.
      e. As for occurrences of side effects, the number of those cases was large/small.
      f. As for the advantages of the new treatment, the number of those is large/small.
      g. ... the advantages of the new treatment, the number of which is large/small.
      h. *The number of every/all/most/both lawyers is large/small.
      i. *The number of some people is large/small.
      j. *The number of Fred, John and Frank is large/small.

A further parallel between Q-adjectives and ‘the number of’ is seen in how they combine with bare plurals. With an unmodified plural noun and a verb in the present tense, both a predicative Q-adjective and a noun phrase headed by ‘the number of’ are slightly infelicitous in an out of the blue context (70). But when the noun is modified, and the verb is in a past tense form, favoring an episodic reading, both improve (71).

(70)  a. ?Trees are few.
      b. ?The number of trees is small.

(71)  a. Trees that survived the drought were few.
      b. The number of trees that survived the drought was small.

A similar effect can obtain contextually. For example, in the context of a discussion about the effects of a drought on the plant life of a region, both (70a,b) become quite acceptable.

The relevant factor here is that the plural nominal must have a specific rather than kind interpretation, denoting a particular spatially and temporally bounded plurality of individuals rather than the kind as a whole. That is, in (71a,b) and the felicitous use of (70a,b), it is a specific set of trees, localized in space and time, whose number is characterized as small.

The inability of Q-adjectives to take kind-denoting bare plurals as subjects is further demonstrated by examples such as the following, which suggest that many, unlike its near synonym numerous, does not participate in direct kind predication (Krifka et al. 1995):

(72)  Cockroaches are widespread/extinct/numerous/??many.

10Note that the bare plural subjects in (70) and (71) also do not have the existential reading typically found with stage-level predicates (Carlson 1977b). (71a) for example does not mean that there exists some surviving set of trees whose number is small, but rather that the totality of such trees is small in number. I am not aware of much work on this reading, though it seems related to the universal interpretation of bare plurals discussed by Condoravdi (1994).
Let us take these parallels seriously, and conclude that the felicitous examples in (67a-g) include some null structure corresponding to ‘the number of’. Specifically, the facts can be accounted for if we take the subjects in the grammatical examples to have a null MeasP layer above the DP:

(73) John’s friends are many.
     \[ IP [MeasP \text{Meas} [DP John’s friends]] \text{are many} \]

Recall that the Meas head encodes a measure function:

(35) \[ \text{Meas}^g = \lambda x_\epsilon \lambda d_\mu_S(x) \geq d \]

On this definition, an expression of type \( e \) can saturate the first argument of the measure function, yielding an expression of type \( \langle dt \rangle \) that represents the measure (quantity or amount) of the entity in question. Applied to John’s friends, for example, it returns the set of degrees \( d \) such that the number of John’s friends is at least \( d \) (74). As this is the appropriate type to serve as argument for the Q-adjective, the derivation may proceed. (75a) thus receives the interpretation in (75b), specifying that the number of John’s friends exceeds the relevant contextually determined standard. This accurately captures the sentence’s meaning.

(74) \[ \text{Meas John’s friends}^g = \text{Meas}^g (\sqcup \lambda x.\text{friend}(x, John)) = \lambda d.\mu_S (\sqcup \lambda x.\text{friend}(x, John)) \geq d \]

(75) a. John’s friends are many.
     b. \[ [\text{POS}]([t_1 \text{many}]([\text{Meas John’s friends}])) = \forall d \in \mathbb{N}^\# [\mu^\# (\sqcup \lambda x.\text{friend}(x, John)) \geq d] \]

Thus again the Q-adjective ends up being predicated of a set of degrees. The difference versus the cases discussed earlier is that here, the set is formed not as the result of lambda abstraction over a type \( d \) trace, but rather by the application of Meas to an entity-denoting expression.

This analysis yields insights into the restrictions illustrated in (67). Meas requires a referential (type \( e \)) argument, allowing definites and pronouns in its complement position. Quantificational noun phrases such as all lawyers, by contrast, are of the wrong type (type \( \langle et, t \rangle \)). This might plausibly be resolved via QR, but the trace left behind would be of the type of an atomic individual, which would yield an odd interpretive effect. Specifically, all lawyers are many/few would receive an interpretation that could be paraphrased as ‘for all \( x \) such that \( x \) is a lawyer, the cardinality of \( x \) exceeds (many)/falls short of (few) the neutral range’. But the cardinality of an atomic individual is necessarily one, and as such this sentence is entirely uninformative. Similar reasoning might also account for the restriction to specific rather than kind-denoting bare plurals, if we take kinds to be atomic individuals (cf. Chierchia 1998b), which therefore lack countable subparts. The infelicity of Q-adjjectives with conjoined singular subjects (e.g. (67j)) remains somewhat puzzling on this account. But the parallel here between Q-adjunctive and ‘the number of’ suggests more general syntactic factors may come into play; alternately, one might consider the possibility that such conjoined NPs denote something other than pluralities of individuals (cf. Heycock and Zamparelli 2000).

In Section 1 it was observed that much/little do not occur predicatively in constructions parallel to (1b):
This is further illustrated by the following, which are based on previously discussed examples of predicative many/few. While their felicity varies somewhat, for reasons I do not fully understand, the contrast to the corresponding many/few sentences is quite robust (here I include much in a negated sentence to show that its infelicity is not due entirely to its NPI-like character).

(76)  
   a. *Vegetation that survived the drought was little/was much/wasn’t much.  
   b. *Evidence as to the suspect’s identity was little/was much/wasn’t much.  
   c. ??The benefit of the new treatment is little/is much/isn’t much.  
   d. *John’s patience is little/is much/isn’t much.

This might seem to argue against the possibility of extending the above analysis of predicative many/few to much/little. But note that the following are quite acceptable:

(77)  
   a. The amount of water in the bucket was little/wasn’t much.  
   b. Ten gallons is very little/not much.

The difference is that here the sentential subject is an amount. I take it that in both cases, the amount is derived via the application of a measure function of the form encoded by Meas to an individual, and thus has the formal status of a set of degrees. Thus when the subject has the right semantic type – type ⟨dt⟩ – much and little can occur predicatively.

The issue with (2b) and (76) must then be that the subjects in these cases cannot be interpreted at the appropriate type, in contrast to what is possible in the felicitous many/few examples. As to why this is the case, the following contrast is relevant:

(78)  
   a. Side effects were fewer than expected.  
   b. *Side effects were more than expected.

Fewer is the comparative form of few, and thus specifies that the dimension involved is cardinality. But more is the comparative of both many and much, and thus does not specify a particular dimension (cf. Bale and Barner 2009). This suggests the possibility that what is wrong with (2b) and (76) is likewise that the dimension of measurement is not made overt, in contrast to examples with many and few, which themselves encode cardinality as a dimension. While this is admittedly somewhat speculative, I hypothesize this to be a licensing condition on the null Meas projection: an apparently entity-denoting expression can be interpreted as a MeasP (i.e. can be interpreted as denoting a quantity or amount) only if the dimension of measurement is made overt.

Additional support for the proposal that Q-adjjectives are predicated of something in the domain of degrees is provided by data relating to their occurrence in the complement position of the attitudinal verb consider. As pointed out in Section 2.2, the felicity of predicative many/few in main clauses such as (1b) does not carry over to this position:

(21a)   *I consider the guests many/few.

But once again, when the subject has a degree- rather than individual-based denotation, Q-adjjectives can in fact occur predicatively under consider:
(79) a. (When it comes to students,) I consider twenty many.
   b. I consider 20 kg too much.

The *consider* construction has been used as a test for predication (see e.g. Partee 1987). Thus these facts provide further evidence that in the present case, the predication relation obtains not between Q-adjectives and individuals themselves, but between Q-adjectives and quantities/amounts associated with individuals. As to the source of the ungrammaticality in (21), this can be accounted for if the relevant examples are analyzed not as small clauses, but rather as involving a predication relation encoded by the verb *consider* itself (see Hoeksema 1994 for arguments against a small clause treatment). Taking *consider* to have semantics along the lines of (80), the grammaticality of (79a,b) is expected, while the the ungrammaticality of (21a) follows as the result of a type mismatch.

(80) $[\text{consider}] = \lambda x, \lambda y. y$ holds it to be the case that $P(x)$

To summarize, Q-adjectives can occur predicatively, but several constraints on this possibility point to the conclusion that they are not in fact predicated directly of individuals. Rather, the subject of predication must be the measure of an individual (formalized as a set of degrees), consistent with the overall theory developed here.

5.2 Attributive Q-adjectives

The final and seemingly most problematic case to consider involves the attributive use of Q-adjectives.

(1c) The many/few students who attended enjoyed the lecture.

As noted above, on the present analysis Q-adjectives cannot be interpreted DP internally, but rather must take sentential scope. But there is evidence that such an analysis will not work for examples such as (1c). The relevant facts involve negative polarity licensing. As seen in (81a,b), the negative *few* on its quantificational use licenses NPIs in both its NP sister and the sentential predicate, as is expected given that both are in its scope (cf. (39)). Attributive *few* similarly licenses an NPI in the noun phrase, but crucially not in the sentential predicate (81c,d). Thus in this case the scope domain of the Q-adjective must be within the DP, and not sentential.

(81) a. Few people who ever visited made donations.
   b. Few people who visited ever made donations.
   c. The few people who ever visited made donations.
   d. *The few people who visited ever made donations.

On the surface, these data might seem to favor the cardinality predicate analysis over the present one, in that it would allow the Q-adjective to be interpreted *in situ* within the DP. However, closer scrutiny reveals facts that are not accounted for by this approach.

First of all, an analysis of *few* as a cardinality predicate does not obviously predict the grammaticality of (81c). On this view, the Q-adjective composes intersectively with the remainder of the nominal expression (see (66) above), such that the latter is not within its scope, as would be needed for NPI licensing. Nor can the definite article be
responsible for the felicity of the NPI here, as the itself in this context is not an NPI licensor (cf. *the people who ever visited made donations).

Also unexplained on the cardinality predicate analysis is that attributive Q-adjectives are possible only with a limited set of determiners, namely definites and possessives:  

(82)  
\begin{itemize}
  \item a. The/those few/many students we invited  
  \item b. His few/many friends  
  \item c. *Some/all/both/most/each few/many students we invited
\end{itemize}

Finally, there is evidence that examples featuring attributive Q-adjectives actually have a more complex structure than that implied by the intersective cardinality predicate analysis, and in particular that the relevant DPs contain clausal material. This is most clearly shown by the fact that they may include sentential adverbs:

(83)  
\begin{itemize}
  \item a. The fortunately few families who were displaced by the flood  
  \item b. The frankly few really good computer games released this year  
  \item c. The undoubtedly many business travelers who need wireless internet access
\end{itemize}

Attributive Q-adjectives also can be paraphrased with a clausal structure, namely a relative clause, as in the following paraphrase of our original example. Like (84), (1c) attributes the property of many-ness/few-ness to the cardinality of the maximal group of students who attended.

(84) The students who attended, who (*that) were many/few, enjoyed the lecture.

As an aside, note that the relative clause in (84) is nonrestrictive rather than restrictive, as evidenced by the need for comma intonation and the unavailability of that as a complementizer. This is somewhat at odds with the cardinality predicate analysis, which treats the Q-adjective as a restrictive modifier.

In the properties discussed above, DPs containing attributive Q-adjectives display parallels to a class of relative clause constructions that Grosu (2002) calls ‘maximizing relatives’, whose members include relatives out of there existentials, amount relatives, free relatives, and (in some languages) internally headed relative clauses (see also Carlson 1977a; Grosu and Landman 1998; Heim 1987; McNally 2008). Like the case discussed here, these have a maximal or exhaustive interpretation, and are restricted to occurring with certain determiners, generally definites and universal quantifiers. Furthermore, constructions of this sort optionally allow an attributive Q-adjective, as illustrated below:

(85)  
\begin{itemize}
  \item a. I took with me the (many) books that there were on the table. \textit{there rel.}  
  \item b. We lost the battle because we didn’t have even the (few) canons our enemy had. \textit{amount rel.}
\end{itemize}

\footnote{Here I put aside the collocation \textit{a few}, and the marginally acceptable \textit{some few}. As evidence that these are different from the cases under consideration, note that they do not license NPIs (e.g. *A few people who ever visited). For discussion of the relationship between \textit{few} and \textit{a few}, see Solt (2009).}

\footnote{On the basis of this and other parallels, in previous work I concluded that attributive Q-adjectives are nonrestrictive modifiers, parallel to prenominal adjectives on their nonrestrictive use. I thank a reviewer for pointing out some issues with this approach, leading me to the revised account presented here, which nonetheless preserves some of the same insights.}
c. What (little) rain fell was not enough to save the crops.

This suggests an analysis of attributive Q-adjectives that in some way aligns them to this class of relative clauses more generally.

This connection was already noted by Hackl (2000), who argues that DPs featuring a Q-adjective in attributive position have the underlying structure of there relatives, such that the many/few students is covertly the many/few students that there were. Hackl does not, however, provide a formal analysis of examples of this sort, nor is there a standard treatment of there relatives that readily captures the facts relating to Q-adjectives. Below I sketch out a possible analysis based on one established approach to maximizing relatives, involving E-type anaphora (see especially Shimoyama 1999 for this sort of analysis of internally headed relative clauses, and von Fintel 1999 for a suggestion that it be extended to there relatives). I assume that the basic idea could, if desired, be adapted to some other treatment of the there construction.

Informally, the analysis I propose derives for our original example an interpretation that could be paraphrased as follows, where the Q-adjective is part of the content of a separate assertion.

(86) There were few students, who attended. The students, enjoyed the lecture.

Formally, I propose that the nominal expression containing the Q-adjective originates relative-clause internally, and is reconstructed into its source position (87a). Here, as would be the case in a matrix there sentence, the Q-adjective must QR, yielding (87b) (where for simplicity I show POS + few as a unit).

(87) the few students (that there were) who attended

a. the \[NP\_\_[CP that [IP there were [MeasP POS-few Meas students] who attended]]\]

b. the \[NP\_\_[CP that [IP POS-few, [[IP there were [MeasP t_i Meas students] who attended]]]\]

Taking the postverbal nominal in there existentials to have a property type (McNally 1998), and analyzing there be to in some way introduce existential quantification, the lower IP in (87b) receives the interpretation in (88a). Just as in previous cases, abstraction over the type d trace and application of POS + few yields (88b), which specifies that the number of students who attended was small relative to the context.

(88) a. \[\exists x [^\star student(x) \land \mu\#(x) \geq d_i \land attended(x)]\]

b. \[\forall d \in N_\# [\neg \exists x [^\star student(x) \land \mu\#(x) \geq d \land attended(x)]]\]

Importantly, the Q-adjective here has scope over the entire DP-internal there clause, and can thus license an NPI within it (cf. (81c)), just as would be the case in a matrix there existential (e.g. there were few students who ever attended).

As the last step, the remainder of the DP is interpreted via an overt counterpart of E-type anaphora (Heim and Kratzer 1998), as a definite description based on a null type \(\langle et\rangle\) proform, whose content is recovered from that of the relative clause. In this case, the proform is resolved to the property of being a plurality of students who attended, such that the DP comes to denote the maximal plurality of student attendees, with the relative clause contributing the presupposition that this group is small in cardinality:
As with E-type anaphora generally, the interpretation of the DP is necessarily definite. As such, only the definite article and other determiners that encode definiteness can occur in this position (per (82)).

To summarize, once we conclude that DPs featuring attributive Q-adjectives have an underlying clausal structure, the present degree-based analysis can be applied to these as well. Specifically, the Q-adjective can take scope over the DP-internal clause just as it would in a matrix sentence, from which position it can among other things license an NPI within this clause.

Provisionally I take it that this analysis could be extended in some way to also handle examples such as (85b,c), but I do not attempt to work out the details here. There is however one further case that merits discussion, involving attributive Q-adjectives in possessive constructions, as in (90). Here, the content of the upstairs DP layer is more than a definiteness marker, including also the possessor. This does not fit readily with an analysis patterned after (89). It seems that we must conclude that here the possessor John is interpreted as part of the NP that originates relative clause internally, along the lines of Elbourne (2001), who analyzes a DP such as his paycheck for the purposes of anaphora as the paycheck of him. But note that (90a) as an attempted paraphrase of the DP in (90) is quite awkward. Rather, the appropriate relative clause paraphrase seems to be (90b), suggesting what we in fact have in this case is extraction from a covert relative have clause.

(90) John’s many friends supported him.
   a. ??The many friends of John that there are
   b. The many friends that John has

That this is on the right track is supported by the fact that extraction from the object position of light verb have exhibits the same determiner restrictions as does extraction from there existentials (e.g. the/every/those/*some/*three window(s) that the house had). I leave a more in-depth investigation of this pattern to future work.

To wrap up this section, the predicative and attributive uses of Q-adjectives on the surface provide the strongest support for a cardinality predicate analysis, and the greatest challenge to the theory developed in this paper. A closer look, however, reveals that the relevant data can be accommodated by the present degree-based analysis, and further that this approach can provide explanations for facts not accounted for if Q-adjectives are treated as predicates of individuals. With this, the goal of achieving a unified analysis of the data set in (1)-(5) has been achieved.

6 Type ambiguity reconsidered

The central proposal put forward in this paper is that Q-adjectives have a single lexical entry underlying all of their quite varied uses. In the present section, I would like to return briefly to contrast this to an alternative possible analysis, according to which Q-adjectives have multiple interpretations at distinct semantic types, connected by some set of general type shifting rules. In that such an approach could potentially allow us to
do away with certain more innovative aspects of the present proposal (e.g. the silent but
costful element Meas), it is worthwhile considering how it might be made to work.

The first observation to be made in this regards is that there is to date no type-
shifting-based theory with the empirical coverage of the one developed here. This is not
to say that such a theory could not be developed. But considering just the more limited
proposals available in the literature is enough to illustrate some of the challenges that
such an undertaking is likely to face.

Let us begin with the possibility, discussed earlier, that Q-adjectives have interpreta-
tions as both quantifiers over individuals and predicates over individuals (putting aside
for now the other issues that have been raised about such a view). The typical starting
point for relating such denotations is Partee’s (1987) flexible-type framework, in which
noun phrases have not one but a family of denotations, connected by a small set of type
shifting operations (see especially Partee 1989; De Swart 2001 for work on Q-adjectives,
and Landman 2004 for a proposal focusing on numerical expressions). We have already
seen that such approaches run into difficulties in the case of few: While a predicative
interpretation of a noun phrase with many can be mapped to a quantificational interpre-
tation via Partee’s $A$, the same operation applied to predicative few produces incorrect
results:

\begin{align}
(91) & \quad A([\text{many students}]) = \lambda Q(\exists x [\text{student}(x) \land |x| > n_c \land Q(x)]) \checkmark \\
& \quad A([\text{few students}]) = \lambda Q(\exists x [\text{student}(x) \land |x| < m_c \land Q(x)]) \times
\end{align}

A similar issue arises in the other direction, i.e. quantifier to predicate. De Swart
(2001) introduces a version of Partee’s $BE$ operation which when applied to a plural
quantificational noun phrase derives a predicate over plural individuals. The following
shows its effect on noun phrases based on many and few.\(^{13}\)

\begin{align}
(92) & \quad BE(P_{(\text{et,})}) = \lambda x. (\lambda y. y = x) \\
(93) & \quad BE([\text{many students}]_{(\text{et,})}) = \\
& \quad = \lambda x. [\lambda Q. \exists z [\text{student}(z) \land |z| > n_c \land Q(z)] (\lambda y. y = x)] \\
& \quad = \lambda x. \exists \text{student}(x) \land |x| > n_c \checkmark \\
& \quad BE([\text{few students}]_{(\text{et,})}) = \\
& \quad = \lambda x. [\lambda Q. \exists z [\text{student}(z) \land |z| > m_c \land Q(z)] (\lambda y. y = x)] \\
& \quad = \lambda x. \exists \neg \text{student}(x) \land |x| > m_c \times
\end{align}

In the many case, the result is a predicate that is true of a plural individual if it is
composed of a large number of atoms, each of which is a student; in set terms, it is the
set of large pluralities of students. This is intuitively correct. Furthermore, as discussed
in the preceding section, a predicate of this form can provide the basis for an analysis of
attributive many, if the is analyzed in terms of a maximality operator (cf. the discussion
of example (66)). But in the few case, the predicate derived is, in set terms, the set of
pluralities that are not both composed of students as atoms and of cardinality greater than
$m_c$. That is, we derive a set that contains individuals other than pluralities of students.
This is not only counterintuitive, but also fails to provide for the correct analysis of
attributive few; with the as defined earlier, the few students comes to denote a plurality

\(^{13}\)Note that here the quantificational entries for many and few given originally in (11) have been
replaced by equivalent entries stated in terms of plural individuals.
including non-students as atoms (in fact, if the domain contains any non-students, it denotes the entire domain).

Within the rather small literature on this topic, the typical approach to resolving this problem is to introduce additional or more powerful type-shifting operations to supplement or replace the simple ones discussed above. In the predicate-to-quantifier direction, De Swart 2001 reserves the existential shift (what I have called $A$; her Existential Closure) for lower-bounded predicates such as *many*, and introduces a second operation, Universal Closure, which applies to upper-bounded predicative expressions such as *few*. Somewhat similarly, Landman 2004 supplements Existential Closure with a Maximalization operation, with the two operations together guaranteeing correct results in both the upper- and lower-bounded cases. Conversely, in the quantifier-to-predicate direction, Landman notes that the correct results in both the upper- and lower-bounded cases can be obtained by assuming a more powerful rule that looks ‘inside’ the noun phrase to consider the denotation of the head noun itself (such that (93b) would be strengthened to include the specification that the pluralities in question be pluralities of students).

However, such modifications add complexity to a system which in Partee’s original formulation was much simpler. The issue is not just the number of rules and their individual complexity, but the need to constrain their application appropriately. On an approach like de Swart’s, the challenge is to explain which of the two predicate-to-quantifier shifts will apply to a given predicate: Why, for example, is Existential Closure not able to apply to a noun phrase with *few*, or Universal Closure to one with *many*? In de Swart’s own proposal the quantificational interpretation is the primary one and the predicative one derived from it, such that the choice of closure rule is determined by the monotonicity properties of the original quantifier; but this hinges on it being possible to formulate an appropriate quantifier-to-predicate shift (which we have seen is also problematic), and is of no help if the predicative entries are taken as primary. Landman’s rule here has an advantage, in that it would handle both *many* and *few* with no further stipulations; but Landman must introduce an additional mechanism to prevent the application of Maximalization to noun phrases formed with cardinal numerals (e.g. 3 boys), as this would incorrectly block an ‘at least’ reading.

What makes this undertaking particularly questionable is that the extra complexity that must be introduced – in the form of additional rules and constraints on their application – is of limited benefit beyond the small domain of Q-adjjectives themselves. Recall that the issues came up in the context of the monotone decreasing *few*. While *few* and *little* are not the only monotone decreasing quantificational expressions in English, the others are for the most part morphologically complex, and in other recent work have been subject to more sophisticated analyses that in some way decompose them into a positive expression and a wide scope negative or decreasing element. Expressions of the form at most $n$ have been analyzed as operators over degrees (Nouwen 2010) or speech acts (Cohen and Krifka 2011). Similarly, Hackl (2000) decomposes fewer than $n$ into *many* plus a negative comparative element that must take wide scope (a related approach to that developed in the present paper). Even negative indefinites such as *nobody* and *no doctor* have been decomposed into a positive expression and a sentential negation operator (Jacobs 1980; Zeijlstra 2004; Penka 2011). With these mechanisms available, the more

14 In de Swart’s system these are closure rules rather than shifting operations on noun phrase denotations themselves; but equivalent type shifting rules could be formulated.
complex predicative/quantificational type shifts required to produce the right results for bare few and little have no role beyond these two items.

A second and seemingly more parsimonious approach to implementing a multiple-type approach for bare few and little involves extending the sentential negation analysis to these expressions as well. Along these lines, McNally (1998), following Ladusaw (1992), analyzes few as a cardinality predicate equivalent to many but with the requirement that it occur in the scope of a phonologically null sentential negation operator. Combining this with a type shift of the form of Partee’s A yields the following interpretation for a quantificational example, which (correctly) states that the number of students attending fell short of some contextually determined threshold value.

(94) a. Few students attended the lecture.
   b. \( \neg \exists x \left[ \text{student}(x) \land \text{many}(x) \land \text{attended}(x, \text{lecture}) \right] \\
      = \neg \exists x \left[ \text{student}(x) \land |x| > n_c \land \text{attended}(x, \text{lecture}) \right] \\

Thus the wide-scope-negation approach seems to allow us to begin with a cardinality predicate entry for few and derive the correct quantificational semantics, with no need for type shifting operations beyond a simple existential shift. But a problem reemerges when we consider few modified by very: (95a) receives an interpretation that renders it equivalent to not very many students attended the lecture, clearly not the right result.

(95) a. Very few students attended the lecture.
   b. \( \neg \exists x \left[ \text{student}(x) \land \text{very-many}(x) \land \text{attended}(x, \text{lecture}) \right] \\
      = \neg \exists x \left[ \text{student}(x) \land |x| \gg n_c \land \text{attended}(x, \text{lecture}) \right] \\
      \times \text{‘There was not a group of very many students (i.e. a group numbering much greater than } n_c \text{) who attended the lecture’}

The degree-based account developed in the present paper avoids these issues. We are able to assume just a single shifting operation based on an existential operator, here implemented as existential closure at the IP level. This, coupled with the degree-based semantics of Q-adjectives themselves, is sufficient to yield the correct semantics in the quantificational use. Furthermore, we obtain the correct results even in cases like (95a). Like McNally’s, the present account is a decompositional one, but what has wide scope over the the existential operator is not sentential negation but rather the negative operator few itself, along with its degree morphology. If very is taken to have semantics similar to POS, with the difference that it introduces a symmetrically wider neutral range \( N^+_{\#} \) (per Heim 2006; Stechow 2007), then (95a) receives the interpretation below, which may be paraphrased as saying that the number of students attending fell short of the extended neutral range – an intuitively correct result.

(96) a. very\( _2[t_2 \text{ few}_1[t_1 \text{ students attended the lecture}]] \)
   b. \( \forall d \in N^+_{\#} \left[ \neg \exists x \left[ \text{student}(x) \land \mu_\#(x) \geq d \land \text{attended}(x, \text{lecture}) \right] \right] \\
   c. \text{\# students attending} \\
   \begin{array}{c}
   \overbrace{N_{\#}}
   \\
   \overbrace{N^+_{\#}}
   \end{array} \rightarrow \text{NUMBER}
The preceding discussion has illustrated some of the challenges that arise in relating the hypothesized quantificational and predicative interpretations of Q-adjectives. Another set of issues arises when we attempt to relate their quantificational/predicative uses to their occurrences outside the nominal domain.

To make this more concrete, I will take a recent proposal in this area as a starting point. Brasoveanu (2008) argues that measure nouns such as liter and gram are systematically ambiguous between interpretations as predicates over scalar intervals and as predicates over individuals. Perhaps Q-adjectives participate in the same systematic correspondence, having both the degree-based interpretation proposed here (responsible for their differential and adverbial uses) and an interpretation as predicates over individuals (responsible in some way for their uses in the nominal domain). On the surface, this is an appealing idea – just as 2 liters might alternately describe a measure and a portion of liquid having that measure, so too might many or few alternately describe a number and a group of individuals with that number. A satisfying aspect of this approach is that the facts relating to Q-adjectives would not require a sui generis explanation, but could rather be subsumed under a broader umbrella.

What argues against such a proposal is that the correspondences that are actually observed are far less general than it would predict. If expressions of degree, measure and quantity have both degree-based and individual-based senses that are systematically related, we would expect, all things being equal, that an expression with an interpretation at one type will have a corresponding interpretation at the other type. But this prediction is not borne out. As was discussed in detail in the preceding section, Q-adjectives do not behave like ordinary gradable adjectives even on their predicative use, casting doubt on the claim that they in fact have interpretations as predicates over individuals. Conversely, ordinary gradable adjectives such as tall – the classic case of gradable predicates over individuals – do not obviously have interpretations as predicates of scalar intervals, as evidenced by the lack of a differential use (e.g. *tall taller than Fred). Even measure phrases, which share the differential use with Q-adjectives, also show divergences from many, etc., in particular being at best marginal in predicative position (*the water was 2 liters). In short, any account based on the existence of a systematic correspondence between degree- and individual-based senses of measure/quantity expressions will be faced with explaining the divergences between different subclasses of these items.

Under the analysis proposed here, these divergences derive from the basic lexical semantics of the different classes. Q-adjectives differ in distribution from ordinary gradable adjectives because they have different semantic types: Q-adjectives as gradable predicates of scalar intervals, adjectives of the tall class as gradable predicates of individuals. Regarding measure phrases I will have something to say below.

To summarize, it may be possible to develop a multiple-type theory of Q-adjectives that achieves the same empirical coverage as the present one. But the above discussion has highlighted the fundamental challenge that must be faced. Namely, if a set of type shifting rules is formulated that is powerful enough to account for all of the relevant data, the resulting system overgenerates, requiring additional mechanisms be introduced to constrain its application. To be sure, this challenge could potentially be met; but I suspect that doing so will involve at least as much machinery as the present approach. Until a principled account can be developed that achieves this, the present theory retains an advantage.
7 Conclusions

Q-adjectives are strange sorts of creatures, whose behavior defies attempts to align them to either quantifiers such as every or adjectives such as tall. I have made a case here that the idiosyncratic patterns of distribution and interpretation observed for this class derive directly from their lexical semantics. Q-adjectives are neither ordinary quantifiers nor ordinary adjectives (though they sometimes act like one or the other), but rather have degree-based semantics, denoting gradable predicates of sets of degrees or equivalently gradable quantifiers over degrees. The theoretical assumptions required to implement this basic idea are modest, and its empirical coverage surpasses that of alternate theories. As a whole, the results of this investigation add to other recent work (e.g. Hackl 2000, 2009; Nouwen 2010) in supporting the central role of degrees and scales in the semantics of quantity.

At the start of the paper it was noted that two other classes of quantity expressions, namely number words and measure phrases, show some of the same distributional flexibility that characterizes Q-adjectives, having quantificational, attributive and differential uses. This suggests that the current analysis should in some way be extended to these as well. While it is beyond the scope of this paper to consider this topic in depth, an obvious approach to capturing the similarity would be to analyze these expressions as simply denoting degrees. Twenty, for instance, would denote a degree on the scale of number, two liters a degree on the scale of volume, and so forth. This is a fairly well established view of the semantics of number words, though somewhat less so for measure phrases, where it is often assumed that the measure noun itself (e.g. liters, pounds, etc.) encodes a measure function (see e.g. Krifka 1989).

To briefly sketch out how this might fit within the framework developed here, with interpretations at type $d$ both numerals and measure phrases could saturate the degree argument slot of Meas, or the differential degree argument of the comparative morpheme, accounting for their quantificational and differential uses. The same would be possible in their attributive use, and as such this analysis would not commit us to (though would be compatible with) an analysis of attributive number words/measure phrases as always having an underlying clausal structure, as I have argued is the case with Q-adjectives. Finally, I noted above that measure phrases differ from Q-adjectives in being only marginal in predicative position. This extends to number words as well:

\begin{align*}
(97) \quad & \begin{array}{l}
\text{a. } \text{??John’s friends are three.} \\
\text{b. } \text{??The water was two liters.}
\end{array}
\end{align*}

This could potentially be explained with reference to their type, which in contrast to that of Q-adjectives is not predicative but rather referential.

Whether an account along these lines could be worked out fully is a topic for the future. If it could, though, it would further reinforce the degree-based approach to the semantics of quantity that I have advocated here.
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