THE SEMANTICS OF ADJECTIVES OF QUANTITY

by

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Abstract
THE SEMANTICS OF ADJECTIVES OF QUANTITY
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This work investigates the semantics of the adjectives of quantity (Q-adjectives) *many, few, much and little*, with the goal of providing a unified semantic analysis of this class, and in doing so exploring the semantics of quantity and degree more broadly.

My central claim is that Q-adjectives must be analyzed as degree predicates – gradable predicates of scalar intervals – with much of the semantic content traditionally ascribed to these words instead contributed by a set of null functional elements and operations. This proposal allows a compositional analysis of Q-adjectives across the wide range of syntactic positions in which they occur, including quantificational (*many dogs bark*), predicative (*John’s friends are few*), attributive (*the little rice that remains*), and especially differential (*many fewer than 100 students*), the latter of which is problematic for theories that take Q-adjectives to be either quantifying determiners or cardinality predicates. The same mechanism also accounts for the operator-like behavior of *few* and *little* and the availability of *much* as a dummy element (*much-support*), and allows quantification over individuals to be analyzed via simple Existential Closure, without giving rise to spurious ‘at least’ readings for *few/little*.
I further show that patterns in the interpretation and distribution of Q-adjectives can be related to properties of the scales of whose intervals they are predicated. The vagueness of Q-adjectives and their apparent cardinal/proportional ambiguity can be accommodated via the manipulation of two elements in the scalar representation, the structure of the scale (bounded vs. unbounded) and the location of the standard of comparison, with no need to posit multiple lexical entries. Aspects of scale structure are also responsible for contrasts in distribution among individual Q-adjectives (e.g. *a few vs. *a many; the problems were many vs. *the difficulty was much).

This thesis thus argues for the relevance of degrees and scales to the analysis of natural language meaning, while adding to recent work investigating subtle syntactic and semantic differences between superficially similar quantificational expressions, and developing compositional analyses of complex expressions of quantity. It further supports a view of nominal syntax in which functional elements contribute semantic content.
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Chapter 1
Introduction

1.1 Four Words and Three Puzzles

In simple terms, this work deals with the semantics of four words – many, few, much and little – a class that (for reasons that will become apparent below) I call ‘adjectives of quantity’, or Q-adjectives for short. My goal is to provide a unified semantic analysis of this class, and in doing so, to explore the semantics of quantity and degree more broadly.

Let me begin by outlining why I believe this to be an interesting, and fruitful, topic for investigation. To put it briefly, Q-adjectives are not, on the surface, terribly well behaved. They present a series of empirical puzzles that are both intriguing in their own right and relevant to broader questions in semantic theory. These puzzles are my starting point.

Puzzle 1 (The Distributional Puzzle): Why is it that Q-adjectives in some respects have the distribution of quantifiers (1)-(2), in other respects pattern with adjectives (3)-(4), and in yet other respects diverge from both classes (5)-(7)?

(1) a. Many/few students attended the lecture
   b. Every/some/most/both student(s) attended the lecture

(2) a. Much/little of the food was eaten
   b. All/some/most of the food was eaten

(3) a. John’s friends are many/few
    b. John’s friends are important/rich/eccentric

(4) a. The little food that he ate …
    b. The hot food that he ate…
As these examples show, the distribution of Q-adjectives is broader than that of either quantifiers or ‘ordinary’ adjectives. Such flexibility poses a challenge to a type driven semantics in which syntactic distribution is at least in part determined by semantic type, the question being what semantic type (or types) can be assigned to Q-adjectives to allow them to occur in the wide range of positions in which we find them.

**Puzzle 2 (The Interpretive Puzzle):** If I utter one of the sentences below, how many students am I claiming to have attended the lecture?

(8)  a. Many students attended the lecture  
    b. Few students attended the lecture

There is, of course, no precise answer to this (twenty-eight, or 1000, or four-score-and-seven, or what have you). Rather, Q-adjectives are vague, and their interpretation subject to contextual factors. The question, however, is what sort of factors are involved in the interpretation of sentences such as these, and how this can be represented formally.

In their vagueness, Q-adjectives distinguish themselves from other quantificational expressions, and pattern instead with gradable adjectives such as tall. As such, we might suppose that any adequate theory of vagueness in the adjectival domain could accommodate the facts relating to Q-adjectives as well. But Q-adjectives also exhibit interpretive variability beyond that seen with gradable adjectives, as Partee (1989)
in particular has pointed out. By way of illustration, (9) has two distinct interpretations: it can be read as asserting that a small number of French-speaking senators were at the gala (the so-called ‘cardinal’ reading), or that a small proportion of the totality of French-speaking senators were there (the ‘proportional’ reading).

(9) Few French-speaking senators were present at the gala dinner

Furthermore, the truth conditions of the two interpretations diverge. On the first reading, but not the second, the sentence could be judged true if there are only a small number of French-speaking senators and they all attended the gala.

As evidence that this is not just a dramatic instance of vagueness, it can be shown that the apparent ambiguity is grammatically constrained. For example, (10), which features the individual-level predicate Republicans in place of stage-level predicate present at the gala dinner, has only the second, ‘proportional’, reading (to see this, note that (10) must be judged false in the situation in which there are only a small number of French-speaking senators and they all are Republicans).

(10) Few French-speaking senators are Republicans

We clearly require something beyond the mechanisms developed for the analysis of vagueness in the adjectival domain to account for these facts.

**Puzzle 3 (The Contrasts Puzzle):** What is the explanation for contrasts such as the following?

(11) a. A few/*a many people visited the museum
    b. They visit every few days/*every many days
    c. The problems were many/*the difficulty was much

As these examples demonstrate, Q-adjectives not only resist easy classification as a group, but also display puzzling distributional differences among themselves. Such
contrasts in grammaticality do not have an obvious syntactic origin, and it is tempting to look for an explanation in the semantics of individual Q-adjectives, but there is not, to my knowledge, any existing semantic framework that can account for them. In turn, patterns such as these, though seemingly trivial, have the potential to serve as evidence towards the correct semantic treatment of Q-adjectives more generally. Looking more broadly, there has also been considerable recent interest in exploring the subtle syntactic and semantic differences that exist between superficially similar quantificational expressions (see especially Geurts & Nouwen 2007); work on data such as those in (11) will contribute to this undertaking.

My primary goal in this work is to solve these puzzles, and in doing so, to develop a comprehensive semantic analysis of Q-adjectives. An explicit part of this goal is to start with the broadest set of data possible; it is my intent to formulate a theory of Q-adjectives that is able to accommodate all of their uses, including those in (1)-(7) above, as well as others that will be introduced in what follows.

More generally, the investigation of the questions sketched out above touches on a range of broader questions in semantics and syntax. To list several of the most central:

a) What are the semantic structures that underlie the expression of quantity and amount?

b) What role should degrees and scales play in semantic theory? What explanatory value (if any) does their introduction offer?

c) How does syntactic structure contribute to semantic interpretation, with particular reference to the nominal domain?
d) How can we best characterize the vagueness of gradable expressions in their ‘positive’ (unmarked) forms within a formal semantic model?

I therefore set out also with the intention of using the particular puzzles surrounding Q-adjectives as a lens through which these more general questions can be examined.

Throughout this work, a central issue of concern to me is that of compositionality. That natural language is (by and large) compositional is of course a truism of elementary semantics textbooks; but while the notion has perhaps been bandied about to the extent that it has lost some of its impact, I nonetheless take the main business of semantics to be the development of formal analyses that are likewise compositional. Here, Q-adjectives present some interesting challenges, first on account of their breath of distribution, and secondly in that they form the basis of complex expressions of quantity (e.g. *too many*, *not a few*, *many fewer than 100*). I make it my goal to develop analyses of the relevant data that are fully compositional (though acknowledging that at points I will not be able to achieve this standard).

1.2 A Summary of the Proposal

The central proposal that I develop in this work is that in their semantics, Q-adjectives are neither ordinary quantifiers nor ordinary adjectives, but rather what I call degree predicates. Specifically, they denote gradable predicates of intervals (sets of degrees) on the scale of some dimension of measurement, allowing them to occur in a wide range of contexts in which scales and degrees are present. My account is thus a decompositional one, in that much of the semantic content traditionally attributed to Q-adjectives themselves is instead contributed by a series of null functional elements and
operations. It is the possibility of combining these in different ways that allows Q-adjectives to occur in the variety of syntactic contexts in which we find them.

I further show that patterns in the interpretation and sequencing of Q-adjectives can be related to properties of the scales of whose intervals they are predicated. The range of readings available to Q-adjectives can be obtained via appropriate choice of scale and standard of comparison, with no need to posit a lexical ambiguity, as has sometimes been done. Contrasts in grammaticality such as those in (11) also derive from aspects of scale structure. In particular: i) few and little describe intervals at the bounded, lower ends of scales, while many and much describe their unbounded upper ends; ii) many and few lexically encode the dimension of cardinality, while much and little are unspecified for dimension.

This work as a whole therefore supports the relevance of notions of degrees, scales and scale structure in accounting for patterns in the distribution and interpretation of lexical items in natural language. It furthermore adds to a body of work showing the need for a compositional approach to complex expressions of quantity.

1.3 A Brief Look at What’s Out There

Before proceeding towards the development of the proposal I summarize above, it is appropriate to take a brief look to the past. The issues that I address in this work have not gone unnoticed in the semantics literature, though the amount of attention they have received varies. While I will not attempt a full review of the literature in this introductory chapter, let me here briefly mention several crucial works and topics of inquiry that are particularly relevant in the context of the present work.
Of the three puzzles introduced above, the second, relating to the interpretation of Q-adjectives, has been the focus of the most in-depth investigation. Interest in the topic was prompted especially by observations in Barwise & Cooper’s (1981) influential work on generalized quantifiers. Barwise & Cooper (B&C) include *many* and *few* among the broad class of lexical items that can be analyzed as quantifying determiners, in doing so establishing what has become the most standard framework for the semantic analysis of Q-adjectives. But they also note that these words fail to fit neatly into the categories in which other determiners can be classified. On the basis of their acceptability in *there* sentences (Milsark 1974), as well as formal tests, *many* and *few* can be classified as weak, patterning with other weak determiners such as *a*, *some*, *no* and the cardinal numerals.

(12) a. There is/are a/five/some/few/many/a few/no dog(s) in the garden
   b. There is/are *every/*most/*all/*both dogs in the garden

But these two words fail to exhibit other properties characteristic of weak determiners. One such case is symmetry, defined below (first formally, then more intuitively):

(13) a. A determiner D is **symmetric** iff for all A, B, B ∈ D(A) iff A ∈ D(B)
   b. D A’s are B iff D B’s are A

Prototypical weak determiners such as *some* satisfy this relation, as seen in (14a); it is less obvious that *many* does (14b), and *few* clearly does not, as seen in (14c) and especially (14d):

(14) a. Some senators are lawyers ⇔ some lawyers are senators
   b. ?Many senators are lawyers ⇔ many lawyers are senators
   c. ?Few senators are lawyers ⇔ few lawyers are senators
   d. Few women are great-grandmothers ⇔ few great-grandmothers are women

Similar issues are observed with other properties, such as intersection. In a now well-known comment, B&C note that they disagree between themselves as to whether *many* and *few* share the properties characteristic of weak determiners, with Cooper “inclined to
think” that they do, but Barwise believing they do not (1981, p. 190). But the facts in
(14) seem indisputable evidence that symmetry, at least, is lacking.

Building on B&C’s observations, a number of authors have sought to develop
semantic analyses of many and few that capture the full range of their possible
interpretations, and thereby account for their inconsistent formal properties (see
especially Keenan & Stavi 1986; Westerståhl 1985; Lappin 1988, 2000; Fernando &
Kamp 1996; Herburger 1997). The culmination of this work (in explanatory power if not
chronological terms) is Partee’s (1989) paper Many Quantifiers, which develops an
elegant theory according to which many and few are each ambiguous between a weak or
‘cardinal’ interpretation with adjectival semantics and a strong or ‘proportional’
interpretation with determiner semantics. I discuss Partee’s analysis, and the other
previously mentioned work, in Chapters 2 and especially 4, where I highlight important
questions that remain open.

While the interpretation of Q-adjectives is well-explored territory, their broad
syntactic distribution – arguably the defining feature of the class – has received less
attention. The possibility of sequencing a Q-adjective after a determiner (the many
students, her few friends, the little rice that is left) has been used as key evidence in the
development of layered analyses of noun phrase structure (Jackendoff 1977; Abney 1987;
Zamparelli 1995; Borer 2005; among others). But in the semantics literature much of the
focus has been on their quantifier-like uses (as in (1a)) and in particularly on many/few as
opposed to much/little. There are some important exceptions: Hoeksema (1983) and
again Partee (1989) look at the adjective-like use of these items, while Klein (1982) and
Heim (2006) consider uses of much and little outside of the nominal domain; and Doetjes
(1997) merits particular note for breadth of empirical coverage. But other work has ignored or downplayed the importance of the non-quantificational uses of Q-adjitives (e.g. Hackl 2000), and there is no existing account that provides a unified treatment of their semantics across all of the syntactic contexts in which they occur. A central goal of the present work is to fill this void.

Turning to the third of the three puzzles I introduced above, contrasts such as those in (11) have for the most part escaped serious attention. Here the main exceptions are in the syntactic literature, most notably Kayne (2005), who explores in depth the case of *a few versus its absent counterpart *a many, among related questions. Kayne’s overall theory, in which the Q-adjjectives many and few are analyzed as modifiers of an unpronounced noun number, will be discussed in greater detail in Chapters 2 and 3, where I highlight parallels to the account I develop. But the corresponding semantics of contrasts such as these remain largely unexamined by Kayne or other authors.

Finally, I conclude this section with a brief mention of several works that, while not directly focused on the puzzles I have started with, are nonetheless highly relevant to the overall topic of this thesis. The first of these is Heim (2006), which builds on Heim’s earlier work on degree modification (Heim 2000) to develop an analysis of the Q-adjective little as a generalized quantifier over degrees, expressing degree negation. I discuss Heim’s work in more detail in Chapters 2 and 3; there, I introduce a proposal for the semantics of Q-adjjectives generally which is similar to Heim’s treatment of little, and which I show is able to account for a much broader range of facts than Heim addresses.

A second particularly relevant work is that of Hackl (2000). Hack’s primary focus is on comparative quantifiers such as more than three dogs or fewer dogs than
people, which he shows can, and in fact must, be analyzed compositionally. His analysis is based on a lexical entry for many as a gradable determiner, which may combine with comparative morphology (and other degree modifiers) having the semantics of degree quantifiers. In this approach, Hackl thus assimilates the analysis of Q-adjectives to the degree-based framework that is currently the leading approach to the analysis of gradable adjectives. My account differs in crucial ways from Hackl’s, but it shares with his the degree-based framework, and especially the focus on compositionality.

Finally, Schwarzschild (2006) makes a similar proposal to the one I will develop in the present work, namely that Q-adjectives are predicates of scalar intervals. Schwarzschild’s evidence comes from a very different area than the questions I consider here: parallels in distribution and interpretation between Q-adjectives and ‘partitive’ measure phrases (e.g. two feet in two feet of rope), and distinctions between these on the one hand and ordinary adjectives and ‘attributive’ measure phrases (e.g. two-inch in two-inch rope) on the other. Rett (2006) develops a related analysis of many and much as modifiers of sets of degrees, drawing on evidence from quantity questions in French and Romanian. I will have more to say about Schwarzschild’s and Rett’s accounts in Chapters 2 and 3; to anticipate this discussion, let me here suggest that the convergence of evidence from several different domains in support of the degree-based view is an important point in its favor.

1.4 Some Basics of Q-Adjectives

In this section, I establish some basic facts about Q-adjectives, which I will assume in what follows.
The first point to make is that Q-adjectives are in fact adjectives morphologically and etymologically. *Many* derives from the Old English *manig*, and still maintains the Germanic adjectival ending -y (from the older -ig) found in words such as *thirsty*, *mighty*, *windy* and many more¹. *Much* likewise has adjectival origins, having come to Modern English from Old English *micel* ‘great’ (still occurring in the dialectal *mickle*). *Few* derives from the Old English adjectival form *feawe*, related to the Latin adjective *paucus*, with similar meaning. Finally, *little* derives from the Old English adjective *lytel*, and is still polysemous, with a second sense that is undoubtedly adjectival (e.g. *the little boy*).

The adjectival status of these words is also reflected in the fact that they inflect like adjectives, having what are commonly taken to be comparative and superlative forms (Jespersen 1914/1970; Bresnan 1973), though admittedly only those of *few* are morphologically transparent:

(15) a. many/more/most  
b. much/more/most  
c. few/fewer/fewest  
d. little/less/least

Adjectival inflection is even more evident for the corresponding words in other Indo-European languages. For example, German *viel* ‘many, much’ and *wenig* ‘few, little’ inflect for gender, number and case; Spanish *mucho* ‘many, much’ and *poco* ‘few, little’ likewise inflect for gender and number.

The four English words of interest here divide into two pairs, *many* and *few*, which in simple terms occur in count noun contexts, and *much* and *little*, which occur in mass noun contexts and outside of the nominal domain (in Chapter 2 we will see that the picture is a little more complex than this, but this will do for a start). Within each pair,

the two words act as opposites or antonyms of one another. Specifically, we can classify them as contraries rather than complementaries (Horn 1989), or in the restricted sense of Cruse (1982), antonyms, in that there is an ‘excluded middle’ between the two members of each pair. As evidence, note that both (16a) and (16b) could be judged false, if the number of students attending was too great to be called many, but too small to be called few (similarly for (17a,b), putting aside for now the oddness of bare much here):

(16)  a. Many students attended the lecture  
      b. Few students attended the lecture

(17)  a. ?There is much rice left in the jar  
      b. There is little rice left in the jar

In common with adjectival contraries such as tall and short, Q-adjectives are gradable, in that they describe degrees of some property there can be more or less of. This is evidenced by the existence (previously mentioned) of comparative and superlative forms, and of modified forms such as too much, very few, and so forth. Q-adjectives also pattern with gradable adjectives in exhibiting an asymmetry in question formation. Thus while (18a) and (19a) are neutral requests for information, (18b) and (19b) imply that few students attended and that John is short, respectively.

(18)  a. How many students attended the lecture?  
      b. How few students attended the lecture?

(19)  a. How tall is John?  
      b. How short is John?

To put their antonymy in somewhat different terms, we can say that the positive many and much are defined by their lower bounds, meaning ‘at least’ some value, while conversely the negative terms few and little are defined by their upper bounds, meaning ‘at most’ some value. To see this, consider again the examples in (16). For (16a) to be
judged true, it seems that the number of students attending must exceed some contextually determined standard of comparison; conversely for (16b) to be true, the number attending must be less than some (perhaps different) standard.

Consistent with these interpretive facts, the positive and negative members of each pair exhibit opposite formal behavior. Thus if we choose our examples carefully, we can show many to be monotone increasing, licensing entailments from sets to supersets, and few to be monotone decreasing, licensing entailments from sets to subsets (and similarly for much vs. little, though I do not show the corresponding examples):

(20) a. Many freshmen came to the party ⇒ Many students came to the party
    b. Many students own cars ⇒ Many students own vehicles

(21) a. Few students came to the party ⇒ Few freshmen came to the party
    b. Few students own vehicles ⇒ Few students own cars

As we would expect from this, few licenses negative polarity items, while many does not:

(22) a. Few Americans have ever been to Mongolia
    b. Few Americans who have ever been to Mongolia want to go back
    c. *Many Americans have ever been to Mongolia
    b. *Many Americans who have ever been to Mongolia want to go back

It has been observed (Partee 1989) that depending on the choice of predicate, intuitions about entailments in examples such as (20) and (21) are less than clear, a pattern that goes hand-in-hand with the inconsistent behavior of Q-adjectives on tests for formal properties such as symmetry, as discussed above.

One further question arises in the consideration of entailment patterns with few and little in particular. For the entailments in (21) to go through, it must be possible for few to be none (otherwise, for example, if three sophomores but no freshmen came to the party, the first sentence in (21a) would be true, while the second would be false). Speakers I have consulted often have the impression that this cannot be right, but that
instead few must be at least one (or even two). But diagnostics introduced by Horn (1989) demonstrate that zero is within the range of values encompassed by few. Thus suppose I make you the following bet:

(23) I’ll bet you that few students will come to the party.

If it later turns out that no students come to the party in question, it would seem that I have won the bet, evidence that few can be none. A similar test can be constructed for little, with the same results. I will therefore adopt the position that both few and little encompass zero as a possible value, but that pragmatic factors (i.e. informativeness) make these terms infelicitous if the speaker knows none to be the case.

There is one final point worth mentioning here. Curiously, in quantificational position both many and few in their bare (unmodified) forms are slightly unnatural in colloquial speech: in place of many one tends to hear a lot, and in place of few, not many:

(24) a. Many students attended ≡ A lot of students attended
    b. Few students attended ≡ Not many students attended

This is even more evident with much and little, with some examples featuring much in particular verging on the outright ungrammatical (e.g. (16a) above).

(25) a. ?Much food was eaten ≡ A lot of food was eaten
    b. Little food was eaten ≡ Not much food was eaten

I will have more to say about the oddness of bare many and much in what follows, but I take the quantificational use of bare Q-adjectives to be grammatical, and therefore within the range of facts that must be captured by a formal analysis.

1.5 Outline of the Work

Let me conclude this introductory chapter with a brief overview of what is to come. Chapters 2 and 3 are the core of this work. Chapter 2 focuses on the
distributional puzzle. Here, I examine existing theories of the semantics of Q-adjectives, showing that the class cannot be treated as either quantifiers or predicates over individuals, and then advance the theory of degree predication that is my central proposal. I also present evidence that degree predication is a more widespread phenomenon, occurring with other expressions of quantity and in the adjectival domain as well. Chapter 3 develops a formal implementation of this proposal; I introduce the lexical semantics of Q-adjectives, as well as the null items that they combine with, and show that with this toolkit, the full range of uses of Q-adjectives can be analyzed. In Chapters 4 and 5, I extend the degree-based approach to examine other facts and questions. Chapter 4 investigates the interpretive puzzle, showing that the range of interpretations available to Q-adjectives can be derived via manipulation of two elements of the scalar representation that is invoked, namely the location of the range of values that serves as the standard of comparison, and the structure of the scale itself (whether or not an upper bound is assumed). Contrasts such as those in (11) are discussed at various points in the work, but in Chapter 5 I focus in depth on one of the most intriguing, namely the contrast between *a few and *a many. Finally, Chapter 6 summarizes the main conclusions, and the broader relevance of my findings.
Chapter 2
The Distribution of Q-Adjectives

2.0 Introduction

In the previous chapter, I introduced three puzzles in the semantics of adjectives of quantity (Q-adjectives) that this work seeks to address:

1) **The Distributional Puzzle:** a) Q-adjectives occur in a wider range of syntactic positions than other quantifiers;

2) **The Interpretive Puzzle:** In their unmodified forms, Q-adjectives are vague and context-sensitive, and perhaps ambiguous between two or more truth-conditionally distinct interpretations;

3) **The Contrasts Puzzle:** Q-adjectives exhibit seemingly idiosyncratic contrasts in distribution among themselves.

In this chapter, I investigate the first of these issues, proposing a unified semantic analysis of Q-adjectives in their modified and unmodified forms, across the syntactic contexts in which they occur. In doing so, I introduce the theory of degree predication that forms the core of this thesis. I argue that Q-adjectives denote gradable predicates of sets of degrees (intervals) on the scale associated with some dimension of measurement. Having shown that this approach is able to capture the facts relating to Q-adjectives, I further show that degree predication is a more widespread phenomenon, occurring also with other expressions of quantity and amount, and (perhaps surprisingly) in the adjectival domain as well.
The outline of this chapter is the following. Section 2.1 presents the empirical data that must be accounted for. Section 2.2 examines two established views of Q-adjectives, highlighting some not so trivial shortcomings of each. Sections 2.3 and 2.4 introduce the crucial data supporting the degree-predicate view, relating to the differential uses of Q-adjectives (Section 2.3) and the operator-like nature of few and little (Section 2.4). Section 2.5 outlines the theory that is at the core of this work; here, I show that Q-adjectives across all of their uses can be analyzed as predicates of scalar intervals. Section 2.6 addresses the question of what distinguishes much/little from many/few. Finally, Section 2.7 briefly examines extensions of this analysis to other cases of degree predication, and Section 2.8 wraps up with some conclusions. 2

2.1 The Data

Let me begin by introducing the distributional data that concern me in this chapter. To start with the Q-adjectives many and few, they may occur in positions that could be called quantificational (1a), predicative (1b), attributive (1c) and differential (1d). 3

(1)  
   a. Many/few students attended the lecture quantificational  
   b. John’s good qualities are many/few predicative  
   c. The many/few students that we invited enjoyed the lecture attributive  
   d. Many/few more than 100 students attended the lecture differential

---

2 Earlier versions of the material in this chapter and the next were presented at the 2007 and 2008 LSA Annual Meetings, the 2007 ESSLLI Workshop on Quantifier Modification, and Sinn und Bedeutung 12 (see Solt 2007a, 2007b, 2008a, 2008b).

3 I use these terms in a pre-theoretic sense, as labels for the positions in which Q-adjectives may occur. I do not intend to imply anything about their semantics in these various positions, and in fact I will argue below that they have the same basic denotations in all of these contexts.
Turning to *much* and *little*, they too may occur in quantificational (2a) and differential (2d) positions, and (in the case of *little* but not *much*) in attributive position (2c). They do not, however, occur predicatively (2b), at least in constructions parallel to that in (1b):  

(2)  
(a) (Too) **much/little** water is left in the bucket  
    quantificational  
(b) *The water in the bucket is **much/little***  
    predicative  
(c) The **little/*much** water that is left in the bucket  
    attributive  
(d) **Much/little** more than a gallon of water is left in the bucket  
    differential  

Beyond this, *much/little* also occur as modifiers in adjectival comparatives (3a) and excessives (3b), adverbial modifiers (3c), and modifiers of deverbal adjectives (3d) and of two idiosyncratic ordinary adjectives (3e) (the latter noted by Abney 1987):  

(3)  
(a) **Much/little** taller than his father  
(b) **Much/?little** too tall  
(c) I slept (too) **little/*much**  
(d) **Much** loved/ **little** known  
(e) **Much/little** alike/different  

*Much* also occurs in a context that has been called *much* support (Corver 1997), when an adjective has been pronominalized with *so*:  

(4)  
(a) Fred is fond of Jane; in fact, he is too **much** so  
(b) Fred is fond of Jane; in fact, he is so **much** so that I am worried about him  

The distribution of Q-adjectives was the subject of considerable discussion in the earlier syntactic literature, notably by Bresnan (1973) and especially Jackendoff (1977), who used facts from this domain as crucial evidence towards a theory of phrase structure.  

---

4 Here we already see some of the puzzling contrasts that characterize Q-adjectives: *many/few* but not *much/little* may occur predicatively; *little* but not *much* may occur attributively (Jackendoff 1977); in some positions (e.g. quantificational) *much* is acceptable when modified (e.g. *too much*) but at best marginal in bare form. I will return to discuss these contrasts later in this work; for now, I will put these details aside and focus on developing a framework that is able to accommodate the range of environments in which Q-adjectives (modified or unmodified) may occur.
But there has been little attempt within the semantics literature to provide a unified analysis of the full range of data discussed above. The most notable exception to this is Doetjes (1997), who examines Q-adjectives in English and French, showing that their quantificational and adverbial uses can receive parallel analyses; her account, however, does not extend to all the cases discussed above. Most other work is more limited in the distributional data that is addressed. In particular, *many* and *few* in their quantificational uses (as in (1a)) have been the focus of considerable investigation in the context of work on quantification, but with little attention to their predicative and attributive uses, which set them apart from other quantifiers (though see Hoeksema 1983; Partee 1989; Hackl 2000 for some discussion). The differential uses of Q-adjectives (1d) and (2d), which I will show below yield crucial insights into their semantics, seem to have more or less escaped semanticists’ notice (excepting a brief mention by Schwarzschild 2006).

Quantification in the mass domain has in general been less studied than count quantification (a notable exception being Higginbotham 1995); and while there have been several interesting attempts to develop unified analyses of *many* and *much* (Creswell 1977; Chierchia 1998a, 2005; Schwarzschild 2006), these accounts have again focused primarily on their quantificational uses. From a different perspective, both the syntax and semantics of *much/little* in their adjectival and adverbial uses (as in (3)) have received some attention (e.g. Neeleman et al. 2004; Heim 2006), but without much of a link to the quantification literature (though see Klein 1982 for a link between *much* in adjectival comparatives (3a) and its quantificational uses).

Furthermore, it is not immediately apparent that a unified account is possible. Consider just the examples in (1a-c), in which *many* and *few* occur in quantificational,
predicative and attributive positions. Already it seems we need interpretations of two or three different semantic types: a quantificational type for (1a), a predicative type for (1b), and a predicative or modifier type for (1c). Differential cases such as (1d) present a further complication, as do the mass examples in (2), and especially the adverbial and adjectival cases in (3). One obvious possibility is that Q-adjectives are lexically ambiguous, that is, that there are two or more different *many’s* (and similarly for the other Q-adjectives). But in the interest of avoiding the unnecessary multiplication of senses, it is reasonable to ask whether all of these different uses can be reduced to a single basic lexical entry.

A second question of distribution relates to the sorts of degree modifiers that co-occur with Q-adjectives. All of the terms under consideration (*many, few, much and little*) have comparative and superlative forms, as shown in (5) (where I follow a tradition going back to Jespersen 1914/1970, and more recently Bresnan 1973, in taking *more* to be the spell-out of *many+er* and *much+er*, *most* to be the spell-out of *many+est* and *more+est*, and *less* and *least* to be the spell-outs of *little+er* and *little+est*, respectively):

(5)  
  a. Fred owns *many* books/*more* books than Barney/the *most* books of anyone I know  
  b. Fred owns *few* books/*fewer* books than Barney/the *fewest* books of anyone I know  
  c. Fred drank *much* wine/*more* wine than Barney/the *most* wine of anyone at the party  
  d. Fred drank *little* wine/*less* wine than Barney/the *least* wine of anyone at the party

Q-adjectives may also occur with a range of other degree expressions (with again some curious gaps that I will address later in this work); these include intensifiers such as *very*
and *incredibly* (6a, 7a), the excessive *too* (6b, 7b), the equative *as...as* (6c, 7c), the sufficiency *so* (6d, 7d), and the demonstratives *this* and *that* (7e, 7e):

(6)  
   a. Fred owns *very many*/*incredibly many*/*very few*/*incredibly few* books  
   b. Fred owns *too many*/*too few* books  
   c. Fred owns *as many*/*as few* books *as* Barney  
   d. Fred owns *so many*/*so few* books that he was kicked out of our club  
   e. If Fred owns *that many*/*this many*/*that few*/*this few* books, he’ll be kicked out of our club

(7)  
   a. Fred drank *very much*/*incredibly much*/*very little*/*incredibly little* wine  
   b. Fred drank *too much*/*too little* wine  
   c. Fred drank *as much*/*as little* wine *as* Barney  
   d. Fred drank *so much*/*so little* wine that he was kicked out of our club  
   e. If Fred drank *that much*/*this much*/*that little*/*this little* wine, he’ll be kicked out of our club

In this, Q-adjectives are distinguished from other sorts of quantificational expressions, which do not allow degree modification (*every-er; *as some as; *too most*). Instead, they pattern with gradable adjectives such as *tall*, as exemplified in (8) and (9):

(8) Fred is *tall*/*taller* than Barney/the *tallest* guy I know

(9)  
   a. Fred is *very*/*incredibly tall*  
   b. Fred is *too tall* (to go on the kiddie rides)  
   c. Fred is *as tall as* Barney  
   d. Fred is *so tall* that he hit his head on the door jamb  
   e. If Fred is *that tall*/*this tall*, he can’t go on the kiddie rides

While this issue is perhaps less fundamental than the first (and a solid framework for addressing it has been established by Hackl 2000), these facts must also be captured by a unified analysis of Q-adjectives.
I would like to propose that a unified analysis is in fact possible. All of the examples in (1)-(4) above have one thing in common, namely some component of scalar meaning. In each of these cases, some element within the semantic representation introduces a scale tracking some dimension, whether that dimension be number (of students), volume (of water), physical extent (John’s height), temporal extent (the length of time I slept), or whatever. My main claim will be that it is this that allows Q-adjectives to occur in all of these contexts: Q-adjectives are predicates of intervals on the scale of some dimension, allowing them to occur in a wide range of contexts in which scalarity is present. At the same time, Q-adjectives are themselves gradable expressions, introducing a degree argument that can be saturated or bound by a degree modifier.

2.2 Two Theories of Q-Adjectives – And Their Shortcomings

Before introducing the theory that I will advocate in this work, I will begin by examining two possible alternatives, the two most familiar perspectives on the semantics of Q-adjectives, both of which derive from key works in the literature: i) Q-adjectives are quantifiers (Barwise & Cooper 1981; Westerståhl 1985; Lappin 1988, 2000; Partee 1989; Herburger 1997; Chierchia 1998a; Hackl 2000; Takahashi 2006); ii) Q-adjectives are predicates or noun modifiers (Milsark 1977; Hoeksema 1983; Partee 1989; McNally 1998; Hackl to appear). Here, I will focus in particular on many and few, both because they have received the most extensive treatment in the literature, and because even a restricted set of data (namely that in (1a-d) above) is sufficient to illustrate the questions left unanswered by these two established views. It will be seen that both of the theories considered here get some of the facts right. But neither of them can get all of the facts right; nor can a composite view under which Q-adjectives are ambiguous between these
two types of representations. The (negative) conclusion of this section paves the way for the degree predicate analysis that I develop in the remainder of this chapter.

2.2.1 Q-Adjectives as Quantifiers

If there can be said to be an orthodox view on the semantics of many and few, it is that they are quantifiers (this, for instance, being the perspective presented in elementary semantics textbooks such as Chierchia & McConnell-Ginet 2000, Gamut 1990 and Heim & Kratzer 1998). The quantificational analysis in its most standard form is represented by the Generalized Quantifier Theory (GQT) of Barwise & Cooper (1981), and its later extensions. On this account, many and few are ‘quantifying determiners’ that express relationships between two sets (type \( \langle \text{et}, \langle \text{et}, \text{et} \rangle \rangle \)), members of a class that includes the logical quantifiers every/each/all, some/a and no, other non-logical quantifiers such as most, several, a few and both simple and modified cardinal numerals (three, exactly three, at most three), and even the definite article and demonstratives. In a GQT analysis, many and few have lexical entries along the lines of the following:

(10)  
\[
\begin{align*}
\text{a.} & \quad \left[ \text{many}_{\langle \text{et}, \langle \text{et}, \text{et} \rangle \rangle} \right] = \lambda P \lambda Q. \left| P \cap Q \right| > n, \text{ where } n \text{ is some large number} \\
\text{b.} & \quad \left[ \text{few}_{\langle \text{et}, \langle \text{et}, \text{et} \rangle \rangle} \right] = \lambda P \lambda Q. \left| P \cap Q \right| < m, \text{ where } m \text{ is some small number}
\end{align*}
\]


Hackl (2000) offers an update to the GQT analysis according to which many is analyzed as a ‘parameterized’ determiner whose first argument is a degree argument that may be saturated by comparative morphology or other degree modifiers:

(11)  
\[
\left[ \text{many} \right] = \lambda d \lambda P \lambda Q. \exists x [ |x| = d \land P(x) = 1 \land Q(x) = 1]
\]
Hackl’s approach has the benefit of establishing a compositional relationship between bare *many* and *few* (1a), simple comparatives (12a), more complex comparatives (12b-e), and constructions with other degree modifiers (13):

(1)  a. Many/few students attended the lecture

(12)  a. More than 100 students attended the lecture
         b. More students than professors attended the lecture
         c. More students read than write
         d. More students attended the lecture this year than last year
         e. More students attended the lecture than there were seats in the hall

(13)  a. Too many students attended the lecture
         b. As many students as professors attended the lecture
         c. So many students attended the lecture that there weren’t enough seats

For other quantificational analyses of Q-adjectives, see also Diesing (1992), Kamp & Reyle (1993) and Herburger (1997).

The obvious attraction of the quantificational approach is that it captures the distributional and interpretive similarities between *many/few* and other quantifiers (or in the language of GQT, quantifying determiners). In uses such as (1a), *many* and *few* look like quantifiers. They could be replaced with other words that are traditionally analyzed as quantificational:

(1)  a. Many/few students attended the lecture

(14)  a. Every student attended the lecture
         b. Most students attended the lecture
         c. Both students attended the lecture
         d. Some students attended the lecture

And like these words, *many* and *few* can be interpreted as specifying the number of elements in a set (here, the set of students who attended the lecture), or its size in proportion to that of another set (here, the total contextually relevant set of students).
But of course the shortcoming of the quantificational view (either in its original form or Hackl’s parameterized extension) is that it has little to say about the non-quantificational uses of Q-adjectives, such as our original (1b-c):

(1)  
   b. John’s good qualities are many/few  
   c. The many/few students that we invited enjoyed the lecture

In the predicative example (1b), *many* and *few* cannot easily be treated as quantificational determiners, since the logical form does not provide two appropriate sets that could serve as their arguments. Rather, there is only one entity or set available as an argument, that denoted by the nominal expression *John’s good qualities*. And in the attributive example in (1c), *many* and *few* are likewise not in any obvious way responsible for introducing quantificational force, given that they could be removed without affecting the overall grammaticality of the sentence (*the students that we invited enjoyed the lecture*).

The standard approach to resolving this sort of issue is via type shifting. In the classic work in this area, Partee (1986) proposes the type shift BE, which derives a predicative interpretation from a quantificational NP. In (15) is given the semantics of BE, and its application to a quantificational interpretation of *a student*:

(15)  
   a. \( BE(\varphi_{et,t}) = \lambda x. \varphi(\lambda y[y=x]) \)  
   b. \( BE(\text{a student}_{et,t}) = \lambda x. \text{student}(x) \)

Predicative interpretations of the form derived via (15) have been argued to play a role in a number of constructions, including most obviously predicate nominals (16), but also *there*-sentences (17) (see especially McNally 1998) and object position of light verbs (18) (de Hoop 1992).

(16) I consider John competent and an expert on unicorns

(17) There is a book on the table
(18) This room has a window

Let us examine how the type shifting analysis fares in accounting for the non-quantificational uses of *many* and *few*. While Partee (1986) does not explicitly address the application of BE to plural quantificational NPs (e.g. *five students, many students, few students*), McNally (1998) and de Swart (2001) propose extensions of this sort that draw on Link’s (1983) lattice-theoretic analysis of plurality to derive predicates of plural individuals from the corresponding generalized quantifiers. Applied to a quantificational interpretation of the noun phrase *many students*, BE yields the following (where ‘*’ indicates a predicate over plural individuals):

\[
\text{(19)} \quad \text{BE(many students}_{\text{et,t}}) = \text{BE}(\lambda Q. \exists x[*\text{student}(x) \land Q(x) \land |x| > n])
\]

\[
= \lambda x. *\text{student}(x) \land |x| > n
\]

On its predicative interpretation, *many students* in (19) can thus be thought of in set terms as the set of pluralities whose atoms are students, and whose cardinalities exceed some contextually determined value \(n\). To take one case of how this interpretation is relevant, we can achieve the correct semantics for *there*-sentences involving *many* via application of an existential operator to a predicative expression derived via BE (20):

\[
\text{(20) a. There are many books on the table}
\]

\[
\text{b. } \left[\text{there be}\right] \left( \left[\text{(many books)}_{\text{et,t}}\right] \cap \left[\text{on the table}\right] \right)
\]

\[
\exists x[*\text{book}(x) \land |x| > n \land \text{on-the-table}(x)]
\]

The predicate derived in (19) also appears to give us what we need for an analysis of attributive *many*. Without concerning ourselves at present with the formal details, take *the* to denote a supremum operator (Link 1983; Landman 2004). Applying it to a predicate of the form in (19) yields the intuitively correct result in (21):
(21) a. The many students that we invited enjoyed the lecture
   
   b. enjoyed-the-lecture(sup(\(\lambda x. \#\text{student}(x) \land \text{invited}(\text{we}, x) \land |x| > n))
   
   where sup(P) = \(\forall x[P(x) \land \forall y[P(y) \rightarrow y \subseteq x]]\) if defined;
   
   c. ‘The maximal group of students we invited, whose number was many,
   
   enjoyed the lecture’

But now observe what happens when we extend this to \textit{few}. De Swart (2001)

derives the following predicative interpretation for a noun phrase containing \textit{few}:

\[
(22) \quad \text{BE(few students})_{\text{et},t} = \text{BE(}\lambda Q.\neg\exists x[\#\text{student}(x) \land Q(x) \land |x| > m])
= \lambda x.\neg[\#\text{student}(x) \land |x| > m]
\]

Here, something seems to have gone wrong. The predicate derived in (22) is, in set

terms, the set of pluralities that are not both composed of students as atoms and of

cardinality greater than the contextually determined value \(m\). In other words, we derive a

set that contains individuals other than pluralities of students (for example, the set in (22)

would contain, among other unlikely members, the planet Venus, my doormat, and the

plurality consisting of all the giant pandas at the National Zoo). An unwelcome

consequence of this analysis is that a \textit{there}-sentence such as (23a) can no longer be
analyzed as involving simple existential quantification over a predicate of the form in
(22), as in (23b), as this would simply assert the existence of something that is not a large

group of books on the table. Instead, de Swart proposes that in the case of predicates
formed from monotone decreasing quantifiers, \textit{there}-sentences (and predicate-to-
quantifier type shifts more generally) should be analyzed in terms of universal
quantification, as in (23c):

\[
(23) \quad a. \quad \text{There are few books on the table}
\]
\[
b. \quad \exists x[\neg[\#\text{book}(x) \land |x| > m \land \text{on-the-table}(x)]]
\]
\[
c. \quad \forall x[\text{on-the-table}(x) \rightarrow \neg[\#\text{book}(x) \land |x| > m]]
\]
Regardless of what one might think of de Swart’s solution to this problem, note that the predicate derived in (22) also does not provide for the correct analysis of attributive few; extending the approach in (21) to few yields the unlikely (24):

(24)  a. The few students that we invited enjoyed the lecture
      b. enjoyed-the-lecture(sup(\(\lambda x. \neg (*\text{student}(x) \land |x| > m \land \text{invited(we,x)})\)))
      c. ‘Everything that isn’t a large group of students who we invited enjoyed the lecture’

From this logical form it follows (incorrectly) that doormats, giant pandas and the like enjoyed the lecture. Instead, the predicative interpretation we seem to want for few students is more along the lines of the following:

(25)  \([\text{few students}_{(c_0)}]\) = \(\lambda x. *\text{student}(x) \land |x| < m\)

But this interpretation cannot be derived via BE, or by any other type shift that can be generalized to the case of many (see Landman 2004 for a similar point). We will see that this is the first of several places where few give us trouble, in that an approach that yields intuitively correct results for many cannot be extended to few.

An even more fundamental issue with DP type-shifting as the means to account for the non-quantificational uses of Q-adjectives is that it makes incorrect predictions regarding their predicative uses. Recall that the sort of type shifts we are considering apply to DPs, not to the quantifying determiners themselves. But on this view we would then expect to find DPs of the form many NP or few NP in predicative position, just as we do singular indefinites, a prediction that is not borne out:

(26)  a. *John’s friends are many students
      b. *John’s friends are few students
      c. John is a student

On the other hand, many and few themselves do occur as predicates, as in the original example (1b). Under a Partee-style type shifting approach, we would seemingly need to
analyze these as involving a phonological null noun, such that an example such as (27a) would have the structure in (27b):

\[(27) \quad \begin{align*}
&\text{a. John’s friends are few} \\
&\text{b. John’s friends are few N}
\end{align*}\]

But we would then be forced to explain why (27a) is grammatical, while the corresponding examples with an overt noun (as in (26a,b)) are not.

Instead, it seems that *many* and *few* themselves require predicative interpretations, a possibility that is not provided for under a quantificational analysis augmented by a theory of DP type shifting. In the next section, I turn to an alternate approach in which this is taken as the basic semantics of Q-adjectives.

### 2.2.2 Q-Adjectives as Predicates

A second well-established view on the semantics of the Q-adjectives *many* and *few* holds that they have adjectival semantics, being like ‘ordinary’ adjectives in denoting predicates or modifiers (perhaps in addition to having a quantificational interpretation).

In what might be considered a precursor of this view, Link (1983) begins his seminal paper on mass terms and plurals with an anecdote about German magazine publisher Rudolph Augstein, who when asked what quality he most appreciated in his friends, replied “that they are few.” Link remarks:

> Clearly, this is not a property of any one of Augstein's friends; yet, even apart from the *esprit* it was designed to display the answer has a straightforward interpretation. The phrase ['that they are few'] predicates something *collectively* of a *group* of objects, here: Augstein's friends. (1983; p. 302)

Link does not return to this example to present a formal semantics for *few* as a predicate of groups or pluralities, but proposals of this nature are found elsewhere in the literature. The earliest expression of this view is by Milsark (1974, 1977), who argues that *many*
and few in their weak uses, like unstressed some, are not quantifiers but cardinality expressions. Milsark does not, however, attempt a formalization of this notion, noting that it does not lend itself to a predicate-logic representation. The predicative view is stated more explicitly by Hoeksema (1983), who proposes that many and few, like cardinal numerals, are adjectival elements that specify the cardinality of a group of individuals; on this view, many books therefore would denote the set of all groups in the domain consisting of many books. Partee (1989) builds on Hoeksema’s and Milsark’s work to argue that many and few are ambiguous between a so-called proportional reading with GQT-type determiner semantics and a cardinal reading with adjectival semantics (I discuss Partee’s proposals in more detail in Chapter 4). More recent proposals for predicative semantics for many and few are found in Kamp & Reyle (1993) (who follow Partee (1989) in taking them to be ambiguous between quantificational and predicative interpretations), McNally (1998), and more recently Kennedy & McNally (2005a).

The ‘adjectival’ or predicative analysis thus gives many and few the semantics of cardinality predicates: predicates of groups or pluralities that hold if the number of individuals in the group/plurality exceeds (many) or falls short of (few) some contextually determined standard:

\[
\begin{align*}
\text{a. } [\text{many}] &= \lambda x. |x| > n, \text{ for some contextually determined value } n \\
\text{b. } [\text{few}] &= \lambda x. |x| < m, \text{ for some contextually determined value } m
\end{align*}
\]

A related approach takes many and few to be attributive modifiers, functions from predicates to predicates (Hackl to appear).

It should be noted that the adjectival analysis of the Q-adjectives many and few aligns to a much broader tradition in which cardinal numerals are analyzed not as
quantifiers but as cardinality predicates or noun modifiers. Early expressions of this view
are found in Hoeksema (1983) and Partee (1986). More recently, influential proposals of
this nature have been offered by Landman (2004), who argues that they are cardinality
predicates, and by Krifka (1999) and Ionin & Matushansky (2006), who introduce
(somewhat different) analyses of numerals as noun modifiers. I will return to the
parallels between Q-adjectives and cardinal numerals later in this chapter.

On the cardinality predicate view (and with some additional assumptions the
attributive modifier view), the facts that were problematic for the quantificational theory
of Q-adjectives are now readily accommodated. As predicates semantically, it is to be
expected that *many* and *few* occur in attributive and post-copular positions, the same
positions where we find adjectival predicates:

(29) a. John’s friends are many/few (cf. John’s friends are tall)
    b. The many/few students (cf. the smart students)

And the entries in (28) give us what we need to analyze these uses. For predicative
*many*, we would have the following:

(30) John’s good qualities are many
    many(John’s good qualities)
    = |John’s good qualities| > n, for some contextually determined n

Entries of the form in (28) would also allow *many* and *few* to combine intersectively with
the predicate denoted by a plural noun:

(31) a. \[ \text{many students} \] = \[ \text{many} \] ∩ \[ \text{students} \]
     = \( \lambda x. \ast \text{student}(x) \land |x| > n \)

    b. \[ \text{few students} \] = \[ \text{few} \] ∩ \[ \text{students} \]
     = \( \lambda x. \ast \text{student}(x) \land |x| < m \)
A complex predicate of this form now provides the correct input to derive the semantics of attributive examples via the application of a determiner. Again taking the to denote a (partial) maximalization operator over sets of plural individuals (Link 1983, Landman 2004), (32a) receives the correct interpretation in (32b) (cf. (24) for the incorrect results derived via the quantificational analysis augmented by DP type shifting).

(32) a. The few students that we invited enjoyed the lecture  
    b. enjoyed-the-lecture(sup(λx. *student(x) ∧ |x| < m ∧ invited(we,x)))  
    c. ‘The maximal set of students, who were few, enjoyed the lecture’

The question then arises as to how to achieve the correct semantics for many and few in their quantificational uses, when they are not preceded by an overt determiner. This is of course the same issue that arises in any theory of indefinites in which they are analyzed as not inherently quantificational. The standard solution is to take quantificational force to arise via the application of a covert existential operator, with the mechanism variously assumed to be global existential closure (Heim 1982), a type shift (Partee 1986; Landman 2004; a.o.), a null determiner with existential semantics (Krifka 1999), or existential quantification over choice functions (Reinhart 1997). Following this general approach, a quantificational case such as (33a) receives an interpretation along the lines of (33b), which is essentially equivalent to what would be derived from the quantificational entry in (10).

(33) a. Many students attended the lecture  
    b. ∃x[ *student(x) ∧ |x| > n ∧ attended-the-lecture(x)]  
    ‘there was a group students numbering more than n who attended the lecture’

Summarizing what we have seen so far, the cardinality predicate analysis of many and few seems to fare better than the quantificational analysis, in that it is able to capture the quantificational, predicative and attributive uses (1a-c) (though the reader will note
that I have not yet analyzed quantificational few, nor the differential use exemplified in (1d); I return to these below).

Furthermore, if the entries in (28) are modified to include a degree argument, as in (34), many and few then have the same semantic type that is commonly ascribed to gradable adjectives (Cresswell 1977; Heim 2000; Hackl 2000; a.o.), the only difference being that the cardinality function in the former case is in the latter case replaced with a measure function such as HEIGHT:

\[
\begin{align*}
(34) & \quad a. \quad \llbracket many_{(d, et)} \rrbracket = \lambda d \lambda x. |x| \geq d \\
& \quad b. \quad \llbracket few_{(d, et)} \rrbracket = \lambda d \lambda x. |x| \leq d
\end{align*}
\]

\[
(35) \quad \llbracket tall_{(d, et)} \rrbracket = \lambda d \lambda x. \text{HEIGHT}(x) \geq d
\]

This first of all provides an explanation for the identical patterns of degree modification in the two cases (as discussed in section 2.1): both types of predicate include a degree argument that can be either saturated or bound by a degree expression (e.g. too, very, comparative morphology). More fundamentally, under the cardinality predicate analysis, there is no need to posit special or idiosyncratic semantics for Q-adjectives. While they are not of the semantic type of other quantificational expressions (every, most, etc.), they have another commonly occurring type, namely that of gradable adjectives. This is clearly desirable from the point of view of simplicity in the grammar.

Yet while the simplicity and explanatory potential of this account is clear, it also faces several challenges, some of which are now well known. The first issue relates to the derivation of the correct semantics for few in its quantificational uses. Above, it was shown that the quantificational interpretation for many can be analyzed as arising from an operation of existential closure over a predicative nominal expression. But as has been repeatedly observed (van Benthem 1986; Herburger 1997; McNally 1998; Krifka 1999;
Hackl 2000; de Swart 2001; Landman 2004; Geurts & Nouwen 2007), application of an existential operator to a predicative expression including a monotone decreasing cardinality predicate (e.g. *few*) incorrectly produces a lower-bounded ‘at least’ reading (an issue which has come to be known as ‘van Benthem’s problem’). Thus with the definition of *few* in (28b), the sentence (36a) would receive the interpretation in (36b).

(36) a. Few students attended the lecture
b. $\exists x [*\text{student}(x) \land \text{few}(x) \land \text{attended-the-lecture}(x)]$
   $= \exists x [*\text{student}(x) \land |x| < m \land \text{attended-the-lecture}(x)]$
   ‘There was a group of students of numbering less than $m$ who attended the lecture’

But this simply asserts the existence of a group of students numbering less than $m$ who attended the lecture, and crucially does not exclude the possibility that a some larger group (numbering $m$ or more), a superset of the smaller group, likewise attended.

This issue is a substantial one for theories of indefinites in which their quantificational interpretations are derived via existential closure over the corresponding predicates. While some authors (Herburger 1997; Hackl 2000) have cited van Benthem’s problem as evidence against a cardinality predicate theory of numerical indefinites, others have sought to augment the predicative theory with mechanisms that overcome it. Approaches that have been proposed include the use of additional or more complex type shifting rules (de Swart 2001; Landman 2004), semantic decomposition (McNally 1998; Landman 2004 on *no*), and mechanisms of scales of alternatives (Krifka 1999). All of these approaches add complexity to the theory, and some require further stipulations to get the facts right. I will not undertake a detailed discussion of the strengths and weaknesses of these various proposed solutions, since ultimately I will reject the cardinality predicate analysis of Q-adjjectives. But let me briefly look in more depth at
one proposal, namely McNally’s (1998) decompositional analysis, which will prove relevant below.

Generally speaking, semantic decomposition in this domain involves the decomposition of a monotone decreasing predicate (e.g. *few*) into an increasing predicate and a negation operator that takes wider (sentential) scope (see Jacobs 1980 for an early development of a decompositional analysis). With regards to Q-adjectives in particular, McNally (1998) follows Ladusaw (1992) in analyzing *few* as a version of *many* that must appear in the scope of a clausal negation operator. In this approach, (36a) would receive the more intuitively correct interpretation in (37a), paraphrased in (37b):

(37) a. $\neg \exists x [*\text{student}(x) \land \text{many}(x) \land \text{attended-the-lecture}(x)]$
    $\neg \exists x [*\text{student}(x) \land x > n \land \text{attended-the-lecture}(x)]$
    b. ‘There was not a group of many students (i.e. a group numbering $n$ or more) who attended the lecture’

What is attractive about the decompositional analysis in this case is that *few* in many respects behaves like a negative expression. A tradition going back at least to Klima (1964) analyzes *few* as the negation of *many* (see also Barwise & Cooper 1981 for a similar point). In support of this very reasonable view, observe that (38a) can be readily paraphrased as (38b); in fact, *few* is slightly unnatural in colloquial speech, the preference being for the overtly negative *not many*:

(38) a. Few students attended the lecture
    b. Not many students attended the lecture

The negative character of *few* is also supported by the fact that it exhibits several classic patterns characteristic of overtly negative expressions. For example, *few*, like *no*, takes *either* as a tag, in contrast to positive quantifiers (e.g. *many*, *some*) which take *too*:
(39) a. Some men like Brussels sprouts, and some women do, √ too/*either
b. Many men like Brussels sprouts, and many women do, √ too/*either
c. No men like Brussels sprouts, and no women do, *too/*either
d. Few men like Brussels sprouts, and few women do, *too/*either

*Few* also gives rise to negative inversion (Klima 1964), like overtly negative expressions:

(40) a. In few cases were side effects observed
b. In no cases were side effects observed
c. *In many cases were side effects observed

The analysis represented in (37) is thus consistent with the negative characteristics of *few*. But importantly, in the form given here, the decompositional analysis is not entirely adequate. Specifically, the original problem crops up again when we turn to expressions in which *few* occurs with a degree modifier such as *very*: a direct translation of the above analysis to (41a) yields the result in (41b) – clearly not the correct paraphrase for this sentence:

(41) a. Very few students attended the lecture
b. ¬∃x[*student(x) ∧ very-many(x) ∧ attended-the-lecture(x)]
   ‘There was not a group of very many students who attended the lecture’

To obtain the correct interpretation in cases such as these, the decompositional analysis would seemingly need to be supplemented with some additional mechanism for the interpretation of degree modifiers such as *very* in the scope of negation.

To anticipate the discussion later in this chapter, the account I will develop will in essence be a decompositional one, and thus like McNally’s capture the negative character of *few*; but it will also avoid the problem exemplified in (41).

While the previous issue could be considered a technical or theory-internal one, a perhaps more substantive objection to the cardinality predicate analysis of Q-adjectives is that they do not exhibit the same distribution as ordinary gradable adjectives. Hackl
(2000) notes that while many and few may occur predicatively in full clauses, they fail to do so in small clause complements of consider, traditionally taken to be a diagnostic for predicative interpretations. In this, their behavior contrasts with that of ordinary gradable adjectives such as tall, and also with the adjective numerous, which in other respects has much the same semantic content as many:

(42) a. *I consider the guests many
    b. *I consider the guests few
    c. I consider the guests tall
    d. I consider the guests numerous

Conversely, Kayne (2005) notes that many and few allow a subsequent ‘unpronounced NP’, while ordinary adjectives (and numerous) do not:

(43) a. Many linguists like phonology, but many don’t
    b. *Good linguists like phonology, but bad don’t
    c. *Numerous linguists like phonology, but numerous don’t

Many and few also occur in partitives, whereas ordinary adjectives do not (and numerous is at best marginal):

(44) a. Many/few of the students came to the lecture
    b. *Good of the students came to the lecture
    c. *?Numerous of the students came to the lecture

A substantial challenge for a cardinality predicate theory is thus how to explain the different distribution of many and few relative to other predicative expressions.

In short, the cardinality predicate analysis of Q-adjectives is in some respects an attractive one, offering an approach to capturing their quantificational, predicative and differential uses. But this analysis also introduces a new set of problems that do not seem trivial to resolve: i) how to achieve the correct quantificational interpretations for few and its modified forms; ii) how to properly constrain the predicative occurrences of many and few.
2.2.3 Interim Summary

Let us take stock of where we are. With regards to the data in (1a-c), neither a quantificational analysis nor a cardinality predicate analysis can provide an adequate account of the semantics of Q-adjectives. The former (not surprisingly) handles their quantificational uses, but runs into trouble in obtaining the correct distribution and interpretation in their attributive and predicative uses. Conversely, the latter yields correct results for the attributive and predicative cases, but requires additional machinery to handle quantificational *few*; there is furthermore a question as to why *many* and *few* do not have the same distribution as other adjectival predicates. An obvious solution would be to follow Partee (1989) in proposing that *many* and *few* are ambiguous between a quantificational type (responsible for their quantificational uses) and a predicative type (responsible for their predicative and attributive uses), with the two perhaps related by some form of type shifting operation.5 But in the next section, I will discuss a further use of Q-adjectives that cannot be accommodated by either the quantificational or predicative analyses, and therefore also not by a theory that holds them to be ambiguous between the two.

2.3 Differential *Many* and *Few*

The most substantive issue with the quantificational and predicative analyses of Q-adjectives comes from an examination of their differential uses (1d). While it has frequently been observed that *many* and *few* may be modified by a wide range of degree modifiers, it has been less recognized (though see Bresnan 1973; Jackendoff 1977;,

---

5 Though note that this would be a somewhat different sort of ambiguity than that envisioned by Partee (1989), for whom quantificational *many/few* on one reading are analyzed as involving predicative semantics; I explore this further in Chapter 4.
Schwarzschild 2006) that *many* and *few* may themselves serve as modifiers in comparatives formed with *more* and *fewer*, producing the complex expressions *many more, few more* and *many fewer* (though oddly no *few fewer*). Further examples of this ‘differential’ usage of *many* and *few* are given in (45); as evidence that this construction is robustly available, some naturally occurring examples are cited in (46):

(45)  
| a. | There were 100 seats in the lecture hall, but unfortunately many more than 100 students showed up for the lecture |
| b. | The lecture hall has 500 seats, but few more than 100 students attended the lecture |
| c. | The whole class of 100 was supposed to attend the lecture, but many fewer than 100 students actually showed up |

(46)  
| a. | Nearly 4 million Afghan children are enrolled in school, including more than 1 million girls, many more than at any point in Afghanistan’s history |
| b. | Few more than 400 Sumatran tigers survive in the wild |
| c. | The latest attempt to count the number of transient vacation rentals on Maui finds many fewer than previously estimated |

A similar pattern obtains with the excessives *too many* and *too few*:

(47)  
| There were many too many/many too few people in the room |

In their differential uses, *many* and *few* can be modified by the full range of degree modifiers that they occur with in their quantificational uses, including intensifiers, the excessive *too, so*, demonstratives, and even comparative morphology (though in some cases the result is awkward):

(48)  
| a. | Very few more than 100 students came to the lecture |
| b. | There were too many more students than allowed in the room |
| c. | I’m surprised that so many more than 100 students came to the lecture |
| d. | I’m surprised that this many more than 100 students came to the lecture |
| e. | More than 100 more students than expected came to the lecture |

Furthermore, the differential construction can be iterated:

(49)  
| a. | Many more than 100 more students than expected came to the lecture |
| b. | Many more than 100 fewer students than expected came to the lecture |
While I will postpone a more complete discussion of *much* and *little* until later in this chapter, let me briefly recall that the same pattern is also seen with *much* and *little*, which occur not only as modifiers in mass comparatives (50), but also in adjectival comparatives (51):

(50) The bucket holds much more/little more/much less than a gallon of water

(51) a. John is much/little taller than his father
    b. John is much shorter than his father
    c. John is much/?little older than Fred
    d. John is much/little younger than Fred

Importantly, the differential uses of Q-adjectives distinguish them from both ordinary adjectives and quantifiers, neither of which can occur in this position:

(52) a. *Every/most more than 100 students came to the lecture
    b. *John is heavy heavier than Fred

This alone raises the suspicion that Q-adjectives do not have the semantics of either adjectives or quantifiers.

Furthermore, and crucially, neither a quantificational nor a cardinality predicate analysis can accommodate these data. In examples such as those in (45), *many* and *few* first of all cannot be analyzed as quantifying determiners, since the logical form does not provide two appropriate sets of individuals that could serve as their arguments. But in these examples – particularly in (45c) – *many* and *few* also cannot be analyzed as cardinality predicates.

In the case of the *more than* comparatives in (45a) and (45b), this latter point is perhaps less than evident; we might suggest that in these two examples, *many* and *few* are predicated of the group of students in excess of the first 100 who attended the lecture, a notion which could be expressed formally with the entry for *many more than 100* given
below (leaving aside the non-trivial question of how this might be derived compositionally from more basic entries for 100, many and comparative morphology):

\[(53) \quad \left\{ \text{many more than } 100 \right\} = \lambda X. \exists Y \exists Z \left[ X = Y \cup Z \land Y \cap Z = \emptyset \land 100(Y) \land \text{many}(Z) \right] \]

(true of a group X iff it is the union of two non-intersecting subsets Y and Z such that Y has 100 members and Z has many members)

But in the corresponding fewer than comparative in (45c), there is no equivalent group of students to whom the property of ‘many-ness’ can be ascribed. Attempting to extend the analysis of many more than 100 in (53) to many fewer than 100 yields the implausible (54):

\[(54) \quad \left\{ \text{many fewer than } 100 \right\} = \lambda X. \exists Y \exists Z \left[ X = Z - Y \land Y \subset Z \land 100(Z) \land \text{many}(Y) \right] \]

(true of a group X iff it is the result of subtracting from a set of 100 elements a subset consisting of many elements)

Here we first of all have the question of what sort of compositional analysis would allow the move from the relationships of set (non)intersection and set union in (53) to the relationships of set inclusion and set subtraction in (54). More troubling, the entry in (54) defines the property of having cardinality of ‘many fewer than 100’ in terms of two sets whose existence is asserted: a set consisting of 100 individuals, and a subset of that set containing many individuals. This is first of all unintuitive (our original example (45c) does not seem to assert the existence of some group of 100 individuals). More fatally, this analysis renders many fewer than 100 undefined in a model that does not contain 100 individuals, an issue that for practical purposes may be unproblematic in cases such as (45c), but to take an extreme case would render the intuitively true (55) as undefined:
(55) There are many fewer than a googolplex of elementary particles in the known universe.

We might attempt to rescue the cardinality predicate view with an intensional analysis according to which in cases such as (45c), many would be predicated of a hypothetical group, some set of individuals that when added to the group of students who attended the lecture would produce a group of cardinality 100. This strikes me as a questionable undertaking; for example, one would need to explain why (45c) requires an intensional analysis, while (45a) can be analyzed extensionally. If the data in (45) were our only concern, such difficulties could perhaps be overcome. However, when we extend the inquiry a little further, it becomes clear that this solution too is unworkable in the general case.

Specifically, consider again the previously discussed examples of much and little used as modifiers in adjectival comparatives:

(56) a. John is much/little taller than his father
    b. John is much shorter than his father
    c. John is much/little older than Fred
    d. John is much younger than Fred

In (56a,b), there is no individual – real or hypothetical – of which much or little could be predicated. We would not, for example, wish to say that much in (56a) is predicated of that portion of John’s anatomy that extends above his father’s height. And even if we convinced ourselves that such an analysis was plausible, it clearly could not be extended to (56c,d), where there is not even a portion of matter available to serve as the argument for the Q-adjective. Intuitively, much and little in (56) do not describe individuals themselves, but rather the difference between the heights or ages of two individuals.

---

6 A googolplex is 10 raised to the power $10^{100}$, a number whose magnitude far exceeds the quantity of anything in the physical universe.
This, I propose, is the key to a simple and elegant analysis of differential Q-adjectives, one that can accommodate all of the data discussed in this section. In their differential uses, Q-adjectives are predicated not of individuals or sets of individuals, but rather of something like numbers or degrees. To be more concrete about this, consider again the examples in (45), featuring *many* and *few*. (45a) can be read as asserting that the difference between the number of students who attended the lecture and the number 100 has the property ‘many’, which in simple terms we could equate to being ‘large’ relative to some contextually determined standard. Similarly, (45b) can be read as asserting that this difference has the property ‘few’ (small relative to some standard). Finally, the otherwise difficult to analyze (45c) receives an essentially identical analysis: the difference between 100 and the number of students who attended was ‘many’. This analysis can of course also be extended to the adjectival examples in (56): in (56a), the difference between John’s height and his father’s has the property ‘much’ or ‘little’. In fact, this is very similar to the analysis given to differential *much* by Klein (1982).

In short, the differential cases present the crucial evidence of a use of Q-adjectives that cannot be accommodated by either a quantificational theory or a cardinality predicate theory. This is important because it demonstrates that not only are these two theories individually unable to account for the range of data investigated, but that a hybrid quantificational/cardinality predicate analysis would also not be empirically adequate.

In Section 2.5, I take the differential construction as my starting point. I show that these cases lend themselves naturally to an analysis of Q-adjectives as predicates of scalar intervals (sets of degrees), and then show that this analysis can be readily extended
to their other uses. First, however, let me introduce a separate set of data that also supports the view that Q-adjectives are predicates of something in the domain of degrees.

### 2.4 Few and Little are Operators

Consider the example below:

(57) They need few reasons to fire you

The most salient interpretation of (57) is not the wide-scope *de re* reading paraphrased in (58a) (where ‘not a large number’ scopes over *need*) nor the narrow scope *de dicto* reading in (58b) (where ‘not a large number’ is fully within the scope of *need*). Rather, (57) is most naturally paraphrased as in (58c), where *not* scopes over *need*, which in turn scopes over ‘a large number’:

(58) To fire you,…
   a. …there are not a large number of (specific) reasons that they need
   b. …they need there not to be a large number of reasons
   c. …it is not the case that they need a large number of reasons

The pattern exemplified here is an instance of what has come to be known as scope splitting, a phenomenon that has been well documented in the case of negative indefinites (Jacobs 1980; Landman 2004; Geurts 1992; among others) and comparatives formed with *few* and *little* (Hackl 2000; de Swart 2000; Heim 2006), but that to my knowledge has not been previously noted for ‘bare’ *few*. We might characterize it by saying that the semantic content of *few* can seemingly be decomposed into two components that can take scope independently from one another.

A similar point is made by examples such as the following, which show that in sentences involving *few* and a modal, an ambiguity may obtain. Consider (59a), based on examples in Heim (2006). On one reading, namely that paraphrased in (59b), we might
be describing a situation in which our department offers only a small number of advanced classes, such that it is not possible for our students to take a large number of such classes. On the alternate reading (59c), we might be describing a situation in which the standards for graduation are particularly low: students can take just a small (i.e. not-large) number of advanced classes and still graduate.

(59) a. Students can take few advanced classes
    b. It is not possible for students to take a large # of advanced classes
    c. It is possible for students to not to take a large # of advanced classes

The example in (60a) is likewise ambiguous, as evidenced by the disambiguating continuations in (60b,c):

(60) a. I need few interruptions…
    b. …to lose my concentration for the entire day!  Few > need
    c. …if I’m going to finish this project by 8pm!  Need > few

These examples have all the markings of scope ambiguities. Apparently, few itself, or some component of it, is an operator that interacts scopally with other scope-bearing elements.

A leading approach to analyzing split scope readings with negative quantifiers has been to decompose them into a positive term and a classic truth-conditional negation operator that can take separate and higher scope. Along these lines, Jacobs (1980) investigates examples such as the German (61a), whose most natural reading is that paraphrased in (61b) (an interpretation not available for the equivalent English sentence):

(61) a. Alle Ärzte haben kein Auto
    ‘all doctors have no car
    b. It’s not the case that all doctors have a car

Jacobs proposes that these cases be analyzed by decomposing the negative quantifier kein into a negation operator and a standard existential quantifier (a step that is made
particularly plausible by the fact that *kein* can be analyzed morphologically as *k+ein*, where *ein* is the indefinite article). On this analysis, (61a) has the following representation:

\[(62) \quad \text{NOT all doctors have a car} \quad 
\neg \forall x [\text{doctor}(x) \to \exists y [\text{car}(y) \& \text{own}(x,y)]] \]

Recall that in the case of *few*, a similar decompositional approach was proposed by McNally (1998) as a solution to the problem of maintaining the correct upper-bounded interpretation on application of an existential operator to a predicative *few* (van Benthem’s problem). In McNally’s account, *few* is a version of *many* which must appear in the scope of a clausal negation operator (see also Barwise & Cooper 1981 and more recently Takahashi 2006 for other decompositional analyses of *few*).

Suppose we adopt this approach, analyzing *few* as equivalent to ‘not a large number’ (leaving aside for now the technical aspects of how this might be realized):

\[(63) \quad [\text{few}] = \neg \ldots \text{a large number} \]

Split scope readings could then be analyzed by allowing the negation operator to take wide scope, above the intensional operator (64), while ambiguities such as those in (59) and (60) would reflect different possible scope relationships between the negation operator and a modal operator, as in (65):

\[(64) \quad \begin{array}{l}
\text{a. They need few reasons to fire you} \\
\neg (\text{they need a large number of reasons})
\end{array} \]

\[(65) \quad \begin{array}{l}
\text{a. Students can take few advanced classes} \\
\neg \Diamond (\text{students take a large number of advanced classes}) \quad (\text{cf. 59b}) \\
\text{b. Students can take a large number of advanced classes} \\
\Diamond \neg (\text{students take a large number of advanced classes}) \quad (\text{cf. 59c})
\end{array} \]

But in the form represented here, the decompositional analysis is not yet entirely correct. Specifically, the very same problem that we encountered in using wide-scope sentential
negation to solve van Benthem’s problem (Section 2.2.2) reappears in the context of scope splitting. For example, if we replace *few* in (64) with *very few*, we can no longer analyze the resulting sentence in terms of wide scope negation, since of course *very few* cannot be equated to ‘not a very large number’.

(66)  

a. They need very few reasons to fire you  

b. ¬(they need a very large number reasons)  

Instead, it seems that we would require something along the lines of (67), where *very* scopes above negation. But if negation here is interpreted as a simple truth-functional negation operator (a function of type \(〈t,t〉\)), it is not clear what interpretation we would need to give to *very* in order for this to be well-formed.

(67)  

\(\text{very}(\neg (\text{they need a large number of reasons}))\)

If it is indeed the case that *few* is (or contains) a negative operator, as is strongly suggested by the existence of split scope readings and apparent scope ambiguities, that operator cannot be simple truth-functional negation. But then what sort of operator could it be? A solution to this question is found in Heim’s (2006) work on *little*. Heim considers examples such as (68a), which like (59) is ambiguous between two readings that are brought out with the disambiguating continuations in (68b,c). On the first reading, (68a) is interpreted as saying that growing very little is a possibility. On the second, (68a) says that it is not possible for us to grow more than a very little:

(68)  

a. We can grow very little  
b. …or we can grow a lot – it’s entirely up to us  
c. …before we run out of space  

To account for ambiguities such as these, Heim proposes that *little* is a gradable ‘degree operator’ or generalized quantifier over degrees (type \(〈d,〈dt,t〉〉\)), which is interpreted as degree negation:
Very is likewise analyzed as a degree operator that binds the first argument (the degree argument) of little. As quantificational elements, neither little nor very can be interpreted in situ, but rather must raise at LF for purposes of interpretability. Ambiguity arises when there is the possibility of multiple landing sites. Thus the ambiguity of (68a) can be attributed to the two possible LFs in (70a) and (71a), which receive the corresponding interpretations in (70b) and (71b) (where L\_very,C is a range introduced by very as a standard of comparison):

(70)  
a. can very\_1 [t\_1 little\_2 [we grow t\_2]  
   (cf. 68b)
b. □[ very (little(λd.growth(we) ≥ d))]  
   □[very (λd.growth(we) < d)]  
   □[growth(we) < L\_very,C]  
   ∃w[growth\_w(we) < L\_very,C]

(71)  
a. very\_1 [t\_1 little\_2 can [we grow t\_2]  
   (cf. 68c)
b. very ( little ( λd.□[growth(we)] ≥ d))  
   very ( little( λd.∃w[growth\_w(we) ≥ d]))  
   very( λd.¬∃w[growth\_w(we) ≥ d])  
   ¬∃w[growth\_w(we) ≥ L\_very,C]  
   ∀w[growth\_w(we) < L\_very,C]

Without delving into the technical details of these derivations, two important points can be made about the representations in (70) and (71). First, since little is analyzed as an operator (specifically, a degree operator), it can interact scopally with other operators, such as the modal can, resulting in scope ambiguities. Secondly, very is likewise an operator; it is base-generated as a constituent with little and raises for purposes of interpretability, necessarily taking scope above little. In other words, Heim’s analysis provides for the scope relationship shown schematically in (67).

It should be apparent that Heim’s analysis of little as degree negation can be extended to few, if we take the degrees in question taken to be degrees of cardinality, i.e.

(69)  \[ \text{little} = \lambda d \lambda P_{\_d}. P(d) = 0 \]
numbers (see Takahashi 2006 for an extension of Heim’s approach along these lines). We would then have a parallel account for the split scope readings and scope ambiguities found with *few*, as exemplified in (57)-(60), including in the case with an overt degree modifier (66). Perhaps more significantly, the degree operator mechanism holds promise as a way to solve van Benthem’s problem, given that an analysis in terms of a wide-scope truth-functional negation operator runs into the same difficulties in both cases. In short, a degree-operator analysis not only accounts for the scope-related facts discussed in the present section, but also holds the potential to solve one of the crucial flaws of the non-quantificational analysis of Q-adjectives discussed in Section 2.2.

I will expand on the preliminary ideas introduced here later in this work. The crucial point that I would like to make at this stage is the following: We have seen evidence for an analysis of the Q-adjectives *little* and *few* as degree operators, per Heim (2006). Under such an analysis, *few* and *little* – once their first degree argument is saturated – are generalized quantifiers over degrees, type $\langle dt, t \rangle$. But another way to view an expression of type $\langle dt, t \rangle$ is as a predicate of a set of degrees, with those degrees in the case of *few* being numbers. This is almost exactly what I argued was required for the analysis of Q-adjectives in their differential uses. To recap, in Section 2.3 I argued that differential *many*/*few* (and by extension *much*/*little*) cannot be analyzed as cardinality predicates or quantificational expressions; rather, we must take them to be predicated of the ‘gap’ or difference between two numbers or degrees. Suppose we formalize the gap between two degrees as a set of degrees (type $\langle dt \rangle$). Then the differential cases too require that Q-adjectives have a denotation of type $\langle dt, t \rangle$. Thus the examination of the differential uses of Q-adjectives (Section 2.3) and of the scope-splitting facts (this
section) provides two converging sets of evidence for the same analysis of Q-adjectives, in which they are taken to be predicated of something in the domain of degrees.

In the next section, I will present the core of the story that I develop in this work, where, building on the insights of this section and the previous one, I propose that Q-adjectives, in all of their uses, are what I will call degree predicates.

2.5 Degree Predicates

The account that I develop in this thesis holds that Q-adjectives are in fact predicates, but that they are not predicated of pluralities or portions of matter themselves (as would be the case under a cardinality predicate analysis). Rather, they are in essence predicated of a number or degree associated with a plurality, portion of matter or eventuality, with that number or degree formalized as an interval on the scale associated with some dimension of measurement. I will call this type of predication degree predication. Let me make this more precise, beginning first with many and few, and then extending the account to much and little.

2.5.1 Many and Few

Consider again cases such as the following, examples of many and few in their differential use:

(72)  a. Many more than 100 students attended the lecture  
b. Few more than 100 students attended the lecture  
c. Many fewer than 100 students attended the lecture

As was observed in Section 2.3, in these examples many and few can be interpreted as saying something about the extent to which the number of students who attended the lecture differs from 100. In terms of degrees and scales, this can be formalized as a claim that in their differential uses, many and few express properties not of individuals, but of
gaps or intervals on the scale of natural numbers. This can be depicted by introducing a number line as a visual representation of the set of natural numbers:

In (72a), the interval from 100 to the number of students attending the lecture is characterized as large relative to some contextually determined standard; in (72b) that same interval is described as small; and in (72c), the interval from the number of students attending to 100 is described as large. In each case, it is a scalar interval that has the property of ‘many-ness’ or ‘few-ness’. Thus the use of Q-adjectives that is not readily accounted for by the either the quantificational or the cardinality predicates analysis receives a simple account under the degree-predicate view.

Having discussed the differential uses of Q-adjectives, let me return now to the simpler constructions introduced at the start of this chapter in examples (1a-c). While nothing requires that these cases be given a degree-predicate analysis instead of a cardinality predicate analysis (save the previously mentioned challenges faced by the latter), it is important to note that they can be analyzed in this way, simply by considering a scalar interval starting at 0. Thus in (1a) (repeated below), many and few can be analyzed as predicated of the interval from 0 to the number of students who attended the

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7 In Chapter 3, I will provide evidence that the scale in question is in fact not the natural numbers, but rather the rational or real numbers, i.e. a dense scale (cf. Fox & Hackl 2006). As this distinction is not relevant to the present discussion, I will for simplicity consider the scale to be the natural numbers.
lecture, stating that this interval is large (for *many*) or small (for *few*) relative to some contextually determined standard.

(74)  Many/few students attended the lecture  (1a)

Predicative uses receive a similar analysis; for example, in (1b) *many/few* is predicated of the interval from 0 to the number of John’s good qualities:

(75)  John’s good qualities are many/few  (1b)

The attributive cases can likewise be analyzed as degree predicates; (1c), repeated below, can be interpreted as presupposing that the interval from 0 to the number of students we invited was large (for *many*) or small (for *few*), and asserting that the plurality so described enjoyed the lecture.

(1)  c.  The many/few students that we invited enjoyed the lecture

In short, the degree-predicate account has the benefit of allowing for a unified analysis of *many* and *few* in all of their uses (quantificational, predicative, attributive and differential); a cardinality predicate analysis cannot do this, since under this view we would require a separate account for the differential cases. To the extent that it is preferable to identify such a unified analysis, as opposed to positing multiple lexical entries for these terms (and I would argue that a single-type analysis is the most
parsimonious and hence the most desirable), then the degree-predicate analysis is at an advantage over competing theories.

2.5.2 Much and Little

Having considered many and few, let us turn now to their counterparts much and little. Recall that the latter pair can be used both to express amount in the mass domain (as in (2)) and to express degree in the adjectival and verbal domains (as in (3)):

(2) a. (Too) much/little water is left in the bucket \textit{quantificational}
    b. *The water in the bucket is much/little \textit{predicative}
    c. The little/*much water that is left in the bucket \textit{attributive}
    d. Much/little more than a gallon of water is left in the bucket \textit{differential}

(3) a. John is much/little taller than his father
    b. John is much/!?little too tall
    c. I slept (too) little/much
    d. Much loved/little known
    e. Much/little alike/different

The examples in (2) are the counterparts of the previously discussed uses of many and few in (1) above, and thus lend themselves to a similar treatment. The only difference is that here the dimension is not cardinality (number), but rather some dimension on which quantities of substance can be measured (volume being the obvious candidate in these examples), and the scale is not the natural numbers but rather some scale associated with that dimension (here, one possibility being a scale whose points are measured in liters). Much and little can then be viewed as predicated of intervals on this scale. For example,

(76) Much/little water is left in the bucket \hfill (2a)

\begin{center}
\begin{tikzpicture}
\node (bucket) at (0,0) {amount of water left in the bucket};
\node (volume) at (3,0) {VOLUME};
\node (large) at (-1,0) {Large (\textit{much})};
\node (small) at (-2,0) {Small (\textit{little})};
\draw[->] (bucket) -- (volume);\end{tikzpicture}
\end{center}
(77) Much/little more than a gallon of water is left in the bucket  

Furthermore, and importantly, this same treatment can also be extended to the 
adjectival and adverbial cases in (3). As noted previously, (3a) can readily be interpreted 
as saying that on a scale measuring the dimension of height, the gap between the point 
that corresponds to John’s height and that corresponding to his father’s height is large (in 
the case of much) or small (in the case of little) (cf. Klein 1982).

(78) John is much/little taller than his father  

In (3b), much can likewise be viewed as expressing a property of the gap between 
John’s height and the (contextually determined) maximum allowable height. Finally, the 
examples in (3c-e) can also be interpreted as involving scales associated with some 
dimension: temporal extent in (3c); degrees of similarity/difference in (3e); perhaps 
degrees of intensity in (3d). These cases provide additional support for the degree-
predicate view. In the adverbial uses of much and little, there is of course no individual 
or portion of matter of which they could be predicated; rather, their arguments must be 
degrees or extents of some sort. In other words, an extension of the cardinality predicate 
analysis to the mass domain cannot capture these uses of Q-adjectives, just as it could not 
capture their differential uses. Rather, much and little in (3) can only be treated as degree 
constructions.
The last point is the crucial one: Since the adjectival and adverbial cases in (3), as well as the differential cases in (1d) and (2d), must be analyzed as degree predicates, it is only by extending this analysis to the uses of Q-adjectives as measures of quantities and amounts (the data in (1a-c) and (2a-c)) that we are able to achieve a unified analysis of these terms. Even if we were able to address the issues discussed with the quantificational analysis in (2.2.1), or the predicative analysis in (2.2.2), neither of these can be extended to capture examples such as those in (1d), (2d), and (3). It is only the degree-predicate analysis developed here that can provide the unified account that we desire. In what follows, I will assume the degree predicate analysis, developing a formal implementation that is able to capture the full set of data in (1)-(3), while also resolving other issues that characterize the quantificational and cardinality predicate views.

2.5.3 The Proposal

Let me summarize my central proposal. Q-adjectives, across all of the contexts in which they occur, are predicates of scalar intervals. As a preliminary statement of this, we can take the following as the semantics of Q-adjectives in their positive forms:

\begin{align*}
(79) \quad & a. \; \left[\text{many/much}\right] = \lambda I. I \text{ is large (relative to the context)} \\
& b. \; \left[\text{few/little}\right] = \lambda I. I \text{ is small (relative to the context)}
\end{align*}

Here, $I$ is a variable that ranges over scalar intervals. If we take intervals to be sets of degrees (a notion that I will make more precise in the next chapter), then the expressions in (79) are of type $\langle dt, t \rangle$, the same semantic type of Heim’s degree operators.

It will be the goal of Chapter 3 to give some formal meat to this proposal, developing a semantics of Q-adjectives that allows a compositional analysis of their full range of uses. At this point, let me make some general observations about the proposal represented in (79). First, on this view, Q-adjectives are neither ‘ordinary’ quantifiers
(like *every*) nor ‘ordinary’ gradable adjectives (like *tall*). Nevertheless, they have characteristics of both classes of expressions. Q-adjectives are in fact predicates, though predicates of scalar intervals rather than individuals. As such, it is not surprising that they exhibit some properties in common with adjectival predicates, such as occurrence in predicative and attributive positions. At the same time, it is also to be expected that their distribution differs from that of predicates over individuals (as discussed in Section 2.2.2). Looking at it from a different perspective, Q-adjectives can also be viewed as having a quantificational type: while they are not ordinary generalized quantifiers, an element of type \( \langle dt, t \rangle \) can be considered a generalized quantifier over degrees (Heim 2000). Thus it is not surprising that Q-adjectives exhibit some behavior typical of quantificational expressions, in particular scope interactions with other operators. Finally, as will be discussed in greater detail later in this work, the semantic type I propose for Q-adjectives is the same type that has been proposed for degree modifiers such as *very*, *too* and comparative morphology. Thus while Q-adjectives do not have the semantics of ordinary adjectives or quantifiers, neither is their semantic type an idiosyncratic one.

Importantly, the analysis sketched out here makes an obvious prediction. If Q-adjectives have the lexical semantics of predicates of sets of degrees, then it is to be expected that we would see this possibility reflected overtly. This is precisely what we find. The examples in (80) show Q-adjectives in predicative position, but unlike the previously discussed cases (1b), their subjects are not pluralities but rather numbers or amounts:\(^8\)

\(^8\) For a reason I do not fully understand, examples such as these are preferable when the Q-adjective is modified with *too* or another degree modifier, rather than occurring bare. It may simply be that in the
I propose that examples such as these show us the purest instance of a predicative use of Q-adjectives, where the subject of predication is a number or amount (which may be formalized as a set of degrees).

Furthermore, recall that ordinarily predicative many and few are infelicitous in the small clause complements of consider. But with a numerical or amount subject, this is perfectly felicitous:

(81) a. *I consider the guests many  
     b. *I consider the guests too many/too few

(82) a. When it comes to mistakes, I consider ten many  
     b. I consider twenty too many/too few  
     c. I consider ten gallons too much/too little

Small clause constructions have long been used as a test for predicative interpretations; by this test, the contrast between (81) and (82) provides further support for the claim that Q-adjectives are fundamentally predicated of degrees, not pluralities or portions of matter themselves.

2.5.4 Other Evidence for Degree Predicates

Before proceeding, let me briefly make note of several previous accounts in the literature that are parallel to the present one. From a syntactic point of view, Kayne (2005) argues that many and few are adjectives that modify not plural nouns themselves, but rather an unpronounced noun NUMBER that intervenes between them, such that a noun phrase such as (83a) in fact has the form in (83b):

absence of degree modification on the Q-adjective, a sentence such as twenty is many is preferably read as equative, rather than predicative.
(83)  a. few books  
     b. few NUMBER books  

Kayne argues that a range of distributional differences between *many/few* and ordinary  
gradable adjectives (e.g. *tall*) can be attributed to the presence of NUMBER in the former  
but not the latter case. For example, with regards to the possibility of an ‘unpronounced  
NP’ following Q-adjectives but not ordinary adjectives, as in (43) (repeated below),  
Kayne proposes that the phonologically null NUMBER licenses the elided NP, just as  
does the overt noun *number*, as in *A large number of linguists like phonology, but a large  
number don’t*.  

(43)  a. Many linguists like phonology, but many don’t  
     b. *Good linguists like phonology, but bad don’t*  
     c. *Numerous linguists like phonology, but numerous don’t*  

Kayne does not address the corresponding semantics of Q-adjectives, but an obvious  
semantic translation of his proposal is that *many* and *few* are predicated of a number,  
which is essentially the proposal that I have made above.  

The present account also parallels that of Schwarzschild (2006), who proposes  
that Q-adjectives (*many, few, much, little* and their modified forms), as well as ‘partitive’  
measure phrases (*two inches* in *two inches of cable*), are predicates of intervals on the  
scale of a dimension introduced by a functional head *Mon* in whose specifier position  
they occur. This proposal is motivated by a consideration of differences in the syntax  
and interpretation of two types of adjectives and measure phrases: on the one hand, Q-  
adjectives and partitive measure phrases measure a dimension that is ‘monotonic’ on the  
part-whole relationship relative to the substance noun, in that any proper sub-part of the  
entity described by the substance noun has a lesser degree of the dimension than the  
entity as a whole. By contrast, ordinary adjectives (e.g. *tall*) and ‘attributive’ measure
phrases (two-inch in two-inch cable) measure a dimension that is non-monotonic (for example, a proper part of a piece of two-inch cable may still be two-inch cable). Schwarzschild accounts for this by proposing that monotonicity is encoded in the semantics of the functional head Mon, whose role it is to introduce the dimension and interval that serves as the argument of the Q-adjective or measure phrase. On this view, the degree-based analysis thus provides an explanation for a fundamental interpretive difference between Q-adjectives and ordinary adjectives. Schwarzschild’s account does not, however, extend to the full range of data discussed here.

In a more explicitly type-driven approach, Rett (2006) proposes an account of Q-adjectives in some respects parallel to the one to be developed here. On the basis of interpretations available to quantity questions in English, French and Romanian, Rett argues that Q-adjectives do not introduce quantification over individuals, but rather must be analyzed as modifiers of sets of degrees.

Finally, the previously discussed work of Heim (2000, 2006) on degree operators and especially on little will serve as one of the building blocks in the account that I develop in the next chapter. I extend this account to all Q-adjectives, and show that in doing so we are able to capture a much broader range of facts than those which motivated its development by Heim.

Returning to the proposal offered at the start of Section 2.5.4, there are of course some significant questions that remain to be answered. What, exactly, is a degree? How are degrees introduced into the semantic representation? If Q-adjectives are not ordinary predicates over individuals, how do they compose with such predicates? If they are not quantificational (in the traditional sense), what is the source of quantificational force in
their (apparently) quantificational uses? Briefly put, what are the bits of meaning required for the analysis our original examples in (1)-(3), and how do these bits come together compositionally? These questions will be the focus of Chapter 3. In the remainder of the present chapter, I address a more general question that the entries in (79) give rise, namely the nature of the distinction between many and much (and correspondingly few and little); I then briefly discuss the broader presence of degree predication.

2.6 The Many/Much Distinction

I have proposed that many, few, much and little can all be analyzed as predicates of scalar intervals, with many and much true of an interval if it is ‘large’ relative to the context and few true of an interval if it is ‘small’ relative to the context. The question then arises as to what distinguishes them. Specifically, what determines when many/few will be used, and when much/little will be used? Certainly there is some difference between them, since members of the two pairs are not interchangeable, as a few brief examples demonstrate:

(1) a. Many/few students attended the lecture  
   a'. *Much/little students attended the lecture

(2) a. (Too) much/little water is left in the bucket  
   a'. *(Too) many/few water is left in the bucket

(3) a. Much/little taller than his father  
   a'. *Many/few taller than his father

It is typical to take this distinction to be one of count versus mass (see for example Higginbotham 1995; Chierchia 1998a), but the fact that much and little in particular occur outside of the nominal domain (as in (3)) suggests that it must be stated in other terms.
More generally, if we are going to claim that all of these terms take as their arguments scalar intervals, rather than individuals of some sort, then the explanation for their different distribution must be found in the structure of the domain of degrees rather than the domain of individuals.

Comparing again the examples in (1) (featuring many/few) to those in (2) and (3) (featuring much/little) leads to the hypothesis that the distinction between these two pairs relates to the dimension involved: many and few invoke counting, occurring when the dimension in question is cardinality (number), while much and little are associated with dimensions other than cardinality (in the above examples, volume, height, temporal extent, similarity/difference). However, an alternate possibility is suggested by Chierchia (2005), who proposes that the distinction is one of morphological agreement. On Chierchia’s analysis, the pairs much/many and little/few are each single quantifiers; many and few are their plural forms, while much and little are their corresponding (suppletive) singular forms. As evidence, the distinction mirrors that between singular and plural demonstratives:

(84) a. Much water/this water
    b. Many dogs/these dogs

Further support for this comes from the Romance languages, where the corresponding distinction is expressed via number morphology (e.g. Spanish mucho ‘much’ versus muchos ‘many’). Chierchia’s account also provides an explanation for the exceptional status of much and little as the only (apparent) exclusively mass quantifiers in English: they are in fact not mass-only quantifiers, but rather morphological variants of mass/plural quantifiers much/many and little/few (differing only from other mass/plural quantifiers such as all, most and some in having separate singular and plural forms).
In what follows, I will argue that while agreement may play some role, the crucial determiner of the *many/much* distinction is dimension.

Notice first that pluralities of entities can be measured on dimensions other than cardinality (number), as exemplified below:

(85)  
| a. Five potatoes | NUMBER |
| b. Two pounds of potatoes | WEIGHT |

But now note that a question formed with *many* is only felicitously answered by a numerical answer, that is, by one that supplies a cardinality. Thus in answer to (86a), (86b) is acceptable, but (86c) is not (we can almost hear Speaker A responding with ‘yes, *but how many??*’)

(86)  
| a. SPEAKER A: How many potatoes did you buy? |
| b. SPEAKER B: Five |
| c. SPEAKER B: #Two pounds |

A similar pattern can be observed in comparatives and equatives. Consider the following scenario:

(87) Fred has 7 potatoes weighing 4 pounds in total  
John has 10 (smaller) potatoes weighing 3 pounds in total

In this situation, (88a) is true, while (88b) is false:

(88)  
| a. Fred does not have as many potatoes as John | TRUE |
| b. John does not have as many potatoes as Fred | FALSE |

Similarly, (89a) is true, while (89b) is false (note that the corresponding comparative based on *many* is uninformative in this respect, since the *many/much* distinction collapses in the comparative, with both sharing the form *more*):

(89)  
| a. Fred has fewer potatoes than John | TRUE |
| b. John has fewer potatoes than Fred | FALSE |
If the presence of _many/few_ versus _much/little_ were determined simply by morphological agreement with a plural noun, then we would expect that their interpretation could reference any dimension on which the given plurality could be appropriately measured. But the examples in (87)-(89) show that this is not the case; rather, _many/few_ can only reference cardinality. The conclusion must be that _many_ and _few_, and the complex expressions formed from them, encode cardinality.

Further evidence that number agreement cannot be the only determining factor is provided by examples where both _many_ and _much_ may occur in what appears to be the same syntactic environment. A particular example is the differential construction. In (90) and (91), both _many_ and _much_ are at least marginally acceptable:

(90) a. We invited many more than 100 people  
    b. ??We invited much more than 100 people

(91) a. ?We waited for many more than twenty minutes  
    b. We waited for much more than twenty minutes

But _many_ is preferable to _much_ in (90), while the opposite is the case in (91) (though I should note that there seems to be some speaker variation in this respect). The difference, I would argue, relates to the dimension that is referenced: in (90) the dimension is cardinality, while in (91) it is duration.

Perhaps the strongest evidence that the difference is not primarily one of agreement is provided by idiosyncratic cases where a plural noun describes a mass-like entity, examples being _scrambled eggs_ and _mashed potatoes_. Note first that these expressions take plural agreement both within the noun phrase and externally to it:

(92) a. Everyone likes these/*this mashed potatoes  
    b. The mashed potatoes are/*is cold
But to the extent that they can be quantified with a Q-adjective, it must be with

*much/little*, not with *many/few*:

(93)  
a. How much mashed potatoes should I make?  
b. *How many mashed potatoes should I make?*

(93a) is perfectly acceptable, while (93b) is odd, except on the reading where we are talking about individual potatoes that will be mashed. Here we have a mismatch between morphological number (plural) and the dimension on which the entity denoted can be measured (cardinality not being a possibility); it is the latter that determines the choice of Q-adjectives. This is the most conclusive evidence that the determining factor in the selection of Q-adjective is the dimension that is involved, not the morphological singularity or plurality of the substance noun.

Before proceeding, however, it is worth noting that there is other evidence that agreement does play some role in the *many/much* distinction. Consider again the examples in (86) and (88). For myself, replacing *many* with *much* yields results in ungrammaticality (though there again seems to be some speaker variation here):

(94)  
*How much potatoes did you buy?*

(95)  
*John does not have as much potatoes as Fred*

Oddly enough, short of a periphrastic construction, there is no way to ask what amount (by weight) of potatoes was purchased, or to express John’s having a lesser amount (by weight) of the tubers. *Many* is excluded because the dimension is not cardinality, but *much* is also excluded, apparently due to some sort of number mismatch. Note also that *much* improves if the syntactic environment is changed: if the noun is elided (96a), or in a partitive construction (96b), in which cases the feature mismatch is eliminated:

64
The preceding evidence demonstrated that *many/much* need not agree with the subsequent noun in any strict sense; but the infelicity of examples such as (94) and (95) suggests there may be some looser form of agreement involved.

To summarize, the distinction between *many/few* and *much/little* is primarily one of dimension: *many* is associated with cardinality, while *much* is associated with other dimensions. I will assume this distinction in what follows.

### 2.7 Degree Predication Elsewhere in the Grammar

The discussion in this chapter has shown that Q-adjectives must be analyzed as predicates of scalar intervals, rather than quantifying determiners or cardinality predicates. Degree predication, as I have called this sort of semantics, is thus one means by which natural language expresses quantity. So just how widespread is degree predication? While my focus in this work is on Q-adjectives, in this section I briefly present evidence that degree predication is not limited to this class, but is also found with other quantity expressions, and in the adjectival domain as well.

#### 2.7.1 Some Extensions

Earlier, I cited the differential uses of Q-adjectives (e.g. *many fewer than 100; much lighter than Fred*) as crucial evidence that they denote predicates of scalar intervals rather than predicates of individuals, since the comparative construction does not make available a plurality or portion of matter that could serve as argument. The comparative can therefore serve as a test for degree-based interpretations: expressions that occur as
modifiers in comparatives must have interpretations that reference degrees or scalar intervals, rather than individuals.

If we apply this test to cardinal numerals (97) and measure phrases (98)-(99), we see that they too must have degree-based semantics (at least optionally), as they can occur as modifiers in comparatives:

(97)  
  a.  **Twenty** more students than expected came to the lecture  
  b.  **Two hundred** fewer runners finished the race this year than last year

(98)  
  a.  We have **five gallons** more gasoline than we need  
  b.  I bought **three grams** less than an ounce of gold

(99)  
  a.  The Empire State Building is **506 feet** taller than the Chrysler Building  
  b.  John is **twenty pounds** lighter than Fred

An important consequence of this observation is that recent analyses of cardinal numerals as cardinality predicates (type ⟨et⟩; e.g. Landman 2004) or noun modifiers (type ⟨et,et⟩; e.g. Krifka 1999, Ionin & Matushansky 2006) cannot tell the full story. Since cardinal numerals (like Q-adjectives) can occur in a context in which there is no plurality to serve as argument, they must have interpretations of a semantic type that does not need one. In fact, the cardinal numerals in examples such as (97) lend themselves to the very same analysis that I proposed for Q-adjectives, namely as predicates of scalar intervals. For example, (97b) is true iff the interval between the number of runners finishing this year and the number finishing last year is (at least) of length 200. (I should note, however, that the comparative facts do not provide evidence to decide in favor of this analysis over one in which numerals simply denote degrees. I will have more to say about cardinal numerals later in this work.)

A similar point can be made about measure nouns such as gallon, pound, and so forth. It is rather common (see e.g. Landman 2004) to treat such nouns as encoding
measure functions. On this view, gallon has the interpretation in (100), allowing the
derivation of the correct semantics for something like five gallons of gasoline as in (101):

\[
(100) \; \llbracket \text{gallon} \rrbracket = \lambda d \lambda M. \mu_{\text{gallons}}(M) \geq d, \text{ where } M \text{ is a variable ranging over portions of matter}
\]

\[
(101) \; \llbracket \text{five gallons of gasoline} \rrbracket = \left( \llbracket \text{gallons} \rrbracket \left( \llbracket \text{five} \rrbracket \right) \right) \cap \llbracket \text{gasoline} \rrbracket = \lambda M. \text{gasoline}(M) \& \mu_{\text{gallons}}(M) \geq 5
\]

But the data in (98)-(99), which show that measure phrases occur as modifiers in
comparatives, demonstrate that they too must have denotations that reference degrees, not
individuals (perhaps in addition to a measure-function denotation of the form in (100)).

The differential test can also be extended to vague quantity nominals such as a lot. Consider the following examples of pseudopartitives:

\[
(102) \; \begin{align*}
a. & \; \text{A group of people came to the party} \\
b. & \; \text{A lot of people came to the party} \\
c. & \; \text{A couple of people came to the party}
\end{align*}
\]

While these cases are superficially similar, results diverge when we move to a
differential construction:

\[
(103) \; \begin{align*}
a. & \; *\text{A group fewer people than expected came to the party} \\
b. & \; \text{A lot fewer people than expected came to the party} \\
c. & \; \text{A couple fewer people than expected came to the party}
\end{align*}
\]

By this test, a couple and a lot can be analyzed as degree predicates; a group cannot.

Other nominals that behave like a lot are a bunch, a ton, lots, a shitload, and many more.

In short, degree predication is common in the expression of quantity and amount.

But I would like to suggest that the phenomenon is even more widespread than this.

Specifically, as will be seen in the next section, gradable adjectives may also in certain
circumstances denote degree predicates.
2.7.2  A Problematic Case – And Its Consequences

A pattern that has rarely been noted (though see Zamparelli 1995 for a brief mention) is that Q-adjectives may be conjoined with ordinary adjectives, as seen below:

(104)  a. Professor Jones’ many and important contributions to syntactic theory
       b. The fans were many and loud
       c. The stains on the shirt were few and small
       d. Air connections to Europe are few and expensive
       e. The ingredients are simple and few

On the standard view that conjunction is restricted to elements of the same semantic type (Partee & Rooth 1983), these examples are problematic for the theory I have developed here. Since ordinary adjectives of the sort exemplified above are typically analyzed as (perhaps gradable) predicates of individuals, we would not predict that they could be conjoined with expressions that denote predicates of intervals on a scale. Instead, these facts would seem to support the cardinality predicate account, under which each of the examples in (104) would involve a simple conjunction of two first-order predicates.

However, a closer look shows that these conjunctions do not behave precisely like conjunctions of two ordinary adjectives. First, what has not (to my knowledge) been observed up to now is that only gradable adjectives may be conjoined with Q-adjectives:

(105)  a. The fans were many and loud
       b. ??The fans were many and American

(106)  a. The tables in the hall were many and large
       b. ??The tables in the hall were many and octagonal
       c. ??The tables in the hall were many and wooden

(107)  a. The stains on the shirt were few and small
       b. ??The stains on the shirt were few and green

Already this distinguishes Q-adjectives from ordinary gradable adjectives. While there are some constraints on how adjectives may be conjoined (a topic that has not to my knowledge received serious attention in the literature), there is no general prohibition
against the conjunction of gradable adjectives with their non-gradable counterparts. Size adjectives are particularly felicitous in this sort of conjunction (108a-c), and other adjectives are possible as well (108d-f):

(108)  a. The tables in the hall were large and octagonal  
       b. The stains on the shirt were small and green  
       c. The aliens were tall and blue  
       d. The floor was wooden and smooth  
       e. I got the shoes because they were red and cheap  
       f. We had heard that most of the guests were American and rude

A second and more subtle difference is that in examples of the form in (104), the conjoined Q-adjective and adjective are in a sense interpreted jointly. In each of these cases, the adjective is interpreted as in some way amplifying the cardinality established by the Q-adjective (or vice versa). In (104a), Prof. Jones’ contributions were not just many in number, but were important as well; in (104b) the fans were not only numerous, but loud as well; in (104c), not only were the stains few in number, but they were small as well; in (104d), not only are there few air connections to Europe, but those that exist are expensive; and so forth.

To put this another way, in each case the Q-adjective/adjective conjunction can be interpreted as positioning the subject relative to some complex dimension formed on the basis of cardinality and a dimension consistent with the gradable adjective. For example, (104a) says something about the total importance of Prof. Jones’ contributions (a function of their number and their individual importance). Similarly, (104c) describes the total area of the stains (a function of their number and the size of each). The other examples can likewise be analyzed with reference to a compound dimension; we might take the relevant dimensions to be those described below:
(109)  a. many and important (contributions)
TOTAL IMPORTANCE (# $\times$ INDIVIDUAL IMPORTANCE)

b. many and loud (fans)
TOTAL LOUDNESS (# $\times$ INDIVIDUAL LOUDNESS)

c. few and small (stains)
TOTAL AREA (# $\times$ INDIVIDUAL SIZE)

d. few and expensive (air connections)
ACCESSIBILITY (# $\times$ AFFORDABILITY)

e. simple and few ingredients
OVERALL SIMPLICITY (# $\times$ INDIVIDUAL SIMPLICITY)

As evidence that this sort of relationship is actually required for the interpretation of Q-adjective/adjective conjunctions, consider the following example:

(110) ?The senators supporting the proposal were many and tall

On first reading (110) is bizarre, there being no obvious way that the height of the senators amplifies their ‘many-ness’. But the sentence improves considerably when we imagine a scenario in which the number of votes a senator has is proportional to his or her height, such that a group of senators who are many and tall has greater voting power than a group who are many in number but average height or short. In other words, the sentence is felicitous in a context in which we can infer the relevant compound dimension, namely voting power.

Again, we do not observe the same effect with gradable adjectives. In the conjunctions of gradable and non-gradable adjectives in (108), there is no sense in which the second conjunct amplifies the first. Conjunctions of two gradable adjectives also need not have this sort of interpretation: in (111a), beautiful does not amplify small; there is no obvious sense in which villages that are small and beautiful possess more of some compound property than those that are small but not beautiful. Similarly, in (111b),
conversations that are short and friendly are not taken to have any other property than shortness and friendliness.

(111) a. The villages in the surrounding area are small and beautiful
    b. Our conversations were short and friendly

Simply speaking, the conjunctions of ordinary adjectives exemplified in (108) and (111) can be analyzed as the ordinary Boolean conjunction of two predicates:

\[(tall \text{ and blue}) = [\text{tall}] \cap [\text{blue}] = \lambda x. \text{tall}(x) \land \text{blue}(x)\]

\[(\text{short and friendly}) = [\text{short}] \cap [\text{friendly}] = \lambda x. \text{short}(x) \land \text{friendly}(x)\]

But when it comes to Q-adjective/adjective conjunctions, something more complicated seems to be going on.

The explanation I would like to propose is the following. Q-adjective/adjective conjunctions are degree predicates. They denote predicates of ‘intervals’ on the scale of some compound dimension formed on the basis of cardinality and the dimension associated with the adjective. As a concrete example, consider (104c). Above I proposed that this example references the dimension total area (formed as the product of number and (individual) size). We might express this by saying that few and small is predicated of a two-dimensional interval on the scale of this dimension (the interval corresponding to the number of stains and their size), returning true if that interval is small. Visually:

(114) The stains on the shirt were few and small  (104c)
The other examples lend themselves to a similar treatment.

The restrictions discussed above on Q-adjective/adjective conjunctions now start to make sense. Only gradable adjectives may be conjoined with Q-adjectives, because gradability is necessary to introduce a dimension that can combine with cardinality to produce the required compound dimension. And if a relationship between cardinality and the dimension in question cannot be established (as in (110)), the result is odd, because we are unable to infer an appropriate compound dimension to which the conjunction can apply.

In summary, rather than being a potential problem for the theory of degree predication that I have developed in the present paper, the patterns of conjunction of Q-adjectives and ordinary adjectives actually support this analysis.

But importantly, the analysis presented here requires that in an example such as few and small, small must in some sense be predicated not of the stains themselves, but of their size. In other words, the gradable adjective small must be interpreted as a degree predicate. This at first seems at odds with the orthodox view of gradable adjectives as predicates of individuals (as discussed earlier in this chapter). But in fact, the conjunction cases are only one example of a broader pattern in which gradable adjectives may describe a dimension associated with an individual, rather than the individual itself.

For example:

(115)  a. John’s tall height made him a natural choice for the basketball team  
      b. Although the size of the stains was small, they were so obvious that I couldn’t wear the shirt  
      c. Fred was wise despite his young age  
      d. the girl’s beautiful appearance (cf. the beautiful girl)
Gradable adjectives may also be predicated of measures on a scale of the dimension with which they are associated:

(116)  a. Seven feet is tall (cf. Shaq is tall)
       b. Twenty pounds is light (cf. the pack is light)
       c. Forty is young (Fred is young)

That this is in fact predication is demonstrated by the fact that the corresponding small clauses are acceptable as the complements of *consider*:

(117)  a. I consider seven feet tall
       b. I consider twenty pounds light
       c. I consider forty young

This is the same pattern that was observed with Q-adjectives (section 2.5 above), and that I argued was supportive of the degree predicate analysis.

The consequence is that in addition to an interpretation as gradable predicates of individuals, gradable adjectives require a secondary interpretation as gradable predicates of scalar intervals, one that is parallel to those given earlier for Q-adjectives:

(118)  a. \([\text{tall}_{(d,e)}] = \lambda d \lambda x. \text{HEIGHT}(x) \geq d\)
       b. \([\text{tall}_{(d,(d,t))}] = \lambda d \lambda I_{\text{Height}}. d \in I\)

In short, degree predication – which I have argued plays a crucial role in the semantics of number and amount – is also one of the means by which degree may be expressed in the adjectival domain. There is clearly a non-trivial question as to how widely available are the degree-predicate interpretations of gradable adjectives. As noted above, gradable adjectives are not allowed in differential constructions (e.g. *tall taller than 6 feet*), evidence that their degree predicate interpretation is not freely available. I must leave this topic as a question for future study. But the general point, I believe, holds: degree
predication is found beyond the realm of Q-adjectives, and beyond the expression of
number and amount; this type of predication is found with ‘ordinary’ adjectives as well.

2.8 Conclusion

The starting point for this chapter was the broad distribution of Q-adjectives,
which distinguishes them from other quantificational expressions, and which poses a
challenge for a unified semantic analysis. I have argued that such a unified analysis is in
fact possible. Q-adjectives, across the range of their uses, can be analyzed as degree-
predicates: predicates of intervals on a scale associated with some dimension of
measurement. More specifically, many and few are predicates of intervals on the scale of
cardinality (number), while much and little are predicated of intervals on the scales of
another amount dimension (e.g. volume), or a dimension associated with a verbal or
adjectival expression. I have also shown that neither a quantificational nor a cardinality
predicate account is able to match the degree-predicate analysis in empirical coverage.

There is of course an obvious alternative to the account that I have argued for in
this chapter. Perhaps Q-adjectives are ambiguous between a degree predicate
interpretation and an interpretation at a different semantic type: a quantifier, or a
cardinality predicate. Such an analysis would be less parsimonious than the one that I
have proposed here. But depending on one’s opinions of type ambiguities, that might not
be considered a particularly serious drawback, and in any case Q-adjectives would not be
alone in this respect (conjunction offering an obvious example of lexical items that
require interpretations at multiple semantic types).

However, recall that both the quantificational and the cardinality predicate
analyses are not free from problems, even in accounting for the data that we would expect
them to handle. The quantificational analysis does not easily accommodate the
predicative and attributive uses of Q-adjectives, while the cardinality predicate analysis
runs into difficulties with quantificational *few* and *little*. Furthermore, if we take Q-
adjectives to have interpretations as predicates of individuals, we must then explain why
they do not have the same distribution as other predicative expressions. If it can be
shown that solutions to these problems fall out naturally from the degree-predicate
analysis, then this approach will have an advantage over a multiple type analysis not just
in its parsimony, but in its explanatory power. In the next chapter, I will argue that this is
in fact the case.

In this chapter, I have sketched out the degree predicate theory in conceptual
terms. The real challenge will be whether it can be implemented compositionally. I turn
to this next.
Chapter 3  
A Formal Implementation  

3.0 Introduction  

In the last chapter, I presented an argument that Q-adjectives be analyzed as degree predicates, that is, predicates of scalar intervals. In the present chapter, I develop a formal implementation of this proposal. Here, I show that much of the semantic content that is traditionally ascribed to Q-adjectives themselves is instead contributed by a set of null functional elements and semantic operations; this includes quantification over individuals, the introduction of degrees and scales, and the introduction of a contextually determined standard of comparison. These elements are able to compose in different combinations and syntactic configurations, and it is this that allows Q-adjectives to occur in the wide range of positions in which we find them. This analysis will therefore support a view in which syntactic structure contributes to semantic interpretation.

The organization of this chapter is the following. Sections 3.1 and 3.2 lay the syntactic and semantic groundwork for the account that I develop. Section 3.3 introduces the building blocks of the analysis, giving the basic lexical entries for Q-adjectives themselves as well as the other elements that will constitute components of the analysis. Sections 3.4-3.8 develop compositional analyses of the semantics of Q-adjectives across the positions in which they occur: quantificational (3.4), differential (3.5), predicative (3.6), attributive (3.7), and adverbial (3.8). Conclusions are summarized in Section 3.9.
3.1 Syntactic Preliminaries

The basic syntactic framework that I will assume in this work that of Principles and Parameters, and when the level of detail warrants, its current instantiation as Minimalism (Chomsky 1995 and followers). To the extent possible, though, it is my intent that the account developed here be independent of any particular version of the theory.

Since much (though not all) of what I will cover in this section relates to noun phrases, let me say something more about my assumptions in this area. Here, I follow Abney (1987) in taking the structure of the nominal domain to closely parallel that of the sentential domain: the NP (the projection of the lexical category N) is dominated by a series of functional heads and their maximal projections, the uppermost of which are the functional head Determiner (D) and its maximal projection DP (just as the verb phrase VP is dominated by a series of functional projections, the highest of which is IP, or in more recent terms, TP). The structure I assume is thus the following:

\[
(1) \quad [DP \ldots [FP_1 \ldots [FP_2 \ldots [FP_n \ldots [NP \ldots]]]]]
\]

There have of course been numerous proposals as to the number and nature of the functional heads that intervene between D and the NP. To summarize a few of the most influential, Abney (1987) makes use of the functional category QP (which houses quantificational elements including many and few), dominated by the category DegP (in whose head position degree morphology resides). Zamparelli (1995) introduces the Kind Phrase (KiP), Predicative Determiner Phrase (PDP) and Strong Determiner Phrase (SDP), with each of these layers corresponding to a different semantic type at which the nominal

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9 I use the term ‘noun phrase’ in a pretheoretic sense to refer to the extended projection of the noun, without specifying which layer(s) of functional structure, if any, are present.
expression can be interpreted. Based on data from Mandarin and Cantonese, Cheng & Sybesma (1999) propose the existence of a Classifier Phrase (ClP) dominated by a Numeral Phrase (NumP). Borer (2005) argues for three projections, ClP, #P and DP, each of which introduces an open value which must be assigned a range by an element in its head or specifier position. From another perspective, Cinque (1994) proposes a series of Agr heads in DP, in whose specifier positions adjectives are generated, an approach intended to capture the well-known restrictions on adjective ordering. In a recent review, Svenonius (2008) undertakes a consolidation of recent work in this area. Drawing on cross-linguistic patterns in the distribution and ordering of elements in the noun phrase (adjectives, determiners, demonstratives, classifiers, numerals, plural morphology), Svenonius proposes the following sequence of functional heads dominating the NP: $n$ (‘little $n$’, which houses nominalizing morphology), SORT/PL (the position of sortal classifiers and plural morphology), UNIT (the positions of numerical classifiers, numerals themselves being in the specifier position of its maximal projection), ART (apparently equivalent to D) and DEM (for demonstrative).

Without attempting a detailed comparison of these various accounts, let me point out one element that they all share, namely a projection that is below D (if D is present) but above the attachment site of ordinary adjectives, which is the locus of numerical expressions and/or weak quantifiers more generally. In the above summary, the relevant projections are Abney’s QP, Zamparelli’s PDP, Cheng & Sybesma’s NumP, Borer’s #P, and Svenonius’ UNITP. Across accounts, the basic motivation for such a projection is the co-occurrence of determiners, numerals and adjectives, with the most common word
order cross-linguistically being Det Num Adj N (Greenberg 1963; Cinque 2005; Svenonius 2008).

(2) the three hungry unicorns

In what follows I assume the existence of a projection of this nature, which for reasons that will become clear below I name MeasP. I take MeasP to be the location of not just numerals, but also Q-adjjectives, as well as certain measure phrases. To refine (1), the structure I assume is therefore the following (remaining neutral on what additional projections may be present between D and Meas, or between Meas and the NP):

(3) [DP … [MeasP… [NP …]]]

Let us turn now to the syntax of Q-adjjectives themselves. Current syntactic theory takes prenominal descriptive adjectives to head maximal projections that are base-generated in (or move to) the specifier position of some functional head within the DP (see especially Cinque 1994). With regards to Q-adjjectives, there is less consensus. Some authors have taken them in at least some of their uses to be heads on the main line of projection from NP to DP (notably Abney 1987) or dominating the DP (Giusti 1997). Others (Bresnan 1973; Jackendoff 1977; Zamparelli 1995; Schwarzschild 2006) analyze Q-adjjectives as heads of their own maximal projections. Zamparelli (1995) discusses this question in detail, and offers several points of support for the latter view. Most basically, the existence of obvious morphological parallels between ordinary adjectives and Q-adjjectives make it unlikely that the former would be maximal projections but the latter heads. This is further supported by the possibility of conjoining the two (a construction I considered in Chapter 2); we would not expect heads to conjoin with maximal projections.
The required repairs were many and expensive.

The fact that Q-adjectives can be modified by degree modifiers (as, too, very, etc.) also suggests that they must be part of phrasal constituents; in particular, Zamparelli notes that the possibility of iterating very in an example such as (5c) suggests that a sequence such as very few is not itself some sort of complex head:

(5) a. too much water
b. the nearly as many people
c. (very) very few students

Perhaps most conclusively, Zamparelli points out that in Romanian (data from Giusti 1992), Q-adjectives do not block N-to-D movement, a fact that would be unexpected if they were heads of a functional projection between N and D, but expected if they were maximal projections located in a specifier position.

To these syntactic points we may add a semantic argument. Recall that Q-adjectives along with their degree morphology can be interpreted as taking scope above their surface position; for example, (6) can be read with very few scoping over can (the reading according to which it is not possible for students to take more than a very small number of advanced classes):

(6) Students can take very few advanced classes

If few were in a head position between N and D, and very in the next-higher head position, it is not clear how this scope relationship could come about; but if we take very and few to constitute a maximal projection, then LF movement of this phrasal category above the modal can establishes the desired scope relationship.

I therefore follow Zamparelli in concluding that Q-adjectives together with their degree morphology are phrasal categories, which are located in the specifier position of
the Meas head. Specifically, I take Q-adjjectives head a maximal projection QP (for Quantity Phrase), parallel to the Adjective Phrase (AP).

Degree modifiers can likewise be analyzed as heads of a maximal projection DegP. This includes both free morphemes (*very, so, as*) and bound comparative and superlative morphemes (*-er, -est*). Within this approach, there are then two possibilities for how ordinary adjectives and Q-adjectives combine with their degree morphology, both of which are well-represented in the literature. On the first, the adjective or Q-adjective is the head of the overall phrase, with the DegP situated in its specifier position (Bresnan 1973; Jackendoff 1977; Bhatt & Pancheva 1995; Heim 2000). On the other, the degree modifier itself is the head of the overall phrase, selecting for an Adjective Phrase or Quantifier Phrase as its complement (Zamparelli 1995; Kennedy 1997; Corver 1997). These two alternatives are depicted in (7):

(7)  

In what follows, I assume the first of these options, on both syntactic and semantic grounds. The semantic considerations I will discuss below. Regarding the syntax, the structure in (7a) provides the necessary syntactic positions for all of the elements that are required for the analysis of Q-adjectives (or ordinary adjectives) and their degree morphology. Specifically, I take a differential comparative to have the structure in (8):
Here, the *than* clause is merged as the complement of the Deg head -er, and the
differential Q-adjective is situated in SpecDegP. By contrast, the structure in (7b) makes
only one position, SpecDegP, available for the *than* phrase and the differential Q-
adjective; to accommodate both, some additional layer of structure would be required.

With regards to (8), we are of course faced with a not so trivial issue, namely that
the syntactic structure does not map to the word order found in the comparative (or other
degree modifier constructions). Rather, the *than* phrase occurs further to the right, either
immediately following the Q-adjective (when the *than* phrase directly names a degree) or
further to the right (in the case of a more complex *than* phrase):

(9)   a. Fewer than 100 students attended the lecture
     b. Fewer students than professors attended the lecture
     c. Fewer students attended the lecture than the concert
     d. Fewer students attended the lecture this year than last year

Here, I will follow a common (if not entirely satisfactory) solution in taking the *than*
phrase to be extraposed to a right-adjoined position (see e.g. Heim 2000), with the
adjunction site being the QP in the case of numerical than phrases such as (9a), or a higher position in the DP or outside of the DP in cases such as (9b-d). As for how the comparative morpheme comes to attach to the Q-adjective, it is common to take this to occur via a spell-out rule in the PF component (e.g. Bhatt & Pancheva 2004). A more theoretically grounded approach would be to take the Q-adjective to merge with the comparative morpheme -er already attached, and to check features against an abstract comparative feature in the Deg head. I assume this to be the case, but for clarity show the comparative morpheme under the Deg head.

3.2 Semantic Preliminaries

3.2.1 The Structure of the Domain

Throughout this work, I assume a type-theoretic framework in which we have an (infinite) set of semantic types that is built up via the following definition:

(10) The set of types is the smallest set such that:
    a. It contains some set of basic types, including e, t and (as will be described below) d
    b. If α and β are types, then ⟨α, β⟩ is a type

With regard to elements of type e, I follow Link (1983) and Landman (2004), among others, in taking the domain of individuals De to have the structure of an atomic mereology, consisting of a set of atoms ATOM(De) plus a set of pluralities formed via the sum operator ⊔ and partially ordered via the part-of relationship ⊑. In the simple

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10 I do not address the question of why this extraposition occurs, or what determines the choice of landing site. If one wants to avoid rightward movement, the relationship between the surface position of the than phrase and the element in the complement position of the DegP can be viewed as one of coindexation; alternately, one could follow Bhatt & Pancheva (2004) in taking the than phrase to merge in its surface position after countercyclic movement of the DegP.

11 I assume there are additional basic types beyond these three (e.g. the type of events), but only these will be needed for the analysis that follows.
case in which the universe contains four atoms $a$, $b$, $c$ and $d$, the structure is therefore that depicted in (11):

\begin{equation}
\text{(11)}
\end{equation}

By way of terminology, in this framework $a \sqcup b$, $a \sqcup b \sqcup d$, $a \sqcup b \sqcup c \sqcup d$, etc. can be called plural individuals or pluralities, each formed as the sum over some set of atoms (atomic individuals).

More formally, the structure is the following:

\begin{equation}
\text{(12)}
\end{equation}

$D_e = <D_e, \sqcup, \sqsubseteq>$, a set of elements $D_e$ formed via the sum operator $\sqcup$ and partially ordered via the ordering relationship $\sqsubseteq$, which satisfies the conditions in (i) and (ii):

i) **Atomicity:**
For all $a \in D_e$, there exists $X \subseteq \text{ATOM}(D_e)$ such that $a = \sqcup X$

ii) **Closure:**
For all $a,b \in D_e$, $a \sqcup b \in D_e$

where the following definitions are employed:

$a \sqcup b$, the sum of $a$ and $b$, is the smallest element $c \in D_e$ such that $a \sqsubseteq c$ and $b \sqsubseteq c$

$\sqcup X$, the sum of the set $X$, is the smallest element $a \in D_e$ such that for all $x \in X$, $x \sqsubseteq a$

$\text{ATOM}(D_e) = \{x \in D_e : \forall y \in D_e [y \sqsubseteq x \rightarrow y = x]\}$
Having discussed the structure of the domain of individuals, it remains to relate it
to expressions of natural language whose extensions contain individuals or sets thereof.
To this end, I follow Link (1983) in introducing the pluralization operator *, defined as
follows for any one-place predicate P:

\[(13) \quad *P = \{x \in D_c: \exists Z \subseteq P: x = \sqcup Z\}\]

I further assume that morphological pluralization corresponds to semantic pluralization,
such that a singular count noun denotes a set of atomic individuals (a subset of
ATOM(\(D_c\))), while the corresponding plural denotes the set of plural individuals formed
from it via the * operation:

\[(14) \quad a. \quad \llbracket \text{book} \rrbracket = \lambda x. \text{book}(x)\]
\[(14) \quad b. \quad \llbracket \text{books} \rrbracket = \lambda x.*\text{book}(x)\]

Note that with the definitions in (13) and (14), atoms are included in the denotation of a
plural count noun; for example, the denotation of the noun books contains individual
(singular) books. This will not prove crucial to the story developed below, and if desired
these definitions could be revised to exclude atoms from the denotations of plural nouns.

With regards to mass terms, I take the mass domain to also have a lattice
structure, which is closed under sum formation, and whose elements are partially ordered
via a part-of relationship. I remain neutral with regards to the relationship between the
mass and count domains, and to the question of whether the mass domain is atom-less or
contains atoms that may be vague or not perceptible (see Chierchia 1998a for discussion).

3.2.2 Degrees and Scales

As the discussion in Chapter 2 has no doubt made clear, I am following the
tradition of Cresswell (1977) and many later authors (von Stechow 1984, 2006;
Wilkinson 2002; Heim 2000, 2006; Hackl 2000; Bhatt & Pancheva 2004; Fox & Hackl 2006) in analyzing gradability and measurement via reference to scales and degrees. Up to this point, however, I have been using the terms *dimension*, *scale* and *degree* in a rather loose way. Before proceeding it is necessary to make these notions more precise.

To start, a dimension in the broadest sense can be thought of as a parameter on which an entity can be measured. But what is a parameter, and what exactly do we mean by ‘measuring’ an individual? A more neutral way to put this is to say that a dimension is a property that an individual can have more or less of, and therefore that two individuals can be compared on.

We can distinguish two types of dimensions. The first type are dimensions of amount. These are the monotonic dimensions of Schwarzschild (2006), dimensions that track the part-whole relationship associated with an entity, in that any proper subpart of an entity will have a lesser amount of the dimension than does the entity as a whole. Volume, weight, mass, and crucially number (cardinality) are all monotonic in this sense. For example, suppose we have a set $S$ of books whose cardinality is $n$. Then for any proper subset $S'$ of this set, the cardinality of $S'$ will be $n' < n$. The second type of dimensions are the non-monotonic ones, those that do not track the part-whole structure associated with an entity. Examples are temperature, height, beauty and many others. For example, a proper subpart of a quantity of fifty-degree water still has a temperature of fifty degrees; likewise, a vertical slice of a 100-foot-tall building is still 100 feet tall.

In contrast to a dimension, a scale is an abstraction, a formal object whose structure tracks some dimension. For example, we might visualize the scale associated
with the dimension of cardinality (number) as in (15a), and the scale associated with the
dimension of height as in (15b).

\[(15)\]

\[\text{a. CARDINALITY} \quad \begin{array}{c} 0 \ 1 \ 2 \ 3 \ 4 \ \ldots \ n \end{array} \]

\[\text{b. HEIGHT} \quad \begin{array}{c} 0 \ 1 \text{ ft} \ 2 \text{ ft} \ 3 \text{ ft} \ \ldots \ n \text{ ft} \end{array} \]

A degree is a point on this scale. Measurement can then be viewed as the association of
an individual with a degree; comparison as the evaluation of the relationship between the
degrees associated with two individuals.

In formalizing this account, I assume an ontology that includes degrees as a
primitive type (type \(d\)), an approach found in Kennedy (2001, 2007), Heim (2000), Hackl
(2000) and others. Degrees are organized into scales; a scale \(S\) is a set of degrees \(D\)
ordered by an ordering relationship \(>\) relative to some dimension \(DIM\); conversely, a
dimension \(DIM\) can be associated with one or more scales \(S\). Individuals may be related
to degrees via measure functions: functions that take individuals as arguments and return
degrees as values (type \(\langle ed \rangle\)). Below I give the general form of a measure function
associating an individual with a degree on the scale associated with some dimension \(DIM\)
(16a), as well as a specific example in which the dimension in question is height (16b):

\[(16)\]

\[\text{a. } \mu_{DIM}(x) = d \]

\[\text{b. } \mu_{HEIGHT}(Fred) = 5'9'' \]

I take degrees to be points on a scale, contra the recent interval-based theories of
Kennedy (2001) and Schwarzschild & Wilkinson (2002). However, as discussed in
Chapter 2, the analysis of Q-adjjectives makes crucial reference to intervals: the gaps or
differences between two points on a scale. We may allow for this by formalizing an
interval as a set of degrees. Specifically, for any two degrees \( d \) and \( d' \) such that the \( d > d' \), the interval between \( d \) and \( d' \) is the set of degrees between these two points, that is, the set \( \{d'': d \geq d'' \geq d'\} \).\(^{12}\)

Formally, an interval can be defined as a convex set of degrees:

\[
(17) \quad \text{A set of degrees } I \in D_{(d)} \text{ is an interval iff } \forall d, d', d'' \in D_d \text{ such that } d > d'' > d', (d \in I \land d' \in I) \rightarrow d'' \in I
\]

On this view, the interval is not a primitive type, but rather an element of the domain \( D_{(d)} \) that possesses a certain property (see Büring 2007 for a similar approach).

In what follows, I also sometimes make use of the following notation for intervals:

\[
(18) \quad \begin{align*}
a. \quad & [d, d'] & \text{the closed interval from } d \text{ to } d' \text{ (containing both } d \text{ and } d') \\
b. \quad & (d, d') & \text{the open interval from } d \text{ to } d' \text{ (excluding } d \text{ and } d') \\
c. \quad & [d, d') & \text{the lower-closed interval from } d \text{ to } d' \text{ (containing } d; \text{ excluding } d') \\
d. \quad & (d, d'] & \text{the upper-closed interval from } d \text{ to } d' \text{ (excluding } d; \text{ containing } d')
\end{align*}
\]

Let us turn now to the expressions of natural language whose analysis requires reference to degrees and scales. Expressions that I take to denote something in the domain of degrees include cardinal numerals (five dogs) as well as measure phrases (three pounds of rice; two feet tall).\(^{13}\) I take gradable expressions to include a degree argument, that is, an argument of type \( d \). In particular, I follow a tradition going back to Cresswell (1977) in taking gradable adjectives to express relationships between individuals and degrees:

\[
(19) \quad [\text{tall}] = \lambda d \lambda x.\mu_{\text{HEIGHT}}(x) \geq d
\]

\(^{12}\) At this stage it is not crucial to specify whether or not the endpoints are included in the interval.\(^{13}\) I will return to a more detailed discussion of the semantics of cardinal numerals later in this work.
Here *tall* associates an individual *x* with a set of degrees (an interval) on the scale of height, namely those degrees that are less than or equal to the height of *x*.

The degree argument of gradable adjectives allows them to compose with degree modifiers, terms that combine with gradable expressions to constrain in some way the range of degrees being referenced. Expressions that are typically considered to be degree modifiers include the following:

(20) a. High degree adverbs: *very, incredibly, extremely,* etc.
    b. Excessive: *too*
    c. Sufficiency: *so*
    d. Equative: *as...as*
    e. Comparative: *-er*
    f. Superlative: *-est*
    g. Demonstratives: *this, that (this tall, that short)*

With regards to the semantics of degree modifiers, I adopt an approach developed by Heim (2000) (see also Hackl 2000; Bhatt & Pancheva 2004; Takahashi 2006; von Stechow 2006) that draws on the notion of a parallel between the domain of individuals and the domain of degrees. Just as there are expressions of natural language that denote individuals (type *e*; e.g. *Fido*) and those that denote generalized quantifiers over individuals (type ⟨*et,t*⟩; e.g. *every dog*), so are there expressions that denote degrees (type *d*), and also those that denote generalized quantifiers over degrees (type ⟨*dt,t*⟩). In the first class we have cardinal numerals (*five*) and measure phrases (*six feet*), which can directly saturate the degree argument introduced by a gradable expression:

(21) a. ⟦six feet⟧ = 6'
b. ⟦six feet tall⟧ = ⟦tall⟧ ( ⟦six feet⟧ )
   = λdλx.µHEIGHT(x) ≥ d (6')
   = λx.µHEIGHT(x) ≥ 6'

The remaining degree modifiers in (20) fall in the second class, denoting generalized quantifiers over degrees (type ⟨*dt,t*⟩), allowing them to bind a degree variable that
saturates the degree argument of a gradable expression. As a concrete example, (22) gives a first approximation of the semantics of the comparative:

\[(22) \quad [-er] = \lambda P_{(dt)} \lambda Q_{(dt)} \exists d [Q(d) \land \neg P(d)]\]

Here, -er is a two-place predicate that takes two sets of degrees as arguments. The first is provided by the than phrase (recall that at the relevant level of interpretation, the than phrase is the syntactic complement of the degree head -er). In the case where the than phrase directly names a degree we can take this to derive from the semantics of the degree-denoting expression itself; more complex than phrases can be taken to involve deletion under identity, with null operator movement resulting in a lambda abstract. For example:

\[(23) \quad \text{John is taller than Fred is tall} \]

\[\quad \left[ \text{Op, Fred is t, tall} \right] = \lambda d, \mu_{\text{HEIGHT}}(Fred) \geq d\]
\[\quad \left[ -er \text{ than Fred is t, tall} \right] = \lambda Q_{(dt)} \exists d [Q(d) \land \neg \mu_{\text{HEIGHT}}(Fred) \geq d]\]
\[\quad \left[ \text{John is taller than Fred} \right] = 1 \iff \exists d [\mu_{\text{HEIGHT}}(John) \geq d \land \neg \mu_{\text{HEIGHT}}(Fred) \geq d]\]

Note also that other degree modifiers likewise allow (or require) complement clauses, which I similarly take to saturate their first argument:

\[(24) \quad \begin{align*}
    \text{a. John is too tall to go on the kiddie rides} \\
    \text{b. John is as tall as Fred} \\
    \text{c. John is so tall that he bumped his head on the door}
\end{align*}\]

It is beyond the scope of this work to attempt an in depth analysis of all of the degree modifiers listed in (20). I refer the reader to Meier (2003), Neeleman et al. (2004) and Kennedy & McNally (2005b) for work in this area. Below, I will return to a more detailed discussion of the comparative.
3.2.3 Some Comments on Scale Structure

Before proceeding, let me comment a little further on the structure I assume for the measurement scales invoked in the semantics of ordinary adjectives and especially Q-adjectives. I first of all follow Fox & Hackl (2006) in taking all measurement scales to be dense, in that for any two distinct degrees on a scale, we can find a third degree that lies between them. Density as a property of scales is defined formally as follows:

\[(25) \text{ A scale } S \text{ is dense iff for all } d,d' \in D \text{ s.t. } d > d', \exists d''[d > d'' > d]\]

In the case of dimensions involved in the measurement of amounts of matter (volume, weight, etc.), as well as those invoked in the interpretation of gradable adjectives (e.g. height, length, temperature etc.), the claim of scale density is intuitively correct. As an example, for any two quantities of substance of different weights, there can be a third quantity whose weight falls between them. Similarly, for any two individuals of different heights, there can be a third individual whose height falls between that of the first two.

There is, however, one apparent exception to this generalization, namely the dimension of cardinality, whose associated scale is typically taken to be the (discrete) set of natural numbers. For two consecutive natural numbers \(n\) and \(n+1\), there is of course no natural number that falls between them. But despite the intuitive naturalness of this picture, Fox & Hackl (2006) have recently argued for the Universal Density of Measurement, namely that all scales involved in natural language measurement – including the scale of cardinality – are formally dense. Evidence for this claim comes from the parallel behavior of expressions of number versus other dimensions with respect to constructions whose semantic interpretation invokes a (direction-sensitive) maximalization operator. Consider for example the parallel below:
(26)  a. *How much doesn’t John weigh?
b. *How many children doesn’t John have?

Fox & Hackl argue that the ungrammaticality of (26a) derives from the fact that the *how much* question asks for the maximally informative degree $d$ (in this case, the minimum degree $d$) such that John doesn’t weigh $d$. On the assumption that the scale measuring weight is dense, there is no such minimum degree: if John weighs (say) exactly 150 pounds, there is no minimal degree $d > 150$ pounds such that John doesn’t weigh $d$. But this account can only be extended to (26b) if we assume that the scale of cardinalities is also dense. To see this, note that if we assume the discrete scale of natural numbers, the set of degrees $d$ such that John doesn’t have $d$ children does have a minimal member (for example, if John has 3 children, that minimal degree is 4). From this, and similar parallels in other domains in which some form of maximalization plays a role, Fox & Hackl conclude that the scale of cardinality, like that of other dimensions, is formally dense.

Independently of Fox & Hackl’s arguments, the claim that the scale of cardinality is formally dense receives further support from even simpler facts. There are several contexts in which cardinalities can be described with reference to a dense scale, notably with average values (27a) and count nouns in measure phrases (27b). Even when it comes to counting entities, a dense scale may be invoked; for example, while talking about fractional children is odd (27c), other types of countable entities are more readily viewed as divisible (as in (27d)):

(27)  a. The average American owns 2.14 televisions
b. The rod is 3.785 feet long
c. ??John has 3½ children
c. John ate 3½ / 3¼ / 3⅛ / etc. pies
We might express this by saying that it is not the domain of degrees of cardinality that is discrete; rather, it is the domain of individuals that is in many cases discrete, such that pluralities of certain individuals (e.g. people) are most naturally mapped to scale points that correspond to whole numbers, that is, members of the set of natural numbers. On the basis of this evidence, I will assume following Fox & Hackl that all scales of measurement, including that of cardinality, are formally dense.

Secondly, I assume measurement scales may differ with respect to their endpoints, a line of argument developed in Kennedy & McNally (2005a). On this view, scales may be either closed (having an endpoint) or open (lacking an endpoint); open scales may be further distinguished by whether they are bounded or unbounded. As an example, the scale associated with the dimension FULLNESS is closed on both ends: a glass, for instance, may either have a maximal degree of fullness or a complete lack of fullness; the corresponding scale therefore has both minimum and maximum points. By contrast, the scale associated with the dimension HEIGHT is open on both ends. For any entity (person, tree, building, etc.) that has a certain degree of height, there can always be an entity that has a greater degree of height, and also an individual that has a lesser degree of height. This scale, therefore, has neither a maximum point nor a minimum point. It is, however, bounded on the lower end (in that that the height of an entity may approach but not reach 0), while being unbounded at its upper end. Finally, the scale associated with the dimension DIRTINESS is closed at its lower end but open and unbounded at its upper end: a table, for instance, can exhibit a complete lack of dirtiness (i.e. be completely clean), but cannot exhibit a maximal level of dirtiness (no matter how
dirty it is, we can always make it dirtier). These distinctions in scale structure can be depicted as follows:

(28) a. Closed at both ends (FULLNESS)
     empty                                   full

b. Open at both ends; lower bounded (HEIGHT)
   short               tall

c. Lower closed (DIRTINESS)
   clean                dirty

Adverbial modifiers serve as diagnostics for the structure of the scale invoked by a gradable adjective. Adjectives associated with the open ends of scales can be modified by *very*, but not by *completely* or *perfectly*, while those associated with closed ends of scales can be modified by *perfectly*, and are often (though not always) awkward with *very* (see also Kennedy 2007). For example:

(29) a. Closed Scale: FULLNESS (empty/full)
     completely empty/?very empty
     completely full/?very full

b. Open Scale: HEIGHT (short/tall)
   *completely short/very short
   *completely tall/very tall

Kennedy & McNally do not discuss scales associated with dimensions of amount (i.e. monotonic dimensions), but we can apply these same diagnostics to them:

(30) a. *completely many/*completely few people
b. *completely much/*completely little water
c. very many/very few people
d. very much/very little water

None of the Q-adjectives *many, few, much* and *little* accept modification by *completely*, and all at least marginally allow modification by *very* (though *many* and *much* are slightly odd with *very*). According to this test, the scales in question are therefore open at both
ends. More specifically, at the lower end the relevant scales are bounded by, but do not
include, 0; at the upper end they are unbounded.14

3.2.4 Semantic Composition

I take Functional Application to be the primary mode of semantic composition.

Functional Application can be defined as follows:

\[(31) \quad \text{Functional Application:} \]
\[
\text{If } f \text{ is an expression of type } \langle \alpha, \beta \rangle \text{ and } a \text{ is an expression of type } \alpha, \text{ then } f(a) \text{ is an expression of type } \beta
\]

I further assume an additional mode of composition, under which two functions that share
an argument of the same semantic type may compose with one another without the
saturation of that argument. Kratzer (1996) introduces an operation of this nature under
the name of Event Identification, which allows two expressions that share an event
argument to compose via identification of events. Kratzer invokes Event Identification in
the analysis of verb argument structure, proposing that the external argument be ‘severed’
from the semantic content the verb and introduced instead by a higher functional head,
with its content linked to that of the verb via identification of the event argument.

McClure (2004) extends Kratzer’s analysis to individual arguments, under the name of
Variable Identification. From a different perspective, Chung & Ladusaw (2003) propose
the operation of Restrict, which allows a property-type indefinite to compose with a
predicate, without saturation of the individual argument.

In the account developed in this chapter, I will make use of a similar mode of
composition, which following McClure (2004) I will call Variable Identification:

---

14 Note however that in Chapter 4 I will present evidence that the scales invoked in the semantics of Q-
adjjectives can in certain contexts acquire an upper bound, with an important interpretive consequence.
(32) **Variable Identification:**
Two functions whose first argument is of the same semantic type may compose via conjunction:

\[ (\lambda a\ldots f(a)) (\lambda a\ldots g(a)) = \ldots \lambda a.f(a) \land g(a), \]

where the ellipses (…) reflect the possibility of additional arguments

On this definition, the two functions that compose in this manner do not need to be of the same semantic type, but only to share a first argument of the same type. To give a concrete example that will be relevant below, the two functions in (33a) and (33b) may compose by Variable Identification as in (33c):

(33) a. \( \lambda x.e.f(x) \) (type \( \langle et \rangle \))
    b. \( \lambda x, \lambda d.d.g(d)(x) \) (type \( \langle e,dt \rangle \))
    c. \((\lambda x.e.f(x))(\lambda x, \lambda d.d.g(d)(x)) = \lambda d.d\lambda x.e.f(x) \land g(x)(d) \) (type \( \langle d,et \rangle \))

Here I follow a convention from Chung & Ladusaw in taking the argument targeted by Variable Identification (here, the individual argument) to be demoted to the lowest position (i.e. the last to be saturated).

3.2.5 **Some Housekeeping**

Let me conclude with some brief matters of housekeeping. In what follows I will use \( x,y,z \) as variables ranging over individuals (type \( e \)), \( P,Q \) as variables ranging over predicates (type \( \langle et \rangle \)), \( d,d',d'' \) as variables ranging over degrees (type \( d \)), and \( I,J,K \) as variables ranging over scalar intervals (type \( \langle dt \rangle \), as defined in (17)). For simplicity, I will omit an event argument, though I believe that the analysis to be developed here could be restated in an event-semantic framework. I ignore tense as not relevant to the central questions under investigation. Finally, unless relevant I omit time and world variables.
3.3 The Building Blocks

In this section, I introduce the core building blocks that will form the basis for the compositional analysis of the full range of constructions in which Q-adjectives occur. To take a step back, in a Generalized Quantifier framework augmented by a lattice-theoretical approach to plurals, a sentence featuring the Q-adjective *many* in its quantificational use, such as (34a), would receive an analysis along the lines in (34b). *Many* itself would then have the entry in (34c).

(34) a. Many students attended the lecture
    b. \( \exists x [\text{*student}(x) \land \text{attended-the-lecture}(x) \land |x| \geq d_{\text{Standard}}] \)
    c. \( \lambda P \lambda Q. \exists x [P(x) \land Q(x) \land |x| \geq d_{\text{Standard}}] \)

In the account I develop, (34a) will receive an analysis that is equivalent to that in (34b). But *many*, itself, will not be represented as in (34c). Rather, the semantic content in (34c) will be decomposed, with only part of it encoded by the Q-adjective itself, with the remainder contributed by three additional elements and operations. It is these four separate pieces that I introduce here, starting with the lexical entries for Q-adjectives themselves.

3.3.1 Q-Adjectives

The central conclusions of Chapter 2 can be summarized in the following three statements about the semantics of Q-adjectives:

a) Q-adjectives are predicates of scalar intervals

b) *Many* and *few* are predicated of intervals on the scale of cardinality, while *much* and *little* are predicated of intervals on the scale of some dimension other than cardinality.

c) Q-adjectives are themselves gradable expressions, allowing modification by a range of degree modifiers.

Formally, these insights can be captured with the following lexical entries:
Lexical Semantics of Q-Adjectives

a. \[ \text{[many]} = \lambda d \lambda I_{\#}.d \in I \]
b. \[ \text{[few]} = \lambda d \lambda I_{\#}.d \in \text{INV}(I) \]
c. \[ \text{[much]} = \lambda d \lambda I_{\text{non-\#}}.d \in I \]
d. \[ \text{[little]} = \lambda d \lambda I_{\text{non-\#}}.d \in \text{INV}(I) \]

Here \( I \) is a variable that ranges over intervals (as defined in (17)). The subscript \( \# \) indicates the interval \( I \) is on the scale associated with the dimension of cardinality, with \( \text{non-\#} \) indicating that the dimension is one other than cardinality. Finally, \( \text{INV} \) is a function that maps an interval to the join-complementary interval on the same scale:

Thus \( \text{many} \) and \( \text{much} \) relate intervals to the degrees that are contained within them, while \( \text{few} \) and \( \text{little} \) relate intervals to the degrees contained within their join-complementary intervals. In a sense, then, \( \text{few} \) and \( \text{little} \) in (35) are defined negatively; they predicate a property of an interval \( I \) by expressing a property of its join-complementary interval \( \text{INV}(I) \). This view is on the same lines as Heim’s (2006) previously discussed analysis of \( \text{little} \) as expressing degree negation (where the negation of a set of degrees is its complement set). I should note, however, that \( \text{INV} \) does not express precisely degree negation, the difference being that any (semi)closed interval \( I \) shares a point with \( \text{INV}(I) \) (for example, if \( I = (0, d] \), then \( \text{INV}(I) = [d, \infty) \), with the point \( d \) common to both).

In previous chapters I noted the parallels between Q-adjectives and gradable adjectives, in that both can combine with the same degree modifiers. This parallel is captured with the entries in (35). To see this more clearly, let us compare the entry for the Q-adjective \( \text{many} \) with that previously proposed for a gradable adjective such as tall:
The two expressions in (37a) and (37b) have in common that their first argument is a degree argument; the parallel between the two is even more evident if we give tall the equivalent though less concise representation in (37b'). It is this that allows Q-adjectives to combine with the same degree modifiers that occur with ordinary gradable adjectives. In this respect, the present account builds on that of Hackl (2000), who likewise proposes the presence of a degree argument in the representation of many as a means to capture the parallel to gradable adjectives. However, the analysis developed here differs crucially from Hackl’s in that he maintains quantificational semantics for many (cf. the discussion in Chapter 2).

The key difference, then, between Q-adjectives and gradable adjectives is that in the latter case the second argument comes from the domain of individuals, while in the former case it comes from the domain of scales and degrees. We will see that it is the similarities and differences between the representations in (37a) and (37b) that account for the similarities and differences in the behavior of Q-adjectives and ordinary gradable adjectives.

3.3.2 Degree Modifiers and POS

Having discussed the semantics of Q-adjectives themselves, let me turn now to the elements that they combine with. The first topic I will cover relates to the saturation of the first argument of the expressions in (35), namely the degree argument.

As I noted above, it is the presence of a degree argument in the lexical entries of Q-adjectives that allows them, like ordinary gradable adjectives, to co-occur with a wide
range of degree modifiers. Recall that in the framework I have adopted, degree modifiers head DegPs that occur in the specifier position of the gradable expression (Q-adjective or gradable adjective). Semantically, they denote degrees (type $d$) or (once their internal argument is saturated) generalized quantifiers over degrees (type $\langle dt, t \rangle$); as an example of the latter, the following was proposed earlier as a first approximation of the comparative:

$$[-er] = \lambda P_{\langle dt \rangle} \lambda Q_{\langle dt \rangle}. \exists d [P(d) \land \neg Q(d)]$$

It should be apparent that this same approach extends to Q-adjectives, given that they too feature a degree argument that can be saturated by a variable bound by a degree quantifier, allowing parallel analyses of examples such as these:

(39) a. fewer than 100  a.' shorter than 6 feet
b. too many mistakes to pass  b.' too unprepared to pass
c. as much money as John (has)  c.' as tall as John (is)
d. so little money  d.' so short

We are now left with the question of how the degree argument in (35) is saturated in the case of the bare ‘positive’ forms many, few, much and little, where there is no overt degree morphology. Here, I follow a long tradition in the analysis of gradable adjectives and other gradable expressions (Cresswell 1977; von Stechow 1984, 2006; Heim 2006; Kennedy 2007) in proposing that these involve the combination of basic gradable entries such as those in (35) with a null ‘positive’ morpheme POS. It is POS that is responsible for the interpretation of large/small relative to the context that characterizes the positive forms of gradable adjectives (including Q-adjectives).

There are numerous proposals in the literature regarding the semantics of the positive morpheme. I will return to a fuller discussion of its semantics in the next chapter; for present, I follow the approach of von Stechow (2006) and Heim (2006) in
taking POS to be a null degree quantifier (like the overt expressions in (20)) that introduces a ‘neutral range’ $N_S$ on the scale $S$ in question – the range of degrees that would be considered neither large nor small with respect to the context:

$$\text{(40)} \quad [\text{POS}] = \lambda I, \forall d \in N_S[d \in I]$$

On this definition, POS is true of an interval if $N_S$ is contained within it. Anticipating a fuller discussion below, the effect of POS as defined in (40) is to establish a range of neutral values intermediate between the positive and negative members of a pair of gradable antonyms: the positive term is true for degrees exceeding $N_S$, while the negative member of the pair is true for degrees falling short of $N_S$. To use the Q-adjectives many and few as an example, the situation that obtains is depicted in (41):

$$\text{(41)} \quad \text{few} \quad \text{“neutral” } N_S \quad \text{many}$$

It is thus the role of POS to contribute the ‘exceeds the standard’ component of the formula in (34c) above.

At this point, let me note that this is only one possible analysis of the relationship between gradable expressions and degree modifiers. The leading alternative is that developed by Kennedy (1997), who argues that gradable adjectives such as tall are functions from individuals to degrees (type $\langle ed \rangle$), and that degree morphology takes gradable adjectives as arguments, producing predicates of individuals. This semantic analysis is paired with a syntactic framework in which the Deg head takes the AP as a complement (i.e. the previously discussed structure 7b). While Kennedy offers a number of arguments in favor of this view, let me point out here that extending it to Q-adjectives
is somewhat problematic. I have argued that ordinary gradable adjectives and Q-adjectives are of different semantic types (the former but not the latter taking individuals as arguments). On a Kennedy-style analysis, we would then need to say that degree modifiers have different semantics in the two cases; that is, we would need to posit a systematic ambiguity in degree modifiers. In the framework I have adopted, this is not necessary. Degree modifiers saturate the degree argument of gradable adjectives and Q-adjectives (or bind a variable in this position); no difficulty is posed by the fact that the two sorts of expressions are otherwise of different semantic type. I assume that the analysis I develop below could be restated in Kennedy’s framework; but the present approach avoids the need to analyze degree modifiers as ambiguous, which I take to be a significant point in its favor.

3.3.3 The MEAS Node

Once their first (degree) argument is saturated, Q-adjectives, as defined in (35), are predicates of scalar intervals, or, equivalently, generalized quantifiers over degrees. In cases such as those below, we then have the question of how Q-adjectives are able to compose semantically with plural and mass nouns, which I analyze as predicates of (possibly plural) individuals.

(42) Many students attended the lecture
POS-many\(^\text{15}\): type (dt,t)
students: type (et)

(43) Little water remains in the bucket
POS-little: type (dt,t)
water: type (et)

\(^{15}\) Here for simplicity I show POS combining directly with the Q-adjectives. With the semantics in (40), POS is actually a quantificational element that binds the degree argument of the Q-adjective. I work this out in detail below.
Here, many and students cannot combine via Functional Application, given that neither is of the correct type to serve as the argument for the other (likewise, of course, for little and water). Nor can they compose via Variable Identification, since they do not share a first argument of the same semantic type.

Here, I build on proposals found in Kayne (2005) and Schwarzschild (2006), and propose that the composition of Q-adjectives and nouns is mediated by a phonologically null functional head, which I call Meas (for ‘measure’). Recall that I argued above that Q-adjectives together with their degree morphology are phrasal categories (QPs), which are located in the specifier position of the functional head Meas:

(44)  
\[
\begin{align*}
\text{a. } [ \text{DP } & [\text{MeasP } [\text{QP } \text{POS}-\text{many}] [\text{Meas'} \text{Meas } [\text{NP } \text{students}]]]] \\
\text{b. } [ \text{DP } & [\text{MeasP } [\text{QP } \text{POS}-\text{little}] [\text{Meas'} \text{Meas } [\text{NP } \text{students}]]]]
\end{align*}
\]

I propose that Meas has a semantic function as a linking element, introducing a degree argument and linking it to the individual argument introduced by the NP.

A relevant parallel is provided by classifier languages such as Chinese. In languages of this type, numerals and other expressions of quantity cannot combine directly with nouns; rather, a classifier must intervene:

(45)  
\[
\begin{align*}
\text{san } & *(\text{zhi}) \text{ xiong} \quad \text{(Chinese)} \\
\text{three } & \text{CL} \text{ bear} \\
\text{‘three bears’}
\end{align*}
\]

Classifiers are typically taken to head a functional projection within the DP (see especially Cheng & Sybesma 1999; Borer 2005; Svenonius 2008). Semantically, the classifier enables the composition of the numeral with the head noun. Here, specific proposals vary, with one common theme being that classifiers individuate atoms for the purposes of counting (e.g. Doetjes 1997; Chierchia 1998b; Borer 2005). A variant of this approach, which is relevant to the present discussion, holds that the classifier also
introduces a degree or number argument that is saturated by the numerical expression. In particular, Krifka (1995) analyzes numeral classifiers in Chinese as introducing a counting measure function:

(46)  
\[ \text{〚zhi〛} = \lambda n \lambda y \lambda i \lambda x [RT_i(x, y) \land OU_i(y)(x) = n] \]
\[ \text{〚san zhi〛} = \lambda y \lambda i \lambda x [RT_i(x, y) \land OU_i(y)(x) = 3] \]
\[ \text{〚san zhi xiong〛} = \lambda i \lambda x [RT_i(x, \text{Ursus}) \land OU_i(\text{Ursus})(x) = 3] \]

Here, RT is a realization function that associates kinds (here, the kind ‘Ursus’ or bear) with the set of (possibly plural) individuals that are members of that kind, and OU is a measure function that counts the number of atoms in a plurality.

I propose that Meas plays a similar classifier-like role, introducing a measure function that links individuals to degrees on the scale of some dimension, and thereby enabling the semantic composition of numerical and quantity expressions (including Q-adjectives) with nouns. On this view, the English numerical DP is more similar to its counterpart in classifier languages than is apparent on the surface. Specifically, both feature a functional head that plays a linking role, the key difference being that this element is phonologically null in English, but overt in languages such as Chinese (here, I leave open the possibility that there may be additional differences in noun denotations in English versus Chinese).

A potential objection to this account comes from the often-noted pattern of complementary distribution between plural morphology and overt classifiers, both across languages and within languages, evidence for a view that it is the English plural morpheme, not some other functional element, that is the equivalent to numeral classifiers (e.g. Doetjes 1997; Borer 2005; Ionin & Matushansky 2006). However, Svenonius (2008) points out the existence of languages in which plural morphology and
numeral classifiers co-occur, evidence that these are reflexes of distinct functional layers.

I assume this position here.

The specific semantics I propose for Meas is given below:

\[
\text{Meas} = \lambda x \lambda d. \mu_\text{DIM}(x) \geq d
\]

Here, \( \mu_\text{DIM} \) is a measure function that associates an individual with a set of degrees (an interval) on the scale associated with some dimension of measurement. On this definition, Meas can combine via Functional Application with an expression of type \( e \).

Alternately, it may combine via Variable Identification with an expression of type \( \langle et \rangle \); it is the latter possibility that allows it to compose with an NP in constructions of the form in (44). At the same time, Meas also introduces a degree argument, which can either be directly saturated by a degree-denoting expression (type \( d \)), or be saturated by a variable of type \( d \) that is bound by a degree quantifier. We will see below that the former route is a possibility for cardinal numerals and some measure phrases, while the latter is the route by which Q-adjectives compose with Meas.

On the definition in (47), Meas does not encode a specific dimension. Rather, the dimension in question is ‘filled in’ on the basis of the NP denotation, the nature of the degree expression that it combines with, and the context of interpretation. In particular, in the presence of \textit{many} and \textit{few}, the dimension must be interpreted as cardinality, since \textit{many} and \textit{few} are themselves lexically specified as predicates of intervals on the scale of cardinality. I take constructions involving measure phrases to likewise involve the presence of Meas, and here they contribute to specifying the dimension. For example, in (48), the dimension is one on which rice can be measured, and whose measure can be 8 pounds, the obvious candidate being weight:
(48)  a. eight pounds of rice  
    b. \[[\text{MeasP eight pounds Meas [XP of [NP rice]]]}\]

In the case of the Q-adjjectives much and little, the possibilities are more varied, since
much and little themselves do not encode a dimension: in little water (more fully, little
MEAS water), it is natural to take the dimension to be volume, while in (too) much money
the most natural choice is monetary value; but the context could make other possibilities
more salient (for example, if we were loading an armored car, we might interpret too
much money as describing the volume or weight of the money). Much and little are thus
underspecified in comparison to many and few; we will see below that this distinction is
relevant to the distribution of the two pairs of Q-adjjectives.

The reader may recall Kayne’s (2005) previously discussed analysis of the Q-
adjjectives many and few as modifiers of an unpronounced noun NUMBER that
intervenes between them and the plural noun (e.g. many NUMBER students); in the
present analysis, the Meas head takes the place of Kayne’s NUMBER. From a semantic
that allow the composition of Q-adjjectives with substance predicates, though as will be
seen, these accounts differ in one crucial respect from the present one.

3.3.4 Quantification over Individuals

The final piece of the puzzle relates to the source of quantificational force in the
case of Q-adjjectives on their quantificational use, as in our original example from
Chapter 2, repeated below:

(49)  Many/few students attended the lecture

As defined in (35), Q-adjjectives are not themselves quantificational (or to be more
accurate, they do not express quantification over individuals, but rather quantification
over degrees). Nor do any of the other elements I have introduced so far play this role. How, then, does quantificational force arise?

An obvious possibility would be to build some sort of quantificational operator into the semantics of the Meas node, so that it not only links individuals to degrees, but also binds the individual variable. This is precisely the approach taken by Schwarzschild (2006) and Rett (2006), for whom the linking element is quantificational. But a look at some further data will quickly show that this is not what we want. Consider again the predicative and attributive uses represented in (50a,b):

(50) a. John’s good qualities are many/few
    b. The many/few students who attended enjoyed the lecture

In (50a), *many/few* must be associated with a plurality of individuals (the maximal set of John’s good qualities), but there is no individual variable that may be bound by a quantificational operator. Similarly, if in (50b) it is the role of *the* to shift a predicative nominal expression to a type that may occur in argument position (either a quantificational or referential type, depending on one’s analysis of *the*), then it cannot be the case that some null functional element also introduces quantification over individuals. Furthermore, as was argued in Chapter 2, certain constructions require a predicative interpretation for noun phrases of the form *many dogs* or *little water*, *there*-sentences being the most well known case (e.g. *there are many dogs in the yard*). We therefore cannot allow the composition of a Q-adjective with a nominal expression via Meas to necessarily involve a shift to a quantificational type. For this reason, I maintain the non-quantificational semantics for Meas given in (47), and look elsewhere for the source of quantification over individuals.
The problem we are then left with is of course the very same one that faces any theory of indefinites that takes them to be basically predicative in their semantics (cf. the fuller discussion of this issue in Chapter 2). The repertoire of possible solutions is now well known: quantificational force may arise via a type shift (Partee 1986; de Swart 2001; Landman 2004), via some sort of global existential closure (Heim 1982; Diesing 1992; Kamp & Reyle 1993), via the application of a null determiner with existential semantics (Krifka 1999), or via existential closure over choice functions (Reinhart 1997).

Among these possibilities, I will propose that in the case of Q-adjectives, quantificational force can be attributed to an operation of Existential Closure, which I define as follows:

(51) **Existential Closure:** Unbound variables are existentially bound at the IP level

This definition bears some similarity to the approach of Diesing (1992). But my account differs from Diesing’s in that I take Existential Closure to be invoked in all quantificational uses of Q-adjectives, and not only in their so-called weak interpretations. I return to a full discussion of this issue in Chapter 4.

Before proceeding, let me briefly discuss why I opt for Existential Closure rather than one of the alternatives. Of the latter, the choice function route is the most distinct, and the easiest to discard. Choice functions were originally introduced into the semantics toolkit as a means to analyze the exceptional wide scope readings available to indefinites. For example, (52a) has a reading (perhaps the most natural one) according to which there is a particular relative of mine (say, Aunt Mabel) whose death would net me a million dollars, as paraphrased in (52b). We could derive this reading via the LF in (52c), where
the indefinite *a relative of mine* scopes out of the *if* clause. But this is unexpected, given that *if* clauses are typically islands for movement.

(52) a. If a relative of mine dies, I’ll inherit a million dollars  
b. ‘There is a particular relative of mine x such that if x dies, I will inherit a million dollars’  
c. [(a relative of mine),][if t, dies, I will inherit a million dollars]

Choice functions provide a mechanism to derive such wide-scope readings, without the need to posit otherwise illicit syntactic movement: a choice function variable \( f \) picks out an element from the set ‘relatives of mine’; this choice function variable can be existentially closed at various points in the derivation, including outside of the *if* clause, yielding the apparent wide-scope reading for the indefinite.

(53) \( \exists f:\text{CF}(f)[\text{dies}(f(\text{relative-of-mine}(x))) \rightarrow \text{inherit}(I, \$1MM)] \)

But importantly, Q-adjectives do not allow exceptional wide scope readings. For example, (54) only has the pragmatically odd reading according to which I will inherit the money if the number of deaths among my relatives does not exceed some threshold value; it cannot be read as saying that there is some particular small group of my relatives (say, Aunts Mabel, Minnie and Marge) whose collective deaths will make me rich.

(54) If few relatives of mine die, I’ll inherit a million dollars

Since noun phrases based on Q-adjectives do not give rise to the readings that could be obtained via choice functions, we must conclude that this is not the mechanism by which their predicative interpretations come to serve as arguments. On the other hand, the reading that is available for this sentence is precisely what one would get if Existential Closure applied within the *if* clause.

The predictions made by other alternatives differ more subtly from those made by an Existential Closure analysis, but there are nonetheless differences that favor the latter.
The first alternative is a syntactic analysis, according to which quantification over individuals arises via the operation of a null existential determiner in the D position, such that a noun phrase of the form in (55a) has the structure in (55b):

(55)  
   a. many dogs  
   b. \[DP \exists [MeasP many Meas [NP dogs]]]\n
A semantics derived via the structure in (55) readily accounts for quantificational occurrences of Q-adjectives, but we would then need to explain why noun phrases of this form are not always quantificational, perhaps by positing that in certain constructions (e.g. *there*-sentences) the DP layer is not present. A more parsimonious approach is to invoke a purely semantic operation, a type shift to a quantificational type that is invoked as a last resort in case of type mismatch. Such a type shift is the operation A of Partee (1986), which shifts predicates to generalized quantifiers:

(56)  
   \([A] = \lambda P_{(dt)} \lambda Q_{(dt)}. \exists x[P(x) \land Q(x)]\]

But both the type shifting approach and the syntactic null determiner approach share the consequence that noun phrases formed with Q-adjectives would have a quantificational type at least some of the time. In fact, there is reason to think that this is not the case. The relevant data comes from patterns of scope taking. It is well known that a comparative quantifier in object position cannot scope over a quantifier in subject position. Thus (57a) and (58a) have only the surface scope readings in (b), and do not allow the inverse scope readings in (c):

(57)  
   a. Every student read more than five books  
   b. every > more than five  
   c. more than five > every

(58)  
   a. Every student read fewer than five books  
   b. every > fewer than five  
   c. fewer than five > every
Perhaps less recognized is that the same pattern also obtains with bare *many* and *few*:

(59)  
   a. Every student read many books  
   b. every > many ✓  
   c. many > every ×

(60)  
   a. Every student read few books  
   b. every > few ✓  
   c. few > every ×

Chung & Ladusaw (2003) discuss a similar phenomenon with bare plurals, which are well known to take obligatorily narrow scope (see also Carlson 1977; van Geenhoven 1998). For example, (61) can only mean that it is not the case that Maxie saw ghosts in the attic, not that there were ghosts that she didn’t see:

(61) Maxie didn’t see ghosts in the attic

Chung & Ladusaw propose that this is a consequence of the mode of composition of bare plurals: they necessarily compose via the non-saturating operation of Restrict (the counterpart of my Variable Identification), which yields a narrow scope reading.

This account extends to the present case. If noun phrases formed with Q-adjectives are never quantificational, but must compose with predicates via Variable Identification, then we would predict that in object position they would necessarily take narrow scope relative to a quantificational DP in subject position, just as is observed in the data above. For this reason, I do not follow an approach under which such noun phrases themselves attain quantificational force via a type shift or combination with a null existential determiner, and instead make use of Existential Closure as defined in (51), under which view quantificational force arises externally to the noun phrase.
3.4 Quantificational Q-Adjectives

3.4.1 The Basic Case

With these pieces in place, we are in the position to provide a full analysis of Q-adjectives across the contexts in which they occur. I begin with their quantificational uses. Consider first our original examples of quantificational many and few. In (52a), I give the relevant surface structure. But with this structure, the Q-adjective cannot be interpreted in situ. Recall that on the definition in (35), the Q-adjective wants an interval (type $\langle dt \rangle$) as an argument; but the Meas' does not have a denotation at this type. As a result, the Q-adjective must raise covertly for purposes of interpretability to take sentential scope, passing first through SpecDP, and leaving behind a trace of type $d$ that is lambda bound. Similarly, POS raises from its base position as a constituent with many/few, also for purposes of interpretability. The resulting LF is thus that in (62b):

(62) Many/few students attended the lecture

a. SS: $[\text{IP} [\text{DP} \emptyset [\text{MeasP POS-many/POS-few [Meas' MEAS [NP students]]}}] \text{ attended the lecture}]

b. LF: $[\text{POS2}[t_2-many_1/t_2-few_1 [\text{IP} [\text{DP} \emptyset [\text{MeasP }t_1 [\text{Meas' MEAS [NP students]]}}] \text{ attended the lecture}]]]

The derivation is shown schematically below, with the semantic type at each stage indicated:
(63) Many/few students attended the lecture

In (64), I give the step-by-step semantic derivation for the case of many:

(64) Many students attended the lecture

\[
\begin{align*}
\llbracket \text{students} \rrbracket &= \lambda x. \text{student}(x) \\
\llbracket \text{Meas} \rrbracket &= \lambda x \lambda d. \mu_{\text{DIM}}(x) \geq d \quad \text{by (47)} \\
\llbracket \text{Meas students} \rrbracket &= \\
&= \lambda d \lambda x. \text{student}(x) \wedge \mu_{\text{DIM}}(x) \geq d \quad \text{by Variable Identification} \\
\llbracket t_1 \text{ Meas students} \rrbracket &= \\
&= \lambda x. \text{student}(x) \wedge \mu_{\text{DIM}}(x) \geq d_1 \quad \text{by Functional Application} \\
\llbracket \text{attended the lecture} \rrbracket &= \\
&= \lambda x. \text{attended}(x, \text{lecture}) \\
\llbracket t_1 \text{ Meas students attended the lecture} \rrbracket &= \\
&= \lambda x. \text{student}(x) \wedge \mu_{\text{DIM}}(x) \geq d_1 \wedge \text{attended}(x, \text{lecture}) \quad \text{by Variable Identification} \\
&\Rightarrow \exists x [\text{student}(x) \wedge \mu_{\text{DIM}}(x) \geq d_1 \wedge \text{attended}(x, \text{lecture})] \quad \text{by Existential Closure}
\end{align*}
\]
Recall that the Meas head introduces the measure function $\mu_{\text{DIM}}$, but that it does not specify a particular dimension of measurement. But *many* itself lexically encodes the dimension of cardinality (number), and as a result, the dimension introduced by Meas must be interpreted as cardinality. I represent this by subscripting the measure function with a #$\mu$ at the stage at which *many* enters the derivation; I use the same notation for the neutral range, which I indicate as $N#$ when the dimension in question is cardinality.

In (65), I give the corresponding derivation (in its relevant differences) for the case involving *few*:

(65) Few students attended the lecture

\[
\llbracket \text{few} \rrbracket = \lambda d \lambda I#.d \in \text{INV}(I) \quad \text{by (35b)}
\]

\[
\llbracket t_2 \text{few} \rrbracket = \lambda I#.d_2 \in \text{INV}(I) \quad \text{by Functional Application}
\]
Here the introduction of the negation operator and the switch from the ‘greater than or equal’ relation to the ‘greater than’ relation reflect the mapping of the original interval to its join-complementary interval via the operation INV.

The final formula in (64) describes the situation depicted in (66), in which the number of students attending the lecture exceeds the neutral range (or equivalently, the neutral range is fully contained within the interval from 0 to the number of students attending the lecture). That in (65) describes the situation depicted in (67), in which the number attending falls short of the neutral range.
These formulae can be paraphrased in the following abbreviated forms, which for the sake of clarity I will sometimes make use of below (here #-Students-AtL is shorthand for the number of students who attended the lecture):

(68) Many students attended the lecture
\[ N_{#} \subseteq (0, \text{-#-Students-AtL}] \]

(69) Few students attended the lecture
\[ N_{#} \subseteq [\text{-#-Students-AtL}, \infty) \]

Finally, while I have been discussing the Q-adjectives *many* and *few*, the same account can of course be extended to their counterparts *much* and *little* in their mass quantificational uses. For example:

(70) a. Little rice remains
b. LF: [POS₂ [t₂ little₁ [t₁ Meas rice remains]]]
\[ \forall d \in N_{5}[\neg \exists x[\text{rice}(x) \land \mu_{\text{DIM}}(x) > d \land \text{remains}(x)]] \]

Since *little* does not encode a particular dimension of measurement, the dimension must be determined via a context; here, possible dimensions would be weight (*little rice remains - only a pound!* or volume (*little rice remains - only two cups!*).

### 3.4.2 Some Problems Resolved

There is an important point to make about the derivations in (64) and especially (65). As a predicate of scalar intervals, the Q-adjective (*many* or *few*) cannot be interpreted *in situ*, since the logical form at that level of the derivation (the MeasP) does not provide an interval to serve as its argument. As a result, the Q-adjective raises covertly, acquiring as an argument the interval (set of degrees) formed by lambda-abstraction over the trace of type \( d \) in its base position. But crucially, this can only occur after the stage at which Existential Closure applies to produce an expression of type \( t \); if lambda abstraction were to occur at an earlier stage, the result would not be an expression
of type \( \langle dt \rangle \), and therefore could not serve as argument to the Q-adjective. The result is that the Q-adjective comes to outscope the existential operator. That is, the scope relationship in (65) is that in (71a), not that in (71b):

(71)  
   a. few > \exists  
   b. \exists > \text{few}

By this means, we are able overcome a significant challenge to the non-quantificational analysis of Q-adjectives. Recall from Chapter 2 that a serious issue faced by the cardinality predicate analysis is that the application of an existential operator (e.g. via Existential Closure) to a cardinality predicate \text{few} incorrectly produces a lower-bounded, ‘at least’ reading (van Benthem’s problem):

(72)  
   Few students attended the lecture  
   \exists x[\ast\text{student}(x) \land |x| < n \land \text{attended}(x,\text{lecture})]  
   ‘there was a group of students numbering less than \( n \) who attended the lecture’

As discussed previously, proposed solutions to this problem have ranged from the introduction of more complex type-shifting rules, to the semantic decomposition of monotone decreasing predicates, to the outright rejection of the non-quantificational analysis of Q-adjectives. The present account avoids this issue, while at the same time maintaining a non-quantificational approach under which quantificational force arises via a simple operation of Existential Closure. Just as in the decompositional analyses discussed in the last chapter (e.g. McNally 1998), the correct upper-bounded interpretation for \text{few} is achieved via a negative operator that takes scope over the existential operator. The difference is that rather than taking this negative element to be a simple truth-functional negation operator, the present analysis takes it to be the degree negation operator \text{few} plus its degree morphology as a whole. The advantages of this are
twofold. First, there is no need to simply stipulate the presence of a wide-scope negation operator; instead, the desired scope relationship is achieved via a movement operation that is motivated by a type mismatch. Secondly, correct results are obtained not just in the case of bare *few*, but also when *few* occurs with a degree modifier such as *very*; recall that this case was problematic for the simpler decompositional analysis discussed earlier.

Taking *very* to be a version of POS that introduces a symmetrically larger neutral range\(^{16}\) \(N_{S}^{+}\) (Heim 2006), as in (73), an example such as (74a) receives the analysis in (74b), which correctly describes the situation in (75).

\[(73) \quad [\text{very}] = \lambda I_{(d_0)} . \forall d \in N_{S}^{+} [d \in I]\]

\[(74) \quad \begin{align*}
    a. \quad & \text{Very few students attended the lecture} \\
    b. \quad & \forall d \in N_{#}^{+} [\neg \exists x [*\text{student}(x) \land \mu_#(x) > d \land \text{attended}(x, \text{lecture})]]
\end{align*}\]

\[(75) \quad \text{\# of students attending} \quad \rightarrow \quad \text{CARDINALITY} \quad \text{\{d: } \exists x [*\text{student}(x) \land \mu_#(x) \geq d_1 \land \text{attended}(x, \text{lecture})]\}

The same mechanism also provides an account of the previously discussed phenomenon of scope splitting. Recall that the example in (76a) is most naturally interpreted as in the paraphrase in (76b):

\[(76) \quad \begin{align*}
    a. \quad & \text{They need few reasons to fire you} \\
    b. \quad & \text{‘it is not the case that they need a large number of reasons to fire you’}
\end{align*}\]

The degree operator analysis of *few* developed here allows for this reading to be derived. Zimmermann (1993) analyzes intensional verbs as expressing a relationship between an

\(^{16}\) Or possibly a modifier of POS, given that it may be sequenced, as in *very very few students* (Kennedy & McNally 2005b).
individual and a property (type \(<s,\langle et\rangle>) (for similar proposals, see Larson, den Dikken & Ludlow 1997; Landman 2004; and van Geenhoven & McNally 2005):

\[
(77) \quad \llbracket \text{need} \rrbracket = \lambda P \lambda x[\text{need}(x, P)]
\]

Drawing on this, we derive the following LF and semantic interpretation for the example in question:

\[
(78) \quad \text{They need few reasons}
\]

**Lexical Form:**

\[
[\text{POS}_2 \ [t_1 \ \text{few}], \ [\text{they need} \ [\text{DP} \ t_1 \ \text{Meas reasons}]]]
\]

\[
\llbracket \text{POS}_2 \rrbracket ( \llbracket (t_2 \ \text{few})_1 \rrbracket (\lambda d_1.\text{need}(\text{they}, \ ^\wedge (\text{reason}^*(x) \wedge \mu_{\text{DIM}}(x) \geq d_1)))
\\forall d \in \mathbb{N} \left[ \neg \text{need}(\text{they}, \ ^\wedge (\text{reason}^*(x) \wedge \mu_{\#}(x) > d)) \right]
\]

\[
\mathbb{N}_{\#} \subseteq [\text{\#-of-reasons}, \infty)
\]

‘the number of reasons they need falls short of the neutral range’

Crucially, *few* must take scope over *need*, because this is the first step in the derivation at which an expression of type *t* is derived, and thus where lambda abstraction over the trace of type *d* will produce an expression of type *<dt>* to serve as its argument.

I should note that while it is difficult to interpret (76a) with *few* scoping under the intensional verb (perhaps for pragmatic reasons), there are parallel examples where this is clearly a possibility. For example, the following example seems to allow both the interpretations paraphrased below:

\[
(79) \quad \begin{align*}
\text{a.} & \quad \text{I need few friends} \\
\text{b.} & \quad \text{‘it is not the case that I must have a large number of friends’} \quad \text{few > need} \\
\text{c.} & \quad \text{‘it must be the case that I have a small number of friends’} \quad \text{need > few}
\end{align*}
\]

To derive the second of these, I follow Larson, den Dikken & Ludlow (1997) in taking it to arise from the underlying clausal structure in (80), which provides a lower expression of type *t* where the Q-adjective can take scope.

\[
(80) \quad \text{I need TO HAVE few friends}
\]
The above remarks pertain primarily to *few*, and by extension, *little*. There is also an important point to make here about the semantics of *many* and *much*. Consider again the derivation in (64), involving quantificational *many*. Lambda abstraction over the trace left by the raising of the QP creates a set of degrees (a scalar interval). This serves as the argument for *many*, after which we again lambda abstract. As can be verified above, the resulting set of degrees is identical to the original one. Thus *many* itself makes no semantic contribution to the derivation; it is in essence semantically vacuous. The same point could be made about *much*. With regards to the semantics of the quantificational *many* and *much* in their positive forms, it is the null morpheme POS that does all work, introducing the neutral range that serves as the standard of comparison.

The claim that *much* in particular is semantically vacuous receives support from the fact that it occurs as a ‘dummy element’, in the previously mentioned phenomenon of *much* support (Corver 1997), which I will discuss in greater detail in Section 3.8.

(81) a. Fred is fond of Jane; in fact, he is too much so
    b. Fred is fond of Jane; in fact, he is so much so that it is surprising.

I propose that the vacuous nature of *many* and *much* is also responsible for the awkwardness of these words in their unmodified forms in quantificational uses. Something like (82a), though certainly grammatical, is much less natural in colloquial speech than the examples in (82b). And (83a) verges on the outright ungrammatical, in contrast to the perfectly felicitous examples in (83b):

(82) a. Many students came to the party
    b. Too many/not many/so many/more than 20 students came to the party

(83) a. ??There is much water in the bucket
    b. There is too much/not much/more than 1 gallon of water in the bucket
The explanation, I suggest, is that in the absence of overt degree morphology, or 
negation, *many* and *much* do not have sufficient semantic content to stand on their own as 
‘quantificational’ expressions. On this view, the contrast between *many* and *much* (i.e. 
(82a) versus (83a)) is also not surprising: *many* carries at least some content, in that it 
specifies that the dimension in question is cardinality, but *much* lacks even that much 
content, being unspecified for dimension of measurement.

### 3.5 Comparatives and the Differential Construction

Having addressed the quantificational use of Q-adjectives, I turn now to their 
differential use, which served as a motivation for the degree predicate analysis that I 
develop in this work. Recall that the Q-adjectives *many* and *few* can serve as modifiers 
of count noun comparatives:

(84)  
\[ \begin{align*} 
& a. \text{ Many more/many fewer/few more than 100 students attended the lecture} 
& b. \text{ Nearly 4 million Afghan children are enrolled in school, including} 
& \quad \text{more than 1 million girls, many more than at any point in} 
& \quad \text{Afghanistan’s history} 
& c. \text{ Few more than 400 Sumatran tigers survive in the wild} 
& d. \text{ The latest attempt to count the number of transient vacation rentals on} 
& \quad \text{Maui finds many fewer than previously estimated} 
\end{align*} \]

*Much* and *little* occur in the same position in the corresponding mass constructions, as 
well as in adjectival comparatives:

(85)  
\[ \begin{align*} 
& a. \text{ We bought much more/much less/little more/?little less than a pound} 
& \quad \text{of rice} 
& b. \text{ John is much taller/little taller/much shorter/?little shorter than Fred} 
\end{align*} \]

Q-adjectives can likewise serve as modifiers in excessives, again with *many* occurring 
with count nouns, and *much* with mass nouns and in adjectival excessive (note that *few* 
and *little* are marginal at best in excessives):
(86)  a. Fred made many too many/many too few mistakes  
     b. We bought much too much/much too little rice  

(87) John is much too tall  

As these examples show, in differential position, just as in quantificational position, the choice of Q-adjective is sensitive to the dimension of measurement: 

\textit{many/few} occur when the dimension is cardinality (as in (84) and (86a)), while \textit{much/little} occur with other dimensions (e.g. weight in (85a) and (86b), height in (85b) and (87)). As evidence for the role of dimension, note that the one ordinary adjective that describes cardinalities, \textit{numerous}, at least marginally allows \textit{many} as a modifier of its comparative form, as a Google search will confirm (e.g. ‘the problems were many more numerous than expected’). Nonetheless, in differential position, the requirement that \textit{many} be used when the dimension is cardinality is less strict. Examples such as \textit{much more than 100 people}, while awkward (to my ear, at least), are certainly better than the corresponding \textit{much people}. Likewise, \textit{much more numerous} is also acceptable (probably more so than \textit{many more numerous}). I do not have a full explanation for this, but it seems to be part of a broader pattern in which the distinction between \textit{many/few} and \textit{much/little} is eroding. Earlier forms of English had a comparative \textit{manier}, which has since been replaced by \textit{more}, the form shared with \textit{much}. The same thing appears to be happening currently with the comparative of \textit{few}, with \textit{less} frequently occurring in place of the prescriptively correct \textit{fewer} (e.g. \textit{less than 100 people}). Perhaps relatedly, one often hears \textit{amount} in place of \textit{number} (e.g. \textit{a large amount of books}). We might speculate that outside of the canonical count noun context (e.g. \textit{many books}), the cardinality-specific \textit{many} and \textit{few} are gradually giving way to the underspecified \textit{much} and \textit{little}. 


Turning to an analysis of differential Q-adjectives, with the approach developed here, this can be accommodated via a simple extension of the treatment of the quantificational cases. To start with the syntax, in the framework I have adopted degree modifiers – including both the comparative morpheme -er and the excessive too – are heads of DegPs that are located in the specifier position of the QP or AP. In the case of the comparative, the than phrase is the complement of the degree head.\footnote{Presumably this is also the location of the modifying clause in excessives, such as too few points to qualify for the finals or too tall to be a jockey; I will not investigate these here.} This leaves the specifier position for a modifier, including another QP. To take the comparative, we thus have the following structure (recall that I take the than phrase to be extraposed):

\begin{equation}
\text{(88)} \quad \text{Many fewer than 100}
\end{equation}

\begin{align*}
& \text{QP} \\
& \quad \text{XP} \\
& \quad \quad \text{DegP} \\
& \quad \quad \quad \text{QP} \\
& \quad \quad \quad \quad \text{DegP} \\
& \quad \quad \quad \quad \quad \text{POS} \\
& \quad \quad \quad \quad \quad \quad \text{many} \\
& \quad \quad \quad \quad \quad \quad \quad \text{Deg' -er} \\
& \quad \quad \quad \quad \quad \quad \quad \quad \text{Q} \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{ti} \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{than 100,}
\end{align*}

Note that among other things, the structure in (88) provides for iteration of Q-adjectives and degree modifiers: in the specifier position of the QP or AP we find a DegP, in whose specifier position another QP may occur; the latter likewise has a DegP in its specifier position, which might likewise contain another QP as specifier; and so forth. Such a structure can accommodate not just the examples in (84)-(87), but also even more complex examples (a similar approach having been discussed by Jackendoff 1977):
a. Much less than a pound of rice
\[ \text{[QP [DegP [QP [DegP POS] [Q' much]] [Deg' -er than 1 lb]] [Q' little]]} \]

b. Much taller than Fred
\[ \text{[AP [DegP [QP [DegP POS] [Q' much]] [Deg' -er than Fred]] [A' tall]]} \]

c. Many too many mistakes
\[ \text{[QP [DegP [QP [DegP POS] [Q' many]] [Deg' too]] [Q' many]]} \]

d. Much too tall
\[ \text{[AP [DegP [QP [DegP POS] [Q' much]] [Deg' too]] [A' tall]]} \]

(90) a. So many fewer mistakes than expected
\[ \text{[QP [DegP [QP [DegP so] [Q' many]] [Deg' -er than expected]] [Q' few]]} \]

b. Many too many more mistakes than expected
\[ \text{[QP [DegP [QP [DegP [QP many] [Deg' too]] [Q' many]] [Deg' -er than expected]] [Q' many]]} \]

Turning to the semantics, I earlier offered the following as a first approximation of the semantics of the comparative:

(91) \[ [-\text{er}] = \lambda P \lambda Q (Q(d) \land \neg P(d)) \]

This would be sufficient for unmodified comparatives:

(92) More than 100 students attended the lecture
\[ \exists d[\exists x [*\text{student}(x) \land \text{attended}(x, \text{lecture}) \land \mu(x) \geq d] \land \neg d \leq 100] \]

But this definition does not allow modification by a Q-adjective or other expression. To address this, I propose the following revised entry for the comparative (based in part on Kennedy 2001):

(93) \[ [-\text{er}] = \lambda I \lambda d \lambda I'.d \in \text{ZERO}(\text{GAP}(I',I)), \text{where} \]

\[
\text{GAP}(I',I) = \begin{cases} I' - I & \text{iff } I \subseteq I'; \\ I' - \text{INV}(I) & \text{iff } \text{INV}(I) \subseteq I'; \\ \emptyset & \text{otherwise} \end{cases}
\]

\[ \text{ZERO is a function that maps an interval to a second interval of the same length whose lower bound is 0} \]

(94) \[ [-\text{er than 100}] = \lambda d \lambda I'.d \in \text{ZERO}(\text{GAP}(I',(0,100])) \]
The effect of the operations of GAP and ZERO is essentially to take the interval between the endpoints of I and I' and ‘slide’ it down the scale so its lower bound is zero. To make this more concrete, the following shows several examples:

\[(95)\]

a. \( I = (0, 100] ; I' = (0, 160] \)
   \[ \text{GAP}(I, I') = I' - I = (100, 160] \]
   \[ \text{ZERO}(\text{GAP}(I, I')) = (0, 60] \]

b. \( I = (0, 100] ; I' = [75, \infty) \)
   \[ \text{GAP}(I, I') = I' - \text{INV}(I) = [75, 100) \]
   \[ \text{ZERO}(\text{GAP}(I, I')) = (0,25] \]

c. \( I = (0, 100] , I' = (0,80] \)
   \[ \text{GAP}(I, I') = \emptyset \]
   \[ \text{ZERO}(\text{GAP}(I, I')) = \emptyset \]

With this definition, a comparative -\textit{er than} \( d \) is a gradable predicate of scalar intervals (i.e. the same type as Q-adjectives themselves), whose function is to associate with an interval \( I \) containing \( d \) a second interval whose lower bound is 0, and whose length is equal to the length of the interval or ‘gap’ between \( d \) and the endpoint of the original interval.

With this framework in place, let us now take a look at our original examples involving comparatives modified by differential Q-adjectives. In (96b), I give the surface structure (minus extraposition) of one relevant example. Here, the differential Q-adjective originates in the specifier position of the DegP. But just as was the case with quantificational Q-adjectives, it cannot be interpreted \textit{in situ}, but must raise covertly, yielding the LF in (96c):

\[(96)\]

a. Many fewer than 100 students attended the lecture
b. SS: \([\text{IP}[\text{QP[DegP[QPPOS-many]]}_{\text{Deg}\text{-er than 100]}]}_{\text{few}}\] Meas students attended the lecture
c. LF: \([\text{POS}_4 [t_4\text{-many}_3 [(t_3\text{-er than 100})_2 [t_2\text{-few}_1 [t_1\text{MEAS students attended the lecture}]])]]\]
The corresponding semantic translation is the following (where it is assumed that the neutral range \( N_\# \) may be different from that invoked in the evaluation of the positive forms of *many* and *few*).

(97) \[ \llbracket \text{POS}_4 \rrbracket \llbracket t_4 \text{ many}_3 \rrbracket \llbracket t_3 \text{ er than 100}_2 \rrbracket \llbracket t_2 \text{ few}_1 \rrbracket \]
\[ ( \llbracket t_1 \text{ MEAS students attended the lecture} \rrbracket ))) \]
\[ = \forall d \in N_\# [d \in \text{ZERO}(\text{GAP}(\lambda d'. \neg \exists [\ast \text{student}(x) \wedge \text{attended}(x, \text{lecture}) \wedge \mu_\#(x) > d'], (0, 100]))]) \]
\[ = N_\# \subseteq (0, 100 - \#-\text{Students-AtL}] \]

The derivation in (97) essentially involves the creation and manipulation of a series of scalar intervals, via successive stages of lambda abstraction and application of degree operators. The derivation proceeds as follows:

(98) Many fewer than 100 students attended the lecture

\begin{align*}
& t_1 \text{ MEAS students attended} \quad (0, \#-\text{Students-AtL}] \\
& t_2 \text{ few} \quad [\#-\text{Students-AtL}, \infty) \quad \text{by (35)} \\
& t_3 \text{ -er than 100} \quad (0, 100 - \#-\text{Students-AtL}] \quad \text{by (93)} \\
& t_4 \text{ many} \quad (0, 100 - \#-\text{Students-AtL}] \quad \text{by (35)} \\
& \text{POS} \quad N_\# \subseteq (0, 100 - \#-\text{Students-AtL}] \quad \text{by (40)}
\end{align*}

Visually, this is shown in (99), which depicts the situation in which the (negative) gap between the number of students attending and 100 is large relative to the context.
Defining the semantics of the comparative to include an extra degree argument, as in (93), thus allows an analysis of modified comparatives such as that in (98), but it would seem to create a problem for unmodified comparatives such as (100a), as there appears to be a degree argument that is left unsaturated. These can be dealt with by taking the degree argument to be existentially bound, as shown in (100b) (cf. Schwarzschild & Wilkinson 2002):

(100) a. More than 100 students attended the lecture
b. \((d\text{-er than }100)_2 \! [t_2\text{-many}_1 [t_1\text{-Meas students attended}]]\)
c. \(\exists d [d \in (0, 100 – \#\text{-Student-AtL}]]\)

The same approach can be extended to the excessive too. Without attempting an in-depth analysis of the semantics of the excessive, we might as a rough approximation propose that too in an example such as (101) has the modalized semantics in (102) (cf. Meier 2003), resulting in the LF and truth conditions in (103):

(101) John bought much too much rice

(102) \(\mathbf{too} = \lambda d \lambda I'. d \in \text{ZERO}(\text{GAP}(I', \lambda d'. \exists w_{\text{Acc}}[P^w(d') = 1]))\)

(103) a. LF: \([\mathbf{too}] = \lambda d \lambda I'. d \in \text{ZERO}(\text{GAP}(\lambda d. \exists x [\text{rice}(x) \land \text{buy}(\text{John}, x) \land \mu_{\text{DIM}}(x) \geq d]^w_0, \lambda d'. \exists w_{\text{Acc}} \exists x [\text{rice}(x) \land \text{buy}(\text{John}, x) \land \mu_{\text{DIM}}(x) \geq d]_w))\]

\(\equiv \text{NS} \subseteq (0, \text{Amt-of-rice-John bought in } w_0 – \max(\lambda d'. \exists w_{\text{Acc}} [\text{John bought } d' \text{ rice in } w]))\)

Here, too references the set of degrees \(d'\) such that there is some acceptable world \(w\) in which the amount of rice John bought was at least \(d'\) (with the dimension on which rice is measured as usual ‘filled in’ on interpretation). The sentence is true iff the amount of rice that John bought in the actual world \(w_0\) exceeds the maximum of this set (i.e., exceeds the maximum amount bought in acceptable worlds) by some large amount.
To summarize, the differential uses of Q-adjectives, which were shown to be problematic for quantificational and cardinality predicate analyses, pose no particular challenge for the present account. Rather, with the appropriate semantics for the comparative and excessive morphemes, differential Q-adjectives can be accommodated by the very same mechanism by which their quantificational uses are analyzed.

3.6 Q-Adjectives as Predicates

3.6.1 The Problem

In Chapter 2, it was pointed out that a potential strength of the cardinality predicate analysis of Q-adjectives (that is, the analysis that I have rejected) is its ability to account for their predicative and attributive occurrences. In order to support the degree-predicate analysis as a valid alternative, it is necessary to show that it too can handle these data. I turn to this now.

Recall that the Q-adjectives *many* and *few* can occur both predicatively and attributively. *Little* and *much* are generally disallowed in predicative position, and only *little*, but not *much*, may occur attributively.

(104) a. John’s good qualities are many/few
    b. The many/few students who attended the lecture were soon bored

(105) a. *The water in the bucket was little/much
    b. The little/*much water that remains in the bucket

*Many* and *few* may also occur predicatively when modified by degree modifiers, though with some constraints that will be discussed below:

(106) a. The errors were too many to count/too few to worry about
    b. Side effects were so many that the clinical was cancelled/so few that they were barely measurable
    c. Despite travelers’ concerns about the hurricane, cancellations were fewer than expected
In pursuit of an analysis of these occurrences of Q-adjectives, let us begin with some patterns of acceptability. What has not to my knowledge been recognized before now is that predicative *many* and *few* are allowed with a restricted class of subjects: bare plurals (107a), definite descriptions and demonstratives (107b,c), possessives (107d,e), demonstratives used pronominally (107f), and *wh*-pronouns (107g). They are disallowed with quantificational subjects (107h), plural indefinites (107i) and conjoined referring expressions (107j):

(107) a. Clues to the suspect’s identity were few/many
   b. The advantages of the new treatment are many/few
   c. As for occurrences of side effects, those cases were few
   d. John’s good qualities are many/few
   e. His friends were many/few
   f. As for the advantages of the new treatment, those are many/few
   g. Let’s talk about the advantages of the new treatment, which are many/few
   h. *All/most/both lawyers are many/few
   i. *Some lawyers are few/many
   j. *Fred, John and Frank are many/few

With respect to these constraints, Q-adjectives already distinguish themselves from ordinary adjectives, which of course can occur predicatively with the latter sort of subject:

(108) a. All/most/both lawyers are greedy
   b. Some lawyers are greedy
   c. Fred, John and Frank are greedy

A similar pattern is observed with the determiners that can occur with attributive *many* and *few*: the definite article, demonstratives and possessives are allowable, while indefinite and quantificational determiners are not¹⁸ (see Hackl 2000 for discussion).

¹⁸ Here I put aside the collocation *a few*, as well as the sequence *every few* in examples such as *every few days*; in Chapter 5 I will show that these have a different origin.
(109)  a. The many/few advantages of the new treatment
   b. Those many/few cases where side effects occurred
   c. John’s many/few good qualities
   d. His many/few friends
   e. *All/most/both many/few lawyers
   f. *Some many/few lawyers

Interestingly, we see an almost identical pattern in noun phrases of the form the number of X, the only difference being that a definite description is awkward here:

(110)  a. The number of clues to the suspect’s identity was small/large
   b. ??The number of the advantages of the new treatment is small/large
      (cf. the number of advantages of the new treatment is small/large)
   c. As for occurrences of side effects, the number of those cases was small/large
   d. The number of John’s good qualities is small/large
   e. The number of his friends was small/large
   f. As for the advantages of the new treatment, the number of those is large
   g. Let’s talk about the advantages of the new treatment, the number of which is large
   h. *The number of all/most/both lawyers is large
   i. *The number of some people is large
   j. *The number of Fred, John and Frank is large

There is a further parallel between the three constructions: in examples involving a definite determiner or, in the case of the predicative use, a bare plural, an unmodified plural noun is awkward in the absence of context, while a modified plural is more felicitous:

(111)  a. ?People were few
   b. ?The people were few
   c. ?The few people ran into the room
   d. ?The number of people was small

(112)  a. People who heard the noise were few
   b. The people who heard the noise were few
   c. The few people who heard the noise ran into the room
   d. The number of people who heard the noise was small
Lest it be thought that this reflects some sort of syntactic constraint (along the lines of that proposed for example by Kayne 1994 to account for contrasts such as *the Paris vs. the Paris of the 30’s), note that in each case the need for modification can be overridden if the context is sufficient:

(113) The desert was barren, with barely enough water to sustain plant life.
   a. Trees were few…
   b. The trees were few…
   c. The few trees were stunted
   d. The number of trees was small

The requirement for modification therefore seems to be pragmatic in nature, rather than syntactic.

With regards to bare plural subjects, Q-adjectives again distinguish themselves from ordinary adjectives. In predicative position the latter can readily take unmodified bare plurals as subjects, as in the well-known pair below:

(114) a. Firemen are altruistic
    b. Firemen are available

There is an interpretive difference as well. In the cases in which a bare plural may occur as subject of a predicative Q-adjective (via either overt modification or contextual support), the interpretation is neither the generic reading found with individual-level predicates (114a), nor the existential reading found with stage-level predicates (114b). Rather, in a case such as (113a), it is the totality of the contextually relevant set of trees that is characterized as few.

There is even, I believe, a contrast between the Q-adjective many and the adjective numerous, whose semantic content is otherwise quite similar to that of many:

(115) a. Cockroaches are numerous
    b. ?Cockroaches are many
(115a) can be read as stating a fact about the cockroach kind, namely that its members exist in large numbers (so-called direct kind predication; Carlson 1977). But to be felicitous, (115b) requires us to infer a context in which there is some particular set of cockroaches that we are talking about.

From these patterns, several conclusions can be drawn. First, despite their occurrence in ‘adjective-like’ positions, Q-adjectives cannot receive the same semantic analysis as ordinary adjectives (a position that is of course also supported by the previously discussed facts relating to their differential uses). Secondly, the predicative and attributive uses of Q-adjectives must be related, a not surprising conclusion. And thirdly, given the parallels to the \textit{the number of X} construction, the semantics of predicative and attributive Q-adjectives appears to involve computing the number of some plurality.

3.6.2 Predicative \textit{Many} and \textit{Few}

With these observations in mind, let us consider predicative Q-adjectives, which I take to be the basic case; in Section 3.7 I show that the semantics of their attributive uses are derived from their predicative semantics. Consider again the data in (107). What distinguishes the grammatical examples in (107a-g) is that (modified) bare plurals, definite descriptions, possessives, pronominals and \textit{wh}-pronouns can be interpreted as group-denoting. For example, in (107a), the relevant group is the maximal set of clues to the suspect’s identity; in (107b) it is the maximal set of advantages of the new treatment; in (107d), the maximal set of John’s good qualities; and so forth. By contrast, in the ungrammatical cases, the subject noun cannot be interpreted as group denoting. In (107h) the subjects express quantification over individuals; in (107i) \textit{some lawyers} is
either quantificational as well, or has predicative semantics. Here, conjoined proper
names such as Fred, John and Frank in (107g) would seem to present an exception, in
that noun phrases of this form are intuitively group denoting. But note that a similar
pattern is observed in partitive constructions. Ladusaw (1982) points out that only group-
denoting (definite) DPs may occur in partitives, while indefinite and quantificational DPs
are disallowed (two of the/those/his/*some/*several/*most/*all dogs). But conjoined
referring expressions, which might be expected to pattern with definites, are bad in
partitives (*some/all/two of Fred, John and Frank). We are led to suspect that conjoined
proper names might require some more elaborate semantic analysis (see Heycock &
Zamparelli 1999 for a discussion of issues in the semantics of conjunction); but I will not
pursue this here.

Returning to the data introduced above, the contrasts observed in the case of bare
plurals are illuminating. A modified bare plural such as people who heard the noise can
readily be understood as referring to a particular group of individuals. The same is the
case for an unmodified bare plural when the context is sufficient to delimit its reference
(as with trees in (113a), which we interpret to mean the totality of trees in the desert
under discussion). But taken out of context, an unmodified bare plural such as people is
not as readily interpreted as denoting a particular group of individuals, but rather tend to
be interpreted as kind-denoting (Carlson 1977; Chierchia 1998b).

These facts can be accounted for if we take the subjects in the grammatical
examples in (107) to have a null MeasP layer above the DP19:

19 This analysis assumes that the Meas head can select for a full DP as well as an NP or a lower functional
projection above the NP. I take this to be unproblematic. See also Cheng & Sybesma for a similar account
in which a phonologically null functional layer is present in bare NP subjects in Chinese.
(116) John’s good qualities are many
\[
[IP [\text{MeasP Meas} [DP John’s good qualities] are many]]
\]

Recall that the Meas head encodes a measure function:

(117) \[
\text{[[Meas]]} = \lambda x.\lambda d.\mu_{\text{DIM}}(x) \geq d
\]

On this definition, a plurality can saturate the first argument of the measure function.

This is precisely what we get in the configuration in (116): the DP denotes a plurality, which can be taken as an argument by Meas; the result is an expression of type \(\langle dt\rangle\), the appropriate type to serve as argument for the Q-adjective:

(118) a. \[
\text{[[John’s good qualities]]} = \sqcup\lambda x. \text{*good-quality}(x, \text{John})
\]

b. \[
\text{[[\text{MEAS John’s good qualities}}\text{]]} = \lambda d.\mu_{\text{DIM}}(\sqcup\lambda x. \text{*good-quality}(x, \text{John})) \geq d
\]

(119) John’s good qualities are many
\[
\text{[[POS1] [t1-many] ( [MEAS John’s good qualities] )}}
\]

\[
\forall d' \in N_{\#}[d' \in \lambda d.\mu_{\#}(\sqcup\lambda x. \text{*good-quality}(x, \text{John})) \geq d]
\]

\[
\equiv \forall d' \in N_{\#}[\mu_{\#}(\sqcup\lambda x. \text{*good-quality}(x, \text{John})) \geq d']
\]

\[
\equiv N_{\#} \subset (0, \#-\text{John’s good qualities}]
\]

(119) thus receives an interpretation according to which the number of John’s good qualities exceeds the contextually determined neutral range \(N_{\#}\) on the scale of cardinality, which accurately captures the meaning of the sentence.

The requirement that the (overt) subject be group-denoting is now clear: only a plurality (an individual) can saturate the individual argument of the measure function.

Hence the similarity between predicative Q-adjjectives and the \textit{number of X} construction: both cases require a group whose members can be counted. The requirement for overt modification in the case of bare plural subjects is also clarified: this facilitates a group-denoting reading for the plural, as opposed to a kind reading. An expression denoting a group or plurality can serve as an argument for the counting measure function introduced
by Meas; but if we take kinds to be atomic individuals (per Carlson 1977, Chierchia 1998b), a kind-denoting expression cannot (or at best the result would be trivial).

Finally, the contrast between many and numerous in (115) suggests that numerous (like most gradable adjectives) can be predicated directly of a kind; there is no need for a Meas layer to introduce a set of degrees as an argument. That is, the analysis is the simple one in (120) (cf. Kayne 2004, who proposes that numerous is an adjective that incorporates NUMBER):

(120) Cockroaches are numerous
    [IP [DP cockroaches] are numerous]
    Numerous(cockroaches)

To summarize, the same basic elements that were involved in the analysis of Q-adjectives in their quantificational use – namely the degree-based interpretations for Q-adjectives themselves (per (35)) and the linking element Meas – can in a different configuration also accommodate their predicative uses. The difference here is that in the quantificational case, Meas occurs between the Q-adjective and the nominal expression, where it has the function of linking the degree argument of the former to the individual argument of the latter; in the predicative case, Meas occurs in the sentential subject, where maps a group-denoting nominal expression to a set of degrees (interval) that can serve as argument for the Q-adjective.

3.6.3 A Variant of the Predicative Use

The data in (121)-(123) represent a variation of the predicative usage of Q-adjectives:
(121)  a. The number of unemployed workers was few/many
    b. The number of unemployed workers was fewer/more than 50
    c. The number of unemployed workers was fewer/more than the number of job openings
    d. The number of unemployed workers was too few to cause concern/too many for the program to be of use
    e. We were surprised that the number of unemployed workers was so many/so few

(122)  a. [When it comes to guests], twenty is many/isn’t many/is few
    b. Twenty is too many (to fit in the van)/too few (to do all this work)
    c. Twenty is more/fewer than needed
    d. Twenty would be many/few

(123)  a. Twenty guests is few/many
    b. Twenty guests is too few/too many
    c. Twenty guests was more/fewer than expected
    d. Twenty guests would be many/few

Each of these cases involves an overt numeral or number in the subject noun phrase. The felicity of these examples is further support that the semantic subject of predicative Q-adjectives is something in the domain of numbers, not individuals. But their analysis poses some questions.

Of these cases, the construction in (121) is the most straightforward, representing an overt counterpart of the null structure proposed in (116) above. Rather than a null functional head taking the overt DP as argument, we have the overt functional noun number doing so. The chief mystery here is that bare many and few are only marginal, while their modified counterparts (too many/few; more/fewer; etc.) are more acceptable; this is repeated to some extent in the examples in (122) and (123) as well. I do not have an explanation for this contrast at present. The intuition is that in (121a) we should use large/small instead of many/few (the number of unemployed workers was large/small). And to be certain, large/small and their inflected forms could also be used in the other examples in (121); but why these also are felicitous with many and few is not clear.
In the examples in (122) it is an overt numerical expression that serves as subject. The question here is what that numerical expression denotes. The simplest view (and the one that I have assumed up until this point) would be to analyze cardinal numerals as denoting degrees:

(124)  Twenty is many  
         \(\text{twenty} \equiv 20\)  \((\text{type } d)\)

But according to the entry in (35a), the Q-adjective many does not want a degree as an argument, but rather a set of degrees (a scalar interval, type \(\langle dt \rangle\)). On the type \(d\) analysis of the numeral, the derivation would crash. It seems that here we need the numeral to denote an interval instead: in the case of twenty in (124), the interval from 0 to 20. We might suggest that cardinal numerals have secondary denotations of type \(\langle dt \rangle\), or that alternately, Q-adjectives themselves have secondary denotations in which they take degrees rather than sets of degrees as arguments. But either of these solutions would be stipulative, and would run the risk of overgenerating.

A more appealing solution is to say that in cases such as (122), the numeral is interpreted as denoting the result of applying the counting measure function to an arbitrary group of individuals of a certain cardinality.

(125)  \(\text{twenty} \equiv (\lambda x. \lambda d. \mu_\varepsilon(x) \geq d)(x_{20})\)  
\(\equiv (\lambda d. \mu_\varepsilon(x_{20}) \geq d)\)  
\(\equiv (\lambda d.20 \geq d)\)  
\(\equiv (0, 20]\)

In essence, (122a) could then be paraphrased as follows: ‘if we count some (contextually relevant) set of individuals and get twenty as a result, that would be many’.
Evidence for this analysis comes from a consideration of the following contrast\textsuperscript{20}:

(126)  
   a. Fifty is much greater than twenty  
   b. Fifty is many more than twenty 

Here, the (a) and (b) examples are both grammatical, but with a subtle difference in interpretation. (126a) simply expresses a comparison between two numbers: the first is substantially larger than the second. But in (126b), the intuition is, I think, that we have counted fifty of something, and are comparing the result to twenty of something (else) that we have counted. This contrast can be captured with the following logical forms. In (126a), \textit{fifty} and \textit{twenty} are treated as individuals, which saturate the individual argument of the gradable adjective \textit{great}, with the comparative morpheme comparing the resulting sets of degrees:

(127)  Fifty is much greater than twenty  
        \[ \text{is great} \]  
        \[ [\text{POS much}] ((\text{[-er]} (\lambda d.\text{great}(20) \geq d))(\lambda d'.\text{great}(50) \geq d')) \]

But in (126b), both \textit{fifty} and \textit{twenty} are interpreted as the result of counting arbitrary groups of individuals (i.e. the analysis proposed for the examples in (122)):

(128)  Fifty is many more than twenty  
        \[ \text{is great} \]  
        \[ [\text{POS many}] ((\text{[-er]} ((\lambda x \lambda d.\mu_{ad}(x) \geq d)(x_{20}))) ((\lambda x \lambda d.\mu_{ad}(x) \geq d)(x_{50})))) \]
        \[ \equiv [\text{POS many}] ((\text{[-er]} ((\lambda d.20 \geq d))(\lambda d'.50 \geq d')) \)

The formula in (128) states that the result of counting fifty of something greatly exceeds (i.e. exceeds by many) the result of counting twenty of something.

Finally, let us consider the cases in (123), where a numerical noun phrase such as \textit{twenty guests} serves as the subject of the predicative Q-adjective. On the surface these examples are puzzling, since they would seem to represent an exception to the restriction

\textsuperscript{20} In (126b), I include the differential \textit{many} to ensure that the dimension is interpreted as cardinality, given that \textit{more} itself corresponds to both \textit{many+/er} and \textit{much+/er}. Interestingly, \textquote{fifty is much more than twenty} (where the dimension is other than cardinality) patterns with \textit{much greater than} in interpretation.
to referential noun phrases as subjects of Q-adjectives: in a sentence such as ‘20 guests is (too) many’, 20 guests cannot be interpreted as referring to some particular group of 20 individuals. But there are reasons to think that these cases require a different analysis than that given to the grammatical examples in (107a-g) in which the subject DP is interpreted as group denoting. First, the singular form of the copula is preferable (twenty guests is too many versus ?twenty guests are too many), just as it is in with bare numeral subjects such as (122). And as with bare numeral subjects, there is a tendency to want a conditional form of the copula (twenty guests would be too many). On the basis of these parallels, I propose that these cases require an extension of the analysis given to the examples in (122). On this view, in a sentence such as (129a) twenty guests is interpreted as denoting the set of degrees that results from applying the counting measure function to an arbitrary group of twenty guests.

(129) a. Twenty guests is too many
   b. \[ \langle \text{twenty guests} \rangle = (\lambda x \lambda d. \mu_{\#}(x) \geq d)(x_{20 \text{ guests}}) \\
      = \lambda d. \mu_{\#}(x_{20 \text{ guests}}) \geq d \\
      \equiv (0, 20]_{\text{guests}} \]

The sentence therefore can be paraphrased as stating that if we have a set of guests and we count them and get twenty as a result, that number is many, which seems to adequately capture its meaning.

A curious consequence of this analysis is that in examples such as these, the numerical noun phrase must be taken to denote something in the domain of degrees, namely an interval on the scale associated with measuring entities of a certain sort. In simple (if not quite technically accurate) terms, twenty guests here denotes twenty as a number of guests. This is of course not the denotation we would ordinarily assign to a noun phrase of this form, which would be more typically taken to denote a generalized
quantifier or (in a predicative analysis of indefinites such as that of Landman 2004) a set of pluralities. As evidence that it is nonetheless a possibility, note that examples such as the following are not at all bad:

(130) Twenty guests is a large number (of guests)

Another case where we seem to require the same sort of denotation for a numerical noun phrase is in differential position, as in (131).

(131) There were 100 guests fewer than expected

In Chapter 2 I proposed that the comparative provides a test for degree-based interpretations, in that expressions that occur as modifiers in comparatives must have (at least optionally) degree-based semantics. The possibility of a numerical noun phrase such as 100 guests in this position is thus further support that these phrases can (in some contexts) be interpreted as degree denoting.

It seems that we have a form of polysemy here, perhaps akin to what is available to certain nouns (e.g. ‘the newspaper is on the front porch’, where we are talking about a physical object, versus ‘the newspaper closed’, where we are talking about the institution). The availability of this sort of secondary interpretation is a topic that would benefit from more in-depth investigation, which I must leave for another time.

3.6.4 Q-Adjective + Noun as Predicates

To turn to another puzzle, in Chapter 2 I pointed out the ungrammaticality of examples such as the following: while Q-adjectives may act as sentential predicates, as can plural nouns, phrases of the form Q-Adjective + Noun cannot.

(132) a. John’s friends are many/few
    b. John’s friends are lawyers
    c. *John’s friends are many lawyers/few lawyers
Again we have a contrast with ordinary adjectives:

(133) John’s friends are rich lawyers

In searching for the reason for this contrast, let us consider first the nature of the predication that would be involved in cases such as those in (132). *Lawyers* must be predicated directly of John’s friends, but *few* or *many* can only be predicated of their number. Furthermore, *lawyers* distributes over John’s friends (each one individually is a lawyer), but the Q-adjective must describe the number associated with their totality.

These mismatches alone might be sufficient to explain the ungrammaticality of cases such as (132c): the same noun phrase cannot express both predication distributed over the atoms of a plurality and predication of the number associated with its totality. But beyond this, there is a structural issue as well. In the examples of predicative Q-adjectives considered up to this point, the Meas head has been in the subject. But in a case such as the ungrammatical (132c), we would require a Meas head in the predicate to link the Q-adjective to the plural noun; furthermore, the subject itself cannot be a MeasP, since this would produce an expression of the wrong semantic type to serve as argument for the plural noun *lawyer*. The contrasting structures are these:

a. [(IP [MeasP Meas [DP John’s friends]] are many)]
b. [(IP [DP John’s friends] are [MeasP many Meas lawyers])]

The structure in (134b) would correspond to the following semantic interpretation:

(135) 〚POS2〛 ( [t2 many1] ( [John’s friends are t1 Meas lawyers] ))
\( \forall d \in \mathbb{N} [*\text{lawyer}(\text{John’s friends}) \land \mu_d(\text{John’s friends}) \geq d] \)

Here, we end up with something that looks questionable: the logical constituent

*lawyer(John’s friends)* is within the scope of universal quantification over degrees, but
it itself does not contain a degree argument. I take this configuration to be ill-formed, and this ill-formedness to be the source of ungrammaticality in cases such as (132c).

There is one apparent counterexample to the claim that a noun phrase of the form Q-Adjective + Noun cannot occur predicatively, namely cases such as the following:

(136) a. The article was fewer than 2500 words
    b. If your first and last names combined are more than twelve characters, only the first twelve will be counted

Here the noun phrases fewer than 2500 words and more than twelve characters in fact act as predicates. But observe that here, the predicative noun phrases can be interpreted as degree-denoting: 2500 words characterizes the length (in words) of the article, and twelve characters describes the possible length (in characters) of your first and last names. The conclusion is that these noun phrases as a whole actually denote degree predicates. The syntactic structure and semantic translation are thus the following

(137) a. [IP [MeasP Meas [DP The article]] was [QP fewer than 2500 words]]
    b. [fewer than 2500 words] (λd.µDIM(the article) ≥d)

These are not, then, counterexamples to the unavailability of Q-Adjective + Noun predicatively, but rather further support for the claim that QPs denoting degree predicates may occur as sentential predicates with a null MeasP layer in the subject noun phrase.

3.6.5 Predicative Much and Little

A final puzzle relating to predicative Q-adjectives is the following: while many and few occur predicatively, much and little do not:

(138) a. The errors in the manuscript were many/few
    b. *The water in the bucket was much/little

What is the source of this contrast?
Here it is relevant to point out again a crucial difference in the semantics of the two pairs. *Many* and *few* encode a dimension of measurement, namely that of cardinality. But *much* and *little* do not. The consequence of this is that in a construction of the following form, the dimension of measurement is identifiable if the Q-adjective is *many* or *few*, but not if it is *much* or *little*.

(139)  \([\text{MeasP DP}] \text{ is Q-adjective}\)

As evidence that this is relevant, note that when a referring DP is replaced by a degree denoting expression, predicative *much* and *little* become acceptable:

(140)  a. *The water is too much/too little*
     b. Ten gallons is too much/little

Here the dimension of measurement is identified not by the Q-adjective, but by its subject: *ten gallons* is a measure of (liquid) volume.

Certain nominal expressions seem to be interpretable as denoting degrees, and here too, *much* and *little* become acceptable predicatively:

(141)  a. We tried to restore the house, but the damage from the flood was too much
     b. The time was too little to examine all aspects of the proposal

In these examples, *damage* can be interpreted as ‘amount of damage’, and *time* as ‘length of time’. In the right context the following is likewise acceptable, with *the shirt* interpretable as ‘the cost of the shirt’:

(142)  The shirt was too much

Finally, and perhaps most conclusively, consider the contrast below:

(143)  a. Side effects were fewer than expected
     b. *Side effects were more than expected
Fewer is the comparative form of few, and thus specifies cardinality. But more is the comparative of both many and much, and thus does not specify a particular dimension. Correspondingly, fewer can occur predicatively here, but more cannot.

The generalization is the following: the structure in (139) is grammatical only if the dimension of measurement is made overt. This condition is fulfilled automatically in the case of many and few, but in the case of much and little, it is only fulfilled if the noun phrase itself provides the dimension. As for why this is relevant in the predicative use of Q-adjectives but not their quantificational use, I take this to be a licensing condition on the null Meas projection: the subject can be interpreted as a MeasP only if the dimension of measurement is specified overtly.

3.7 Attributive Q-Adjectives and Nonrestrictive Interpretation

With the above conclusions regarding predicative Q-adjectives in mind, let us turn now to their attributive uses:

(144)  a. John’s many friends supported him through his illness
       b. [The desert was barren.] The few trees were stunted

Recall that under a cardinality predicate analysis of Q-adjectives, examples such as these would seemingly receive a straightforward analysis: the Q-adjective combines intersectively with the plural noun, with the taking the resulting predicate as an argument (cf. Hoeksema 1983 for an analysis along these lines, and Landman 2004 for a similar account of cardinal numerals in attributive position):

(145)  $\llbracket \text{few trees} \rrbracket = \llbracket \text{few} \rrbracket \cap \llbracket \text{trees} \rrbracket$
       $= \lambda x. \text{few}(x) \land *\text{tree}(x)$

$\llbracket \text{the} \rrbracket = \lambda P_{(\eta)}. \text{sup}(P) \text{ iff sup}(P) \text{ is defined; undefined otherwise}$

where sup(P) = $\exists x[P(x) \land \forall y[P(y) \rightarrow y \subseteq x]]$
\[
\text{the few trees} = \sup(\lambda x. \text{few}(x) \land \text{tree}(x)) \text{ iff } \\
\sup(\lambda x. \text{few}(x) \land \text{tree}(x)) \text{ is defined; } \\
\text{undefined otherwise}
\]

On the other hand, these data look problematic for the degree-predicate analysis I have developed here. As demonstrated in Section 3.4, Q-adjectives cannot combine in situ with a plural nominal expression (that is, with an expression of type \langle et \rangle or \langle d,et \rangle). In their quantificational uses, they must raise covertly to take sentential scope, binding a variable left in their surface position. But it can be shown that in their attributive use, Q-adjectives do have sentential scope. The crucial evidence for this is provided by NPI licensing. As mentioned earlier, quantificational few licenses an NPI in the sentential predicate, demonstrating that it has scope over the predicate; but attributive few does not, evidence that its scope is lower:

(146)  
\begin{align*}
\text{a. Few Americans I know have ever been to Mongolia } \\
\text{b. *The few Americans I know have ever been to Mongolia}
\end{align*}

We apparently require some mechanism by which the Q-adjective can take lower scope, as is achieved via the cardinality predicate analysis in (145).

While it might be tempting on the basis of such examples to reconsider a cardinality predicate analysis for at least some uses of Q-adjectives, I think there is an important reason to believe that this is not the right approach. The analysis sketched out in (145) treats the Q-adjective as a restrictive modifier: from the set of pluralities that satisfy the predicate trees, we pull out those that also exhibit the property of ‘few-ness’. In essence, this would be to treat attributive Q-adjectives on par with other restrictive modifiers, including (most) prenominal adjectives (e.g. a tall man) and as well as restrictive relative clauses (the man that I met). But Q-adjectives in attributive position are not actually interpreted restrictively, but rather nonrestrictively. In simple terms,
(144b) does not state that the group of trees that were few in number (as opposed to some other groups of trees) were stunted. Rather, it states that the totality of the contextually relevant trees, whose number was few, were stunted. But this is not precisely what the final formula in (145) expresses; and it only by finessing the definition of the that this is entailed at all.

It is well known that ordinary prenominal adjectives can also have nonrestrictive (sometimes called appositive) readings (for relevant discussion, see Bolinger 1967; Kayne 1994; Morzycki 2008). To take an example, in the industrious Greeks, industrious can be read either restrictively (the Greeks who were industrious, as opposed to other sorts of Greeks) or nonrestrictively (the Greeks, who as a whole were industrious). Importantly, attributive many and few in several respects exhibit striking parallels to ordinary adjectives on their nonrestrictive readings. First, nonrestrictive readings of prenominal adjectives are possible with only a subset of determiners, and disallowed with other determiners or bare plurals:

(147) a. The industrious Greeks built a powerful empire  
    Restrictive: Of the Greeks, those who were industrious built a powerful empire  
    Nonrestrictive: The Greeks, who as a people were industrious, built a powerful empire

b. Every unsuitable word was deleted  
    Restrictive: The words that were unsuitable (but not necessarily the suitable ones) were deleted  
    Nonrestrictive: The words, all of which were unsuitable, were deleted

c. Helen’s sophisticated friends  
    Restrictive: Those of Helen’s friends who are sophisticated  
    Nonrestrictive: Helen’s friends, all of whom are sophisticated
(148)  a. Some unsuitable words were deleted  
Restrictive: Of the words that were unsuitable, some were deleted  
Nonrestrictive: #Some of the words, all of which were unsuitable, were deleted

b. Twenty unsuitable words were deleted  
Restrictive: Of the words that were unsuitable, twenty were deleted  
Nonrestrictive: #Twenty of the words, all of which were unsuitable, were deleted

c. Most unsuitable words were deleted  
Restrictive: Of the words that were unsuitable, most were deleted  
Nonrestrictive: #Most of the words, all of which were unsuitable, were deleted

d. Unsuitable words were deleted  
Restrictive: Words that were unsuitable (as opposed to other sorts of words) were deleted  
Nonrestrictive: #Of the words, all of which were unsuitable, some (all?) were deleted.

As these examples illustrate, in the presence of a definite determiner, universal quantifier or possessive, both restrictive and nonrestrictive readings are possible for the adjective. But with an indefinite determiner, a quantificational determiner other than the universal, or no determiner, the nonrestrictive reading is not available. With one exception, this is the same pattern in determiners that may occur with predicative Q-adjectives; the exception is the universal quantifier, which is not generally allowed with attributive Q-adjectives.

Another parallel is the following: on their nonrestrictive readings, when an prenominal adjective is paraphrased with a relative clause, the latter must be introduced with a wh-word, and not with that:

(149)  a. The Greeks, who were industrious, established a powerful empire  
   - Restrictive or nonrestrictive (with intonational difference)

b. The Greeks that were industrious established a powerful empire  
   - Restrictive only

147
Correspondingly, a relative clause with a Q-adjective can only be introduced by a *wh*-word; that is, in this respect Q-adjectives again pattern with adjectives on their nonrestrictive rather than restrictive readings:

(150)  a. The trees, which were few, were stunted
       b. *The trees that were few were stunted

An additional if more distant parallel is provided by the Romance languages. It is well known that prenominal adjectives in Romance languages can only be interpreted nonrestrictively, while postnominal adjectives have both interpretations. As an example from Spanish (Morzycki 2008):

(151)  a. Los sofisticados amigos de Maria  nonrestrictive only
       ‘the sophisticated friends of Maria’
       b. Los amigos sofisticados de Maria  restrictive or nonrestrictive

The equivalents of Q-adjectives in these languages likewise occur prenominally (*pocos arboles* ‘few trees’), and cannot appear postnominally, the position where we find restrictive modifiers (*arboles pocos*).

The central insight here is that attributive Q-adjectives are nonrestrictive modifiers, and should be analyzed as such. What we need, then, is a theory of nonrestrictive modification to provide a framework in which to formulate their analysis. Towards this end, there is some relevant work in the literature, including the previously mentioned work discussing nonrestrictive adjective meanings, as well as a broader body of work on nonrestrictive relative clauses and other appositives (Rodman 1976; McCawley 1998; Potts 2003). While we will see that existing frameworks are not completely adequate to handle attributive Q-adjectives (or, for that matter, the nonrestrictive readings of ordinary adjectives), they provide a starting point.
A recurring theme in the work in this area is that nonrestrictive modifiers do not combine with the modified term intersectively (as do restrictive modifiers), but rather express an independent supplementary proposition that is in some way conjoined with or appended to the primary descriptive content of the utterance. A recent development of this idea is found in Potts’ (2003) comprehensive study of conventional implicatures, under which class he includes appositives of all sorts. Since I will draw on the framework developed by Potts, let me sketch out its basic elements.

The crucial notion behind Potts’ analysis is that expressions of natural language convey two levels of meaning: the ‘at-issue’ content (roughly speaking, the primary asserted content) and the conventional implicature content (which is independent of the at-issue content, but nonetheless generates entailments). Potts develops a multi-dimensional logical system under which these two levels of meaning are computed independently. The starting point is an enriched theory of types which makes use of both ‘at-issue’ types (e, f, s, etc.) and conventional implicature (CI) types (c, t, s, etc.). In addition to the standard rule of at-issue functional application, an additional rule of CI application allows an expression of at-issue type to saturate the argument position of a CI functional type, while still remaining available to saturate the argument position of an at-issue functional type. For example:

(152) Lance, a cyclist,

\[
\begin{align*}
\text{lance: } & e^a \\
\text{cyclist(lance): } & f^c \\
\text{lance: } & e^a \\
\text{comma(cyclist): } & \langle e^a, f^c \rangle \\
\text{cyclist: } & \langle e^a, f^a \rangle
\end{align*}
\]
Here, the appositive noun phrase *a cyclist* is marked as expressing CI content by a syntactic feature COMMA. Via the rule of CI application, the referential expression *Lance* saturates the argument slot of this expression, creating a backgrounded proposition (*Lance is a cyclist*), the CI content of the sentence. At the same time, the type *e*\(^a\) expression *Lance* remains available to saturate another predicate. These two levels of meaning are represented in the top node of the above tree, where the bullet (●) separates at-issue content (above) from CI content (below). Semantic composition may now proceed via ordinary at-issue functional application yielding the at-issue content of the expression. For example, in the following, the at-issue content is *Lance is training*:

(153) Lance, a cyclist, is training

![Semantic Tree]

Note here that in Potts’ system it is the entire semantic tree, not just the top node, that is interpreted; as a result, the CI content does not need to be passed up the tree.

Potts extends his analysis to expressive adjectives (*Sheila’s damn dog*) and epithets (*that bastard Chuck*), which he also takes to express CI meaning. With the exception of one brief mention (to be discussed below), he does not discuss nonrestrictive readings of ordinary prenominal adjectives. This challenge is taken up by Morzycki (2008), who considers these within Potts’ framework. Morzycki begins again with the
insight that nonrestrictive readings essentially involve double assertions; on his analysis, (154a) is parallel to, and requires an analysis that renders it equivalent to, (154b):

(154)  a. Every unsuitable word was deleted  
       b. Every word, was deleted. They, were unsuitable.

Here *they* refers back to the plurality consisting of the totality of the set of contextually relevant words quantified over by *every*. Morzycki implements this proposal within Potts’ framework with a rule of expressive predicate modification that introduces a supremum operator that derives the plurality that serves as the argument of the nonrestrictive modifier *unsuitable*.

(155) Expressive Predicate Modification

\[
\beta: \langle e^a, t^a \rangle  
\rightarrow  
\alpha(\text{sup(} \beta)): t^c  
\alpha: \langle e^a, t^a \rangle  
\beta: \langle e^a, t^a \rangle
\]

Applied to (154a), this rule (and subsequent derivation) results in the following as the top node of the semantic tree, which mirrors the paraphrase in (154b) (where \(C\) is a resource domain variable that restricts the extension of the noun *words* to those that are contextually salient):

(156)  \( \text{every(} \lambda x.\text{word}(x) \land x \in C)(\text{deleted}): e^d \)

\[
\rightarrow  
\text{unsuitable(} \text{sup(} \lambda x.\text{word}(x) \land x \in C)\text{): } t^c
\]

With regards to an analysis of attributive Q-adjectives, Morzycki’s approach seems at first promising. The relevant examples can likewise be paraphrased with a plural pronoun:

(157)  a. The few trees were stunted  
       b. The trees, were stunted. They, were few
And the interpretation is parallel as well: it is the totality of the contextually relevant set of trees that is characterized as few. But there is a further complication – in the case of both ordinary adjectives and Q-adjectives – that renders this analysis not entirely satisfactory. Specifically, consider examples such as these:

(158) a. Her valuable books were destroyed in the fire
    b. Fred’s many friends supported him

Here, the argument of the nonrestrictive modifier is not simply the result of applying a supremum operator to the set denoted by the noun phrase, but rather the result of the composition of that NP with the possessive nominal or pronoun. That is, it is her books that are characterized as valuable, and Fred’s friends that are characterized as many.

(159) a. Her books were destroyed in the fire. Her books were valuable.
    b. Fred’s friends supported him. Fred’s friends were many.

While it is less obviously necessary, note that a parallel analysis also captures the intuitive interpretations of examples involving other determiners, as shown in the paraphrases in the (b) examples below:

(160) a. Every unsuitable word was deleted.
    b. Every word was deleted. Every word was unsuitable.

(161) a. The few trees were stunted.
    b. The trees were stunted. The trees were few.

Thus it is not a supremum operator introduced by a compositional rule that derives the plurality that serves as an argument for the nonrestrictive modifier; rather, it is the determiner itself.

Here, we have a bit of a puzzle from the point of view of compositionality. At the level of surface structure, the Det+NP sequence is not a syntactic constituent, but it serves
as a semantic constituent. Furthermore, it plays this role not just once, but twice: once at
the level of at-issue meaning, and once at the level of CI meaning.

Within the relevant literature, there is not, to my knowledge, a fully satisfactory
solution to this puzzle. One might address these facts by proposing that the determiner
originates lower within the DP, where it forms a constituent with the NP at the stage of
the derivation that is relevant to semantic composition. Alternately, we could augment
our semantic repertoire of modes of composition to allow the required predicate-
argument relationship to be established. Potts, in a brief discussion of one relevant
texample, opts for the latter solution. The example he considers is the following, which
exhibits the same puzzle of non-constituency as those discussed above, in that *lovely*
seemingly modifies *his vases*.

(162) Chuck said I could have one of his lovely vases

Potts proposes that in cases such as these, there is a necessary mismatch between the
syntactic tree and the semantic tree, allowing the adjective *lovely* to modify an expression
that is not a syntactic constituent. In simplified form, this would give the structure in

(163a) the ultimate semantic translation in (163b):^{21}

(163) a. \([DP \; his \; [NP \; [AP \; lovely] \; vases]]\)

b. \(his(vases)\; e^a\)

\(\bullet\)

\(lovely(his(vases))\; i^c\)

For simplicity, I will adopt Potts’ general approach. I therefore take the relevant
texamples of attributive Q-adjectives to include a Meas head in whose specifier position

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^{21} Potts’ analysis is actually somewhat more complex than this, in that it involves the lift of *his vases* to
generalized quantifier type. I simplify by taking the possessive pronoun to map a set term to an entity; thus
*his vases* is the maximal plurality that stands in the appropriate relationship with the individual denoted by
*his.*
the QP is situated (just as in the case of quantificational Q-adjectives), and which
introduces a measure function:

(164)  \[ DP \text{ the } [\text{MeasP } QP \text{ Meas } [\text{NP trees}]] \]

I then propose the following rule of Attributive Q-Adjective Interpretation:

(165) **Attributive Q-Adjective Interpretation:** a structure of the form in (a),
where \( \alpha(\beta) \) has referential type, has the semantic translation in (b):

a.  \[ DP \alpha [\text{MeasP } QP \text{ Meas } [\text{NP } \beta]] \]

b.  \( \alpha(\beta): e^a \)

\( QP(\text{Meas}(\alpha(\beta))): f^c \)

I assume the rule in (165) to be a specific case of a more general rule that accounts for the
nonrestrictive readings of ordinary adjectives; but I do not address the general case here.

Applying this rule to the examples under consideration produces the results
below:

(166)  a.  \[ DP \text{ the } [\text{MeasP } QP \text{ Meas } [\text{NP trees}]] \]

b.  the(trees): \( e^a \)

\( \text{POS-few}(\lambda d. \mu(\text{the(trees)}) \geq d): f^c \)

(167)  a.  \[ DP \text{ his } [\text{MeasP } QP \text{ Meas } [\text{NP friends}]] \]

b.  his(friends): \( e^a \)

\( \text{POS-many}(\lambda d. \mu(\text{his(friends)}) \geq d): f^c \)

In both cases, the Q-adjective (or rather, QP) takes as argument the result of applying
Meas to the plurality derived via composition of the material in the DP layer and the NP;
this occurs at the CI level of interpretation, creating a backgrounded proposition in which
the Q-adjective is predicate. At the same time, that plurality remains available for further
composition in the at-issue level of meaning. Thus (168a) has the final translation in (168b), equivalent to the previously discussed paraphrase in (168c):

(168) a. The few trees were stunted
    b. stunted(the(trees)): \( t^a \)
    c. ‘The trees were stunted. The trees were few’

Additional support for the analysis of attributive Q-adjectives as nonrestrictive modifiers come from a further consideration of negative polarity licensing. Recall that attributive \textit{few} (unlike quantificational \textit{few}) does not license NPIs in the sentential predicate (169a). But curiously, attributive \textit{few} does license NPIs within the noun phrase (169b):

(169) a. *The few students had \textit{ever} been to Mongolia
    b. The few students who had \textit{ever} been to Mongolia enjoyed the lecture

The latter pattern is unexpected on the cardinality predicate analysis, under which the NPI would not be in the semantic scope of an appropriate licensor (170a). But it is predicted on the nonrestrictive modifier analysis, where the NPI is in the scope of \textit{few} on the CI dimension of interpretation:

(170) the few students who had \textit{ever} been to Mongolia
    a. sup(\( \lambda x.\)few(\( x \)) \& student(\( x \)) \& ever-been-to-Mongolia(\( x \))
    b. the students who had \textit{ever} been to Mongolia: \( e^a \)

A nice consequence of the analysis developed here is that it accounts for the parallels between the attributive and predicative uses of Q-adjectives. The backgrounded proposition derived in the attributive cases has essentially the same
semantic structure as do sentences involving predicative Q-adjectives: the Q-adjective is predicated of a set of degrees associated with a plurality. Thus the restrictions on possible determiners are the same: only a Det+NP sequence that can be interpreted referentially is allowed, because only this provides the necessary plurality to serve as the argument of the measure function. Hence in both cases below the (a) and (b) examples are fine, while the (c) example is not, because some students cannot be interpreted referentially:

(171) a. The many students enjoyed the lecture
    b. His many students enjoyed the lecture
    c. *Some many students enjoyed the lecture

(172) a. The students were many
    b. His students were many
    c. *Some students were many

There is one exception to this neat parallel that requires comment, namely the case in which no determiner is present. Recall that with modification or sufficient contextual support, a bare plural may occur as the (overt) subject of a predicative Q-adjective (173). But in the absence of a determiner, prenominal Q-adjectives do not allow an attributive reading. Thus (174a) only has the interpretation paraphrased in (174b), which is characteristic of a quantificational role for the Q-adjective; it does not allow the reading paraphrased in (174c), which is what we would get if the Q-adjective received the nonrestrictive modifier interpretation.

(173) Students who enjoyed the lecture were few

(174) a. Few students who enjoyed the lecture made donations
    b. ‘Of the students who enjoyed the lecture, the number who made donations was small’
    c. ‘#Students who enjoyed the lecture made donations. Students who enjoyed the lecture were few’
The definition in (165) deals with this by specifying that lexical material is required in the DP layer to yield an attributive reading. But it would be nice to find some reason behind this stipulation. Here, it is relevant to consider the following example, without the Q-adjective:

(175) Students who enjoyed the lecture made donations

In (175), *students who enjoyed the lecture* has either a generic interpretation (‘in general, students who enjoyed the lecture made donations’) or an existential one (‘there were students who enjoyed the lecture who made donations’). Importantly, it cannot be read with the noun phrase referring to the totality of students who enjoyed the lecture (‘the maximal group of students who enjoyed the lecture made donations’); that is, the noun phrase cannot be interpreted referentially. Exploring the reasons for this would take us far afield from the present topic, into the vast literature on the bare plural. But we can note that if this interpretation is unavailable for the bare plural, it effectively blocks a reading of examples such as (174a) as involving attributive *few*, since a rule along the lines of (165) would derive a referential interpretation for the bare plural in the at-issue dimension of interpretation. In fact, as far as I can tell, the only case in which we can get a referential interpretation for the bare plural is in the case of a predicative Q-adjective, which is distinguished by the fact that the resulting entity only saturates the argument slot of the measure function, and does not have any other role in the semantic translation.

There is undoubtedly more that could be said in this area, but I must leave it at this.

I will wrap up this section with a comparison of the quantificational and attributive uses of Q-adjunctives, and a solution to a question that comes up in this context. Under the analysis developed here, despite their surface similarities quantificational cases
such as (176a) and attributive cases such as (176b) require completely different semantic translations.

(176)  a. Few students enjoyed the lecture  
       b. The few students enjoyed the lecture

Quantificational few in (176a) must be analyzed as a degree quantifier that takes sentential scope at LF, and that takes as argument the set of degrees (scalar interval) formed by lambda abstraction over the trace of type $d$ in its base position. By contrast, attributive few in (176b) has been analyzed as a nonrestrictive modifier, which stands in a predication relationship with (the number of) the students in the CI dimension of interpretation. We have just seen that the nonrestrictive modifier interpretation is not available for the Q-adjective in an example such as (176a), where there is no overt determiner. Conversely, I provided evidence above that an example such as (176b) does not allow a wide-scope degree quantifier interpretation (cf. the discussion of example (146)). But I have not yet demonstrated how this is ruled out. I turn to this now.

Recall from the discussion in Section 3.4 that quantificational Q-adjectives raise covertly from their surface position in SpecMeasP to take sentential scope, in doing so passing through the specifier position of the null DP. I propose that in the attributive cases, such raising is blocked, because SpecDP is otherwise filled. This is most evident in the case of a possessive pronoun or possessive noun phrase (e.g. Fred’s many friends), as these are commonly taken to be in SpecDP. This account can be extended to cases involving a definite determiner (e.g. the few trees) if we take SpecDP here to be occupied by an abstract specificity feature (cf. den Dikken (2007)).

The result is that in the presence of lexical material in the D layer, a Q-adjective can only compose as a nonrestrictive modifier. As a consequence, the ‘quantificational’
interpretations of Q-adjectives and their nonrestrictive modifier interpretations are in complementary distribution.

3.8 Adverbials and More

I will conclude this chapter with a brief discussion of the adverbial uses of Q-adjectives, and of some related matters. Recall from Chapter 2 that the Q-adjectives much and little can serve as modifiers in the verbal domain, and as modifiers of deverbal adjectives and of a couple ordinary adjectives:

(177) a. The company grew too much/too little
    b. Much loved/little liked
    c. Much/little different

The simplest approach here is to take the relevant verbs and adjectives to include a degree argument (cf. Heim 2006):

(178) \[ \text{grow} = \lambda d \lambda x. \text{grow}(x) \text{ to degree d} \]

To establish a closer parallel to the uses of Q-adjectives within the nominal domain, we might refine this view by taking the degree argument to be introduced by some functional head within the extended verbal projection; I do not pursue this question further here, but refer the reader to Doetjes (1997) for a more detailed study of adverbial Q-adjectives.

The cases I would like to consider further are those in (179) and (180):

(179) John is rich; in fact, he is too *(much) so.

(180) a. John is more intelligent (*intelligenter) than Fred
    b. John is smarter than Fred
    c. John drank more wine than Fred

When a gradable adjective is pronominalized with so, as in (179), it cannot combine directly with a degree modifier, but rather a ‘dummy’ much must be inserted (the so-called much support of Corver 1997). In what I will suggest is a related pattern, certain
gradable adjectives do not have inflected comparative (or superlative) forms, but rather require the comparative and superlative to be expressed by a prenominal *more* (*most*), as exemplified in (180a). I will not attempt an explanation for these patterns, but instead focus on the puzzle of the semantic role of *much* in these cases. In (179), if *so* stands in for the gradable adjective, what contribution could *much* be making? As for the examples (180), *more* in quantificational uses such as (180c) has been analyzed as the spell-out of *much*+er (or *many*+er). But if *more* in (180a) is likewise to be analyzed as *much*+er, we again have the question of what *much* is adding to the picture, given that the interpretation of (180a) is essentially parallel to that of (180b). Seemingly *much* is a dummy element here, with no semantic content whatsoever.

Under the present account, there is in fact nothing out-of-the-ordinary about these cases. As was argued earlier, *much* (and likewise *many*) is in a sense semantically vacuous: it associates a set of degrees with the degrees contained within it, allowing it to function as an identity function on sets of degrees.

\[(181) \quad [\text{much}] = \lambda d \lambda \text{non-#}.d \in I\]

We might go as far as to say that the function of *much* is simply as a carrier of degree morphology, precisely the role it plays in the examples in (179) and (180).

To take first the comparative cases, the syntax of an example such as (180a) differs only from that of the morphological adjectival comparative (180b) in that the specifier position of the AP hosts a QP rather than a DegP.

\[(182) \quad \text{John is smarter than Fred} \quad \quad \quad (180b)\]

SS: [John is [AP [DegP –er than Fred is d smart] smart]]
LF: [[[DegP –er than Fred is d smart], [Fred is [AP t1 smart]]]
(180a) John is more intelligent than Fred
SS: [Fred is [\text{AP} [\text{QP}\mathbin{\text{−er than Fred is d}} \text{much intelligent}] much] intelligent]]
LF: [[[\text{DegP} \mathbin{\text{−er than Fred is d}} \text{much intelligent}]]_2 [[t_2 \text{ much}]]_1 [\text{Fred is [\text{AP} t_1 \text{ intelligent}]]]]

At LF, the difference between (180b) and (180a) is that the latter involves not one but two stages of raising: the QP raises to sentential scope (just as in the case of quantificational Q-adjectives), and the DegP raises from there. The corresponding semantic translation is that in (184a):

\begin{align*}
(184) & a. \left[\mathbin{\text{−er than Fred is d } \text{much intelligent}]}_2 \right] (\left[\left[t_2 \text{ much}]}_1 \right] (\lambda d_1. \text{intelligent(John) } \geq d_1)) \\
& b. \left[\mathbin{\text{−er than Fred is d } \text{much intelligent}]}_1 \right] (\lambda d_1. \text{intelligent(John) } \geq d_1)
\end{align*}

The crucial point here is that the degree quantifier \(t_2 \text{ much}\) maps the set of degrees \(\lambda d_1. \text{intelligent(John)} \geq d_1\) to itself, with the result that the formula in (184a) is equivalent to that in (184b), which in turn is directly parallel to the translation that a morphological comparative such as (180a) would receive.

The \text{much} support example in (179) can receive a similar analysis, if we take \text{so} to be a pronominal copy of the adjective that heads an AP in whose specifier position a QP is housed. While \text{much} intervenes between \text{so} and the degree modifier \text{too} in the semantic translation, \text{much} is semantically vacuous, with the result that the composition proceeds as if the degree modifier combined directly with the (pronominal) adjective.

In summary, whatever are the morphological or syntactic reasons behind the obligatory occurrence of \text{much} in these cases, the resulting semantics pose no particular problem, for the reason that \text{much} is essentially transparent to semantic composition. Other authors have accounted for these facts by positing an unpronounced \text{much} in the
representation of ordinary adjectival comparatives (Bresnan 1973) or by taking the
‘dummy’ much in much-support contexts to have different semantics from
quantificational much (Corver 1997). Under the present account, neither of these
solutions is necessary, which I take as an advantage of the present analysis.

3.9 Concluding Remarks

The goal of this chapter was to develop a formal implementation of the degree
predicate theory of Q-adjectives that is able to accommodate the range of syntactic
positions in which they occur. This has been accomplished by an analysis that
decomposes the semantic content traditionally associated with many, few, much and little
into four separate components. Q-adjectives themselves are analyzed as gradable
predicates of scalar intervals. The standard of comparison referenced by Q-adjectives (or
ordinary adjectives) in their positive forms is contributed by a null positive morpheme
POS, which alternates with overt degree morphology. Degree predicates are linked to
predicates over individuals by the functional head MEAS, which introduces a measure
function. Finally, quantification over individuals arises via Existential Closure.

As has been seen in this chapter, these four elements can compose in different
combinations and configurations, and it is this that gives Q-adjectives the flexibility to
occur in the range of positions that we find them in. In the analysis of quantificational
use of Q-adjectives, all four come into play. In their predicative and attributive uses, the
Q-adjective, POS (or other degree morphology) and Meas are involved, but there is no
role for Existential Closure. The differential and adverbial cases require only the Q-
adjective and POS, as the degree argument is provided by the degree morpheme (-er or
too) or the modified adjective or verb.
We have further seen that a consequence of this decompositional approach is that essentially all semantic content has been stripped away from *many* and *much* themselves, enabling them to function as identity functions on sets of degrees. This allows the phenomenon of *much*-insertion to be accommodated without any separate provision for a ‘dummy’ *much*, and also suggests an explanation for the oddness of bare *many* and (especially) *much* in quantificational uses.

A corollary of this analysis is that Q-adjectives in attributive position must be analyzed as nonrestrictive rather than restrictive modifiers. While this represents a departure from a tradition of analyzing attributive *many* and *few* (and cardinal numbers) as cardinality predicates that combine intersectively with the following NP, the nonrestrictive analysis successfully captures the similarities between the attributive and predicative uses of Q-adjectives (notably the determiners that occur in each construction), and also accounts for the previously unnoticed parallels between attributive Q-adjectives and ordinary prenominal adjectives on their nonrestrictive readings.

Finally, in this chapter we have seen the first evidence that facts relating to dimensions, scales and degrees are relevant to explaining patterns in the distribution and interpretation of Q-adjectives. I have shown that the contrast between *many/few*, which occur predicatively, and *much/little*, which (generally) do not, can be attributed to the fact that the former pair but not the latter encodes a dimension of measurement. This theme will be continued in Chapter 4 (where I show that the cardinal/proportional ambiguity of Q-adjectives is accounted for by the structure of the measurement scales invoked) and in Chapter 5, (where it will be seen that the contrast between *a few* and *a many* can likewise be related to a property of scale structure).
Chapter 4
The Interpretation of Q-Adjectives

4.0 Introduction

In this chapter I investigate the second of the three puzzles that I started this work with, namely the interpretive puzzle. Consider the sentences in (1), examples of the Q-adjectives many and few in their quantificational use:

(1) a. Many students attended the lecture  
b. Few students attended the lecture

In simple terms, (1a) seems to assert that the number of students attending the lecture was greater than some standard, while (1b) states that it was less than some (possibly different) standard. But greater than or less than what? There is obviously no fixed value that establishes the cut-off between many and not many, or between few and not few. Rather, these terms are vague, and the truth or falsity of sentences such as those in (1) depends on the context of evaluation: Who is the relevant set of students? What sort of lecture are we talking about (an in-class lecture, the keynote address at a national conference, etc.)? What were our expectations for the number of students in attendance? And so forth.

Furthermore, as will be seen in more detail below, the interpretation of Q-adjectives fluctuates between two (or more) distinct readings, with a sentence potentially judged true under one reading but false under another. For instance, (1a) might be read as asserting either that a large number of students attended, or that a large proportion of some contextually relevant set of students did so. In other words, Q-adjectives appear to be ambiguous, in addition to being vague.
Both these properties of Q-adjectives – their vagueness and their apparent ambiguity – are part of broader patterns. While Q-adjectives are alone among quantificational expressions in demonstrating vagueness of this sort, their behavior in this respect mirrors that of expressions of other syntactic categories, notably gradable adjectives. The ambiguity described briefly above is likewise one instance of a more general pattern by which a certain class of quantifiers (including *many, few, some*, and the cardinal numerals) are seemingly ambiguous between what have come to be known as strong and weak readings. But Q-adjectives also exhibit properties that set them apart from the other members of these classes.

While the facts surrounding the distribution of Q-adjectives have received relatively little attention in the semantics literature, the same cannot be said of their interpretive variability. There is a sizeable body of work on the interpretation of *many* and *few* (notably Barwise & Cooper 1981; Westerståhl 1985; Keenan & Stavi 1986; Lappin 1988, 2000; Partee 1989; Fernando & Kamp 1995; Herburger 1997; Cohen 2001). More generally, the literature on vagueness, and on the strong/weak ambiguity, is extremely extensive. But there are questions that remain open, with regard to both Q-adjectives themselves and the broader phenomena that they partake in.

My goal in this chapter is to investigate these issues from the perspective of the degree-based theory developed in Chapters 2 and 3. I first show that the framework developed here is able to accommodate the range of interpretations that are available to Q-adjectives. Beyond this, I show that this approach yields insights into several known problems in their semantics. I further discuss some more general implications for the understanding of vagueness in natural language.
To anticipate the main proposal of this chapter, my claim will be that the full range of readings that we find with Q-adjectives can be derived via the manipulation of two elements of the scalar representation that the degree-based analysis invokes: i) the structure of the scale itself (whether or not an upper bound is assumed); and ii) the choice of the neutral range that serves as the standard of comparison. In developing this account, my focus will be on many and few, primarily for the reason that they have received the most extensive treatment in the literature; but it will be clear that the same phenomena occur with much and little, and that the account developed here can be extended to capture those as well.

The outline of the chapter is the following. In Section 4.1, I discuss the vagueness of Q-adjectives, relating their behavior to the paradigm case of vagueness, namely gradable adjectives, the subject of much recent interest. In Section 4.2, I summarize the multiple readings that have been attributed to Q-adjectives. In Section 4.3, I introduce the basic proposal that will be developed in the chapter. Section 4.4 reviews several of the accounts that have been proposed to capture these facts, and summarizes key questions that remain open. Sections 4.5 and 4.6 develop the analysis: I show that the degree predication analysis provides a framework for investigating the interpretation of Q-adjectives, and apply this framework to account for both their vagueness and the multiple readings that they allow. Section 4.7 returns to the so-called ‘reverse’ reading of Q-adjectives, and offers some observations towards its correct analysis. Section 4.8 summarizes the conclusions.
4.1 Q-Adjectives and Vagueness

Q-adjectives are vague. In fact, this is often taken to be the defining characteristic of the class, with authors variously referring to them as ‘vague quantifiers’ or ‘vague numerals’ (e.g. Zamparelli 1995; Nouwen to appear). Given that vagueness is central to the class of words under consideration in this work, it is appropriate to examine what exactly it is that we mean by the term.

Vagueness has been the subject of much work in both the semantics and philosophical literature (e.g. Fine 1975; Kamp 1975; Lewis 1982; Williamson 1994; Raffman 1994; Graff 2000; Kennedy 2007). While vagueness can arguably be found in expressions of all lexical categories, including nouns, verbs and prepositions, the paradigm case, and the one that has been most thoroughly studied, is that of gradable adjectives. Kennedy (2007), who offers a review of the literature in this area, describes vagueness as encompassing three phenomena: i) the variability of truth conditions relative to the context; ii) the existence of borderline cases; and iii) participation in what is known as the Sorites Paradox. All of these, as Kennedy shows, are exhibited by gradable adjectives such as expensive or large.

An investigation of the interpretation of Q-adjectives shows that they take part in these same phenomena. With regards to the variability of truth conditions relative to the context, consider again example (1a):

(1) a. Many students attended the lecture

The number of students required to establish the truth of (1a) depends on the context. If we were, for example, discussing the professor’s lecture in an advanced semantics class, twenty students might be considered many; but in the context of a campus-wide lecture
by an eminent and well-known scholar (say, Noam Chomsky speaking on the political situation in the Middle East), those same twenty students likely would not be considered many. Even with respect to the same real-world situation, the truth or falsity of a sentence such of (1a) depends on the perspective or frame of reference of the conversational participants. To take again Chomsky’s lecture, if we were considering the number of students attending in comparison to the much smaller number of students who typically come to campus-wide events, (1a) might be judged true. On the other hand, if our focus was on the number of students in attendance in the context of the much larger number with an interest in Middle East politics, (1a) might be judged false in the very same situation (that is, with the same number of student attendees).

As for the existence of borderline cases, imagine again Chomsky’s lecture, taking place in the largest auditorium on campus. We might feel with complete confidence that in this situation 1000 students in attendance would count as many, and with equal confidence that twenty would not. But what about 50? 100? 200? In any given context, there will necessarily be some values on the borderline, where there is uncertainty as to whether many applies.

Q-adjjectives also give rise to what is known as the Sorites Paradox, as illustrated by (2):

(2) a. If 1000 students attended Chomsky’s lecture, that is many
    b. If n students attending Chomsky’s lecture qualifies as many, then n – 1 students attending Chomsky’s lecture also qualifies as many
    c. 1 student attending Chomsky’s lecture qualifies as many

While the premises in (2a) and (2b) seem valid, and the argument itself sound, the conclusion in (2c) is without doubt false. This unwelcome result seems in some way to derive from the phenomena previously discussed, namely the truth-conditional variability
of many and the existence of borderline cases, though the solution to the problem is far from clear, and a number of approaches have been proposed in the philosophical literature.

Another characteristic particular to some vague gradable adjectives (one not explicitly discussed by Kennedy) is the existence of an apparent gap between the extensions of the positive and negative members of a pair of antonyms. For example, we can imagine a situation in which both (3a) and (3b) would be judged false, namely one in which John’s height was intermediate between what we would consider tall and what we would consider short (see Horn 1989 for discussion).

(3)  
\begin{align*}
\text{(a)} & \quad \text{John is tall} \\
\text{(b)} & \quad \text{John is short}
\end{align*}

Here too we see the same pattern with Q-adjectives (as was touched on in Chapter 3); for example, we can imagine a situation in which both (1a) and (1b) were judged false, if the number of students attending was too great to be called few, but not sufficient to be characterized as many.

I should mention that I am assuming the gap between the positive and negative terms in examples such as (1) and (3) to be part of the semantics; that is, there is some range of possible heights for which both (3a) and (3b) would be literally false. This position, though common, is not universally accepted, with some authors proposing instead pragmatic accounts of these facts (see especially Krifka 2007 for a view that positive and negative antonyms in fact divide up the semantic space entirely, and the apparent gap is pragmatically derived on the basis of epistemic uncertainty as to the precise location of the boundary between them). In section 4.4 I will present evidence
that tends to support the semantic view; for the present, I follow the common view that
pairs such as these are contraries rather than complementaries semantically.

Note also that while I have been basing my examples on many and few, the same
issues also obtain with much and little. As an example, consider the following:

(4) There is little water left

The volume of water that would suffice to establish the truth or (more aptly) the falsity of
(4) depends on whether the context is (say) my water bottle or an Olympic-sized
swimming pool. And even with the context of evaluation fixed, there will be a range of
possible volumes for which (4) is neither clearly true nor clearly false.

In short, the Q-adjectives many, few, much and little exhibit much the same sort of
vagueness as do ordinary gradable adjectives such as tall. As such, it might seem that
current theories of vagueness in the adjectival domain could accommodate these data as
well. But in the next section we will see evidence that the truth-conditional variability of
Q-adjectives involves not just vagueness but an apparent ambiguity.

4.2 Q-Adjectives and Ambiguity

As far back as Milsark (1974, 1977) and especially Partee (1989), it has been
noted that many and few have two distinct readings. Milsark (1977) makes this point
about the following example:

(5) Many unicorns exist

One reading of (5) can be paraphrased as “the universe has a lot of unicorns in it” (p. 20).
The second possible (though certainly implausible) reading could be paraphrased as
follows:
It is a fact about a fairly large number of unicorns, as opposed to certain others, that they exist. For example, unicorns exist, but the rest are merely characters in tapestries and nursery stories. (Milsark 1977, p. 20)

Many N can thus be interpreted as saying something either about the number ofNs, or about certain Ns in contrast to others.

In a perhaps more relevant example (in that it does not involve a restrictor with an empty extension), Partee (1989) proposes that simple sentences such as those in (6) are ambiguous between what she terms ‘cardinal’ and ‘proportional’ readings (terminology that I will adopt here, as it is now fairly standard). On the cardinal reading, (6a) asserts that a large number of aspens died, while on the proportional reading, it asserts that a large proportion of some contextually relevant set of aspens died; the same is true for (6b), modulo the replacement of ‘large’ with ‘small’:

(6) a. Many aspens died  
   b. Few aspens died

The distinction between the cardinal and proportional readings can be detected most clearly with few, where the truth conditions for the two diverge. Partee notes that (7) can be read as true in the situation where all faculty children were at the 1980 picnic, if it was the case that there were only a small number of faculty children in 1980.

(7) There were few faculty children at the 1980 picnic

On the proportional reading few N can never be all the Ns (on the reasonable assumption that proportional few means something along the lines of ‘a small proportion of’). The compatibility of a sentence such as (7) with a situation in which all obtains is thus evidence of a distinct cardinal interpretation under which few essentially means ‘a small number’.
The situation is more complicated in the case of *many*, which does not seem to have a ‘pure’ proportional reading. For example, in the situation where there are four students in my class and three of them have blue eyes, it seems that (8) must be judged false, despite the fact that the predicate is true of a large proportion (75%) of the students. Apparently, *many* must minimally be a considerable number (leaving aside for now what is meant by ‘considerable’).

(8) Many students in my class have blue eyes

Nevertheless, *many* still exhibits a cardinal/proportional ambiguity, though it is more subtle than that for *few*. For example, (9) can be read either as asserting that there are a large number of men in the kitchen, or that a large proportion of some contextually relevant set of men is in the kitchen:

(9) Many men are in the kitchen

As evidence that this apparent ambiguity is in fact more than a dramatic instance of context dependence, note that the availability of one or the other of the two readings is in some cases grammatically determined. Most clearly, as noted by Partee, in a sentence containing an individual-level predicate, only a proportional reading is available. To return to an example that I discussed in the introductory chapter, (10a), with a stage-level predicate (SLP), exhibits the ambiguity in question: it can be read as asserting either that a small proportion of the totality of French-speaking senators were at the gala, or that a small number of such senators were there (even if that group was in fact all of the French-speaking senators in existence). But (10b), with an individual level predicate (ILP), can only mean that a small proportion of French-speaking senators are left handed;
the cardinal reading is absent (in that if there are only a small number of French-speaking senators, and they are all left handed, the sentence must be judged false).

(10) a. Few French-speaking senators were present at the gala
    b. Few French-speaking senators are left handed

I will discuss other grammatical constraints on the availability of cardinal and proportional readings in more detail in Section 4.5. The crucial point here, though, is that the existence of such constraints suggests that the cardinal/proportional distinction is in some way grammatical in nature, and not merely the result of context-dependence in setting the standard of comparison.

The cardinal versus proportional ambiguity for many and few is part of a broader pattern in which so-called weak quantifiers are ambiguous between what have come to be known as strong and weak readings. The classic case involves some. For example, (11a) can be read as asserting that there is a plurality of men in kitchen (weak), or that certain men, as opposed to others, are in the kitchen (strong); here the difference in meaning is signaled by stress, with some unstressed on the first reading and stressed on the second. A similar (though more subtle) effect is observable with cardinal numerals, as in (11b):

(11) a. Some men are in the kitchen
    b. Three men are in the kitchen

The same pattern relative to predicate type is observed in these cases as well: while SLPs such as that in (11) allow both strong and weak reading, ILPs allow only strong readings.

The ambiguity apparent with Q-adjectives, however, differs in a crucial way from the strong/weak ambiguity exhibited by other weak quantifiers. Consider the following minimal pair, which have only strong/proportional readings due to the presence of the ILP (here I again use few because the relevant points are clearer than with many).
The most natural interpretation of (12a) is that there is a plurality of students in my class who have blue eyes, and furthermore that this plurality is a proper subset of the total set of students in my class. The second part, however, must be an implicature (Horn 1989), given that it is cancellable:

(13) Some students in my class have blue eyes; in fact, all of them do

But for (12b) to be true, it is not sufficient that the number of students in my class with blue eyes be small, and that this group be a proper subset of the total set of students in my class. Rather, the number with blue eyes must be small relative to the total number of students in my class. Furthermore, this is not implicated, but entailed, since it is not cancellable:

(14) #Few students in my class have blue eyes; in fact, there are only few students in my class, and they all have blue eyes

Something more is going on in the case of Q-adjectives than in the case of some and other weak quantifiers; in some way, the standard of comparison for the Q-adjective must be set in relationship to the size of the restrictor set (in the examples above, the set of students in my class). As a consequence it is not at all clear that an adequate analysis of the cardinal/proportional ambiguity found with Q-adjectives will fall out from a more general analysis of strong/weak ambiguities with weak quantifiers (e.g. Büring 1996).

While the existence of distinct cardinal and proportional readings for many and few is now widely accepted, and has been assumed by many later authors (e.g. Kamp & Reyle 1993; Higginbotham 1995; Cohen 2001), another reading remains more
controversial. This is the so-called reverse reading, first discussed by Westerståhl (1985).

The classic example is that in (15a):

(15)  a. Many Scandinavians have won the Nobel Prize in Literature
    b. ‘Scandinavians make up a large proportion of the winners of the Nobel
       Prize in Literature’
    c. Many winners of the Nobel Prize in Literature are Scandinavian

The absolute number of Scandinavians who have won the prize in literature is undoubtedly small, and the proportion of all Scandinavians who have done so is miniscule. Nevertheless, if Scandinavians make up a large proportion of the winners, there is a reading of (15a) on which it is true, namely that paraphrased in (15b). That is, the sentence is interpreted as if its arguments are reversed, as in (15c) (hence the term ‘reverse reading’).

On the view that many and few are quantifying determiners, the availability of this reading is troubling, in that it violates what has been proposed to be a semantic universal, namely conservativity. First discussed by Barwise & Cooper (1981) (who use the term ‘lives on’), conservativity means that in evaluating the truth or falsity of a sentence of the form $Q As$ are $B$, one only need look at the $As$ and determine how many of them are $B$; there is no need to look outside of the set $A$ (e.g. at the other things that are $B$). Other than many and few, all English determiners are conservative; Barwise & Cooper propose that all determiners cross-linguistically have this property, a position that is still largely accepted. But the reading of (15a) paraphrased in (15b) is clearly non-conservative, in that it makes reference to the total set of Nobel prize winners (i.e., the set $B$).

There has been considerable dispute regarding the correct analysis of the reverse reading, and even regarding what the relevant facts are (see especially Büring 1995; de Hoop & Solà 1996; Herburger 1997; Cohen 2001). But authors agree (with some
justification, in my opinion) that there is something more complicated involved here than a garden-variety cardinal or proportional reading.

Many and few, then, are not simply vague (as described in Section 4.1); they also allow multiple distinct readings. As Partee (1989) points out, we must say that many and few are apparently ambiguous in addition to being vague, not rather than being vague. Each of the distinct readings that obtain is itself vague, in the sense of exhibiting truth-conditional variability and borderline cases; for example, in the case of (6a) on its cardinal reading, the number of dead aspens that would count as many depends on the context, and for any context there will be cases in which it is uncertain whether many obtains. Both phenomena thus must be accounted for. In the remainder of this chapter, I will work towards an account of these facts.

4.3 The Proposal in a Nutshell

My goal in this chapter is to demonstrate that both the vagueness and the apparent ambiguity of Q-adjectives can be accounted for in the degree-based framework that I have developed in this work. Let me first note that this is not an entirely trivial undertaking. To illustrate this point, consider that in a GQT-style analysis of Q-adjectives, the cardinal and proportional readings of many might be captured via two separate lexical entries that are distinguished by the sets that are referenced in their logical forms:

\begin{align*}
\langle \text{many}_{\text{Cardinal}} \rangle &= \lambda P \lambda Q. |P \cap Q| > n \\
\langle \text{many}_{\text{Proportional}} \rangle &= \lambda P \lambda Q. |P \cap Q|/|P| > k
\end{align*}

Here, the cardinal reading is based on the size of a single set \((P \cap Q)\), while the proportional reading requires reference to two sets \((P \text{ and } P \cap Q)\). But this approach to
capturing the cardinal/proportional ambiguity is not available in the present account, where Q-adjectives are analyzed as predicates of scalar intervals, and sets of individuals are not available as arguments.

Nevertheless, I will show that this ambiguity, as well as the other facts discussed above, can be accommodated in the present theory. Further, I will show that this analysis provides insight into several known problems in these areas. My account will build on the theory developed in Chapters 2 and 3, where Q-adjectives were analyzed as predicates of sets of degrees on some measurement scale, with the semantics of their positive forms making reference to a contextually determined ‘neutral range’ (though the reader will recall that I did not motivate this analysis of the positive form, nor discuss how the neutral range is determined):

\[(17)\]

\[a. \quad [\text{POS-many}] = \lambda I#.N_# \subseteq I^{22}\]

\[b. \quad [\text{POS-few}] = \lambda I#.N_# \subseteq \text{INV}(I)\]

My main claim is the following: the full range of readings available to many and few can be derived via manipulation of two elements: the structure of the scale (whether or not an upper bound is assumed) and the choice of the neutral range on that scale.

Specifically, the claim will be that the proportional reading arises in what I call totalizing environments, where some totality of individuals enters into the semantic derivation, with the consequence that the range of scalar values under consideration is restricted to those less than or equal to the measure of that totality, creating a scale that is effectively bounded on the upper end. For example, on the proportional reading of (6a), the scale in question measures the contextually relevant set of aspens, as presented in

\[22\] Here, for purposes of simplicity, the positive morpheme POS is shown as forming a constituent with the Q-adjective; recall that in a full derivation I have analyzed POS as a generalized quantifier over degrees that binds a variable of type \(d\) that saturates the degree argument of an adjective or Q-adjective.
(18a). Conversely, on the cardinal reading, no contextually specified domain of quantification (set of aspens) is assumed, and the relevant scale is therefore open on the upper end, as represented in (18b) for the cardinal reading of (6a):

(18) a. **Many aspens died (Proportional)**

\[ \text{# of aspens that died} \]

\[ 0 \rightarrow N_p \]

b. **Many aspens died (Cardinal)**

\[ \text{# of aspens that died} \]

\[ 0 \rightarrow N_p \]

The truth-conditional variability of Q-adjectives on either of these readings then derives from the context-dependence of the location of the neutral range, which I will argue is determined as a range of values judged not significantly different from some contextually determined ‘point of comparison’ \( p_c \).

It will further be seen that typical instances of the ‘reverse’ reading can be analyzed as variants of the cardinal reading. A prediction of this analysis is that we should observe a similar pattern with proportional readings; I will show that this prediction is borne out, though without giving rise to what looks like a reverse reading.

In accounting for the interpretations available to Q-adjectives, the real work is then in developing a theory that determines the selection of the scale structure and neutral range in a given context. I turn to this task in Section 4.5 and 4.6. First, however, I will take a brief look at some of the accounts of these facts that are available in the literature, and highlight some important questions that remain open.

**4.4 Some Previous Accounts**

The interpretive variability of *many* and *few* has been treated extensively in the literature. Some of this work has been mentioned at various points in previous chapters,
but as a starting point I will review it as a whole here, with an aim of identifying some still-open questions that the degree-based account has the potential to answer.

For convenience, I will use some standard terminology and notation from the generalized quantifier literature. I use abbreviations of the form in (19) to represent sentences of the form many students came to the party or few children like spinach.

(19) a. many(A,B)
b. few(A,B)

I call the first argument of many or few (i.e., $A$ in the above) the restrictor, and the set it denotes the domain of quantification; I call the second argument of many and few ($B$ in the above) the scope.

4.4.1 Barwise & Cooper (1981)

An early attempt to tackle the vagueness and context-sensitivity of many and few is that of Barwise & Cooper (1981), who in order to avoid issues with vague expressions such as Q-adjectives make use of a simplifying ‘fixed context assumption’. On this view, there is a rich context that fixes the denotations of basic lexical items, including vague terms such as many and few. For example, the context might fix the following as the interpretation of many:

(20) $\llbracket\text{many}(A,B)\rrbracket = 1 \text{ iff } |A \cap B| > 3/10 (|A|) \text{ and } |A \cap B| \geq 30$

By this definition, the number that qualifies as many $A$ gets smaller as the cardinality of $A$ gets smaller, but in no case can be less than 30 (note that this interpretation brings in both a cardinal and a proportional aspect). But it has been pointed out (e.g. by Westerståhl 1985), the fixed context assumption runs into trouble with examples such as (21), where the first many tends to be interpreted as a larger number than the second:

(21) Many men date many women
Apparently, we would need a separate context for each occurrence of *many* and *few*,
which renders the notion of a ‘fixed’ context essentially powerless.

4.4.2 Lappin (1988)

An interesting attempt to develop a more flexible contextualized analysis of *many*
and *few* is that offered by Lappin (1988), who draws on examples such as (22), in which
an explicit comparison is made, to argue that the basic semantics of *many* and *few*
involves the comparison of the size of two sets, the second of which is defined with
reference to a contextually determined comparison set \( C \), as in (23):

\[
\text{(22) Many students are musicians, as compared to bankers}
\]

\[
\text{(23) } \left[ \text{many}(A,B) \right] = 1 \text{ iff } |A \cap B| > |A \cap C|
\]

On this analysis, a sentence such as *many students are musicians* is true if the number of
students who are musicians exceeds the number of students who are members of some
contextually relevant set \( C \) (say, the set of bankers, the set of doctors, etc.). The
variability in interpretation of *many* and *few* can thus be attributed to different
possibilities in the selection of this context set.

What is appealing about this treatment is that on some uses of *many* and *few*, this
sort of comparison seems to be exactly what we have in mind. But it seems more
doubtful that we always are making this kind of comparison when using *many* or *few*.
Furthermore, in the case of *few* (where Lappin proposes that the ‘greater than’ operator
be replaced by ‘less than’), a formula of the form in (23) can only give us a proportional
reading. For example, (24) could only be interpreted as asserting that the number of
solutions to the problem that have been discovered is less than the number of solutions to
the problem that meet some other condition (e.g., that haven’t been discovered); this is
necessarily a proportional interpretation, which is not the only (or even the preferred) reading for this sentence.

(24) Few solutions to the problem have been discovered

It is interesting to note that Lappin considers an alternate entry for many of the form in (25), in which alternatives to the restrictor rather than scope are considered, but rejects it for the reason that it violates conservativity:

(25) \[ \text{many}(A,B) = 1 \text{ iff } |A \cap B| > |C \cap B| \]

An entry of this form actually has a better chance of accurately rendering the truth conditions of (24) (which might be interpreted as saying that the number of solutions to this problem that have been discovered is less than the number of solutions to some other problem that have been discovered).

Below, I will return to a consideration of sentences such as (24) that feature an overt comparison, which I will argue provide crucial evidence towards the correct treatment of the semantics of many and few.

4.4.3 Westerståhl (1985)

Several authors have argued that many and few are actually lexically ambiguous. One of the first of these is Westerståhl (1985), who proposes that many (and likewise few) requires a series of denotations, such that in a sentence of the form many(A,B), the size of the set A \( \cap \) B is compared alternately to some proportion of the size of A (26a), that proportion of As that corresponds to the proportion of Bs in the domain as a whole (26b), some number n which is a function of the size of the domain (26c), or some fixed proportion of the size of B (26d):
While this approach is perhaps descriptively adequate, the consequence is a multi-way ambiguity, with every new reading requiring an additional lexical entry, with no clear upper bound on their number. Westerståhl himself notes that it might be preferable to adopt a simple interpretation of many as ‘at least $n$ (for some contextually defined $n$)’, though this requires the development of some semantic apparatus for factoring in the contextual contribution.

4.4.4 Partee (1989)

A more principled ambiguity analysis, and one that has influenced much later work, is that of Partee (1989). Drawing on Milsark’s observations, Partee proposes that many and few are ambiguous between a cardinal interpretation with adjectival semantics (27) and a proportional interpretation with determiner semantics (28):

(27) **Cardinal:**
\[
\text{a. } [\text{many}] = \lambda x. |x| > n, \text{ for some large } n \\
\text{b. } [\text{few}] = \lambda x. |x| < m, \text{ for some small } m
\]

(28) **Proportional:**
\[
\text{a. } [\text{many}] = \lambda P \lambda Q. |P \cap Q| / |P| > k, \text{ where } 0 < k < 1 \\
\text{b. } [\text{few}] = \lambda P \lambda Q. |P \cap Q| / |P| < j, \text{ where } 0 < j < 1
\]

At first look, Partee’s proposal captures the facts in an elegant way. As discussed in detail in Chapter 2, many and few have both quantifier-like and adjective-like uses. In the former they may have both cardinal and proportional readings (as in (6) above), while in the latter they are typically (though not always) cardinal. In linking these two properties, then, Partee’s analysis has clear explanatory power.

But as I have argued earlier in this work, the account represented in (27) and (28) is not, in this form, entirely adequate. In Chapter 2 I discussed issues that arise from a
type-theoretic perspective; in particular, some uses of *many* and *few* (notably the
differential cases such as *many* in *many fewer than 100*) cannot be analyzed as either
cardinality predicates (as in (27)) or quantifying determiners (as in (28)). Here let me
briefly mention that the interpretive facts also do not line up perfectly, in that we find
proportional interpretations in contexts where Partee’s account predicts that only cardinal
readings should occur. For example, Partee takes the comparative and superlative forms
of *many* and *few* to be based on the cardinal adjectival entries in (27), but notes
problematic cases such as the following, which has a reading on which it is true, possible
only if *few* can receive a proportional rather than cardinal reading.

(29) Fewer people in big cities than small towns know their neighbors

Similarly, Partee proposes that attributive *many* and *few* have only a cardinal
interpretation (consistent with their adjective-like position). But we can find exceptions
to this, such as (30a), where *few* has a proportional reading comparable to that in (30b):

(30) a. The few children who like spinach rarely need vitamin supplements
    b. Few children like spinach

A quantifier/adjective ambiguity thus does not fully account for the patterns of
availability of cardinal and proportional interpretations.

4.4.5 Keenan & Stavi (1986)

Keenan & Stavi (1986) argue that *many* and *few* cannot be treated extensionally at
all, but rather require an intensional analysis. They support this claim using the following
example. Imagine a situation involving an annual meeting of doctors. Suppose that
during the past year every doctor has obtained a law degree, and suppose further that the
total number of doctors attending the meeting this year is much smaller than in past years.
Then despite the fact that doctors and lawyers have the same extensions in this situation, (31a) might be judged true, while (31b) was judged false:

(31) a. Few doctors attended the meeting this year  
b. Few lawyers attended the meetings this year

It looks like the truth of a sentence of the form many\((A,B)\) or few\((A,B)\) cannot be determined purely on the basis of the extensions of \(A\) and \(B\); we need to look at the intensions of \(A\) and/or \(B\) as well. On this basis, Keenan & Stavi conclude that many and few must be excluded entirely from the class of quantifying determiners, and classified instead with ‘value judgment expressions’ such as a surprisingly large number of.

Keenan & Stavi do not develop an intensional treatment of many and few, but analyses of this sort have been developed by other authors; I review these below.

4.4.6 Fernando & Kamp (1996); Lappin (2000)

In an update to his previously discussed contextualized account, Lappin proposes an intensional analysis in which the evaluation of a sentence of the form many\((A,B)\) or few\((A,B)\) involves a comparison of the size of \(A \cap B\) in the actual situation \((sa)\) to its size in a set \(S\) of normative situations \((sn)\), essentially alternate possible worlds:

(32) \[[\text{many}(A,B)]^sa = 1 \text{ iff } S \neq \emptyset \text{ and for every } sn \in S, \]
\[|A]^sa \cap |B]^sa > |A]^{sn} \cap |B]^{sn}|\]

In a related approach, Fernando & Kamp (1995) start from the insight that many\((A,B)\) obtains when it is the case that \(|A \cap B|\) ‘could well have been’ smaller than it is in the actual world. They formalize this via a probability function over possible worlds, as given in somewhat simplified form in (33):

(33) \(|A \cap B| = n\) is many \text{ iff it is probable that } |A \cap B| < n \text{ iff } p(\{w: |A \cap B| < n\}) > c
That is, $n$ As that are $B$ can be considered many if the probability (across worlds) that $|A \cap B|$ is less than $n$ exceeds some threshold value $c$.

There is merit, I believe, in the insight that evaluating a sentence of the form $many(A,B)$ or $few(A,B)$ sometimes involves considering some other way things might have been, i.e. some other possible world. Consider again our original two sentences, in the situation in which the lecture in question was Chomsky speaking to a campus-wide audience on the topic of Middle East Politics:

(1)  
   a. Many students attended the lecture  
   b. Few students attended the lecture

It seems eminently reasonable to say that an evaluation of either of these sentences involves a comparison of the number of students who actually attended to some number who might have attended.

But I am less convinced that either of the implementations outlined above is workable in the general case. To start with Lappin’s account, the obvious issue is in defining what it is we mean by ‘normative’ situations or possible worlds. These do not necessarily need to be worlds consistent with our expectations or desires, or with the past course of events, as evidenced by the felicity of examples such as these:

(34)  
   a. Many students attended the lecture, just like we expected  
   b. Many students attended the lecture, just like we hoped  
   c. Many students attended the lecture this year, just like every year

But how else might we characterize normative? As a further example of a case problematic for this analysis, consider the following:

(35)  Few American men are more than 7 feet tall

If we wish to analyze (35) as expressing the proposition that the number (or proportion) of American men over 7 feet tall is smaller than that number (proportion) in some set of
‘normative’ worlds, then the notion of a normative world must be a very flexible one, and in particular be based on worlds that are very different from the actual one.

The choice of worlds is relevant, and potentially problematic, for Fernando & Kamp’s approach as well. These authors do not specify which worlds are to be considered in the calculation in (33), but depending on the set of worlds selected, the probability function \( p \) could return a value that is either arbitrarily low or arbitrarily high, such that the conditions in (33) either always obtain or never obtain.

More fundamentally, there are some uses of *many* and *few* that do not lend themselves to quantification over worlds other than the actual one. This is particularly the case with mathematical statements. All the examples in (36) are interpretable:

(36) a. Many odd numbers are prime  
b. There are many prime numbers between 1 and 1000  
c. Few three-digit prime numbers end in 7

Yet it does not seem that we should analyze these as involving comparison to the situation in some alternate worlds in which the distribution of prime numbers is different than it is in the actual world.

We must conclude, then, that some uses of *many* and *few* can be analyzed with reference to some world or worlds other than the actual one, but not all should be.

4.4.7 Summary

To summarize this brief review, there are several interesting accounts of the interpretation of Q-adjjectives available in the literature, but none seems to capture all of the readings available to these words, and the constraints on where they can occur. I believe, in fact, that all of these proposals suffer from the same basic issue: considerable specific content is built into the lexical semantics of the Q-adjective itself, with the result
that either some available interpretations are accounted for while others fall through the cracks, or it becomes necessary to posit a lexical ambiguity to get all of the possible readings. From another perspective, all of the works discussed above are situated within the literature on quantification, and as such none of them explicitly address the interpretive parallels between Q-adjectives and ordinary adjectives. The following questions thus remain to be answered:

1) Is it possible to develop a unified account of setting the standard of comparison for Q-adjectives, one that is able to capture all of the interpretations available to these items, without positing a multi-way lexical ambiguity?

2) What is the relationship between the cardinal and proportional readings of Q-adjectives, and how can the constraints on the availability of each be accounted for?

3) How should the parallels between Q-adjectives and ordinary adjectives be accounted for?

Below, I will draw on the scalar analysis developed in Chapters 2 and 3 to formulate answers to these questions.

4.5 POS and the Standard of Comparison

4.5.1 The Positive Form

Let us begin by considering how the standard of comparison is set for *many* and *few* in the case where they most clearly have a cardinal reading, as in the following examples:
In the next section, I will move to a discussion of how the cardinal versus proportional distinction is derived, and show that the treatment developed in this section can be extended to the proportional cases as well.

On the approach that I will take, *many* and *few* in their positive forms are underspecified, in that there is a parameter that must be ‘filled in’ either on the basis of the context or (in some cases) by material present in the linguistic representation. As such, it will avoid the issue of over-specificity that characterizes some of the proposals discussed above. I will show, however, that it is possible to approach systematically the question of how this contextual parameter receives its value; we can go beyond statements of the form ‘determined by the context’. In particular, consideration of constructions where an overt standard of comparison is present will provide insight into strategies invoked in filling in this value. Furthermore, this account will be couched in a framework that can also accommodate ordinary gradable adjectives, thereby capturing the interpretive parallels between these and Q-adjectives.

Recall that in the account developed in Chapters 2 and 3, I followed a degree-based approach to gradability, according to which *many* and *few* (and gradable adjectives more generally) include a degree argument that is saturated or bound by degree morphology. The semantics of their positive (unmarked) forms were analyzed as arising via composition with a null positive morpheme POS, an approach that builds on a long tradition in the analysis of gradable adjectives (e.g. Creswell 1977). In particular, I adopted from von Stechow (2005) and Heim (2006) the notion that POS introduces a
‘neutral range’ \( N_S \) of values that are regarded as neither large nor small with respect to the context\(^{23} \):

\[
(38) \begin{align*}
\text{a. } \left[ \text{many} \right] & = \lambda d \lambda I. d \in I \\
\text{b. } \left[ \text{few} \right] & = \lambda d \lambda I. d \in \text{INV}(I)
\end{align*}
\]

\[
(39) \left[ \text{POS} \right] = \lambda I. \forall d \in N_S[d \in I] \\
= \lambda I. N_S \subseteq I
\]

This approach gives our original sample sentences the following translations, with their truth conditions given in abbreviated interval-based notation below:

\[
(40) \begin{align*}
\text{a. Many students attended the lecture} \\
& \forall d \in N_\#[\exists x[*\text{student}(x) \land \mu_\#(x) \geq d \land \text{attended}(x, \text{lecture})]] \\
& N_\# \subseteq (0, #-\text{Students-AtL}]
\end{align*}
\]

\[
\begin{align*}
\text{b. Few students attended the lecture} \\
& \forall d \in N_\#[\neg \exists x[*\text{student}(x) \land \mu_\#(x) > d \land \text{attended}(x, \text{lecture})]] \\
& N_\# \subseteq [#-\text{Students-AtL}, \infty)
\end{align*}
\]

From a purely technical standpoint, this analysis of the standard of comparison already accounts for one aspects of the semantics of \textit{many} and \textit{few} (and of pairs of gradable antonyms more generally), namely the existence of an intermediate range where neither the positive nor the negative member of the pair applies. As such, this approach has advantages over analyses that define the positive forms of such pairs with reference to a single standard degree (e.g. Cresswell 1977; Kennedy 2001). By way of illustration, suppose we gave (1a,b) the analyses in (41):

\[
(41) \begin{align*}
\text{a. Many students attended the lecture} \\
& d_{\text{Std}} \in (0, #-\text{Students-AtL}] \\
\end{align*}
\]

\[
\begin{align*}
\text{b. Few students attended the lecture} \\
& d_{\text{Std}} \in [#-\text{Students-AtL}, \infty)
\end{align*}
\]

\(^{23}\) Recall that \( N_S \) stands for the neutral range on the scale \( S \). As was done in Chapter 3, in the case involving \textit{many} and \textit{few}, where the scale in question is necessarily cardinality, this is indicated with the subscript \# (\( N_\# \)).
If we assume that \( d_{Std} \) is the same in both (41a) and (41b), then any value less than or equal to this standard would be considered \textit{few}, while any value greater than or equal to this standard would be considered \textit{many}, with no intermediate range between them. On the other hand, if we assume that the standards for the positive and negative members of the pair are different (as in Kennedy 2001), a more serious issue arises: there is nothing that would prevent the positive and negative antonyms from holding true simultaneously. That is, with the appropriate choice of standards, both (41a) and (41b) could be judged true relative to the same context or perspective.\(^{24}\) Defining the positive form in terms of a range rather than a single point avoids this problem.

On this view, the neutral range \( N_S \) is then the locus of the truth-conditional variability of Q-adjjectives (and other gradable expressions) relative to the context. Furthermore, if the neutral range is in some way taken to have fuzzy boundaries, then the existence of borderline cases is likewise accounted for. But what determines the location and extent of this crucial neutral range, and why might its boundaries be fuzzy rather than precise? Here, relevant insight is provided by examples where the standard of comparison is in some way made explicit:

(42)  
\begin{enumerate}
\item a. Compared to yesterday, there are few cars in the parking lot today
\item b. Compared to the Peoria public library, the Northgate College library owns many linguistics books
\item c. Compared to Barney, Fred didn’t make many mistakes
\end{enumerate}

(43)  
\begin{enumerate}
\item a. For a Sunday, there are few cars in the parking lot
\item b. For a small institution, the Northgate College library owns many linguistics books
\item c. Fred didn’t make many mistakes for a beginner
\end{enumerate}

\(^{24}\) In fact, on the analysis in (41), the same issue is present even if \( d_{Std} \) is identical for the positive and negative members of the pair, in that \( d_{Std} \) itself is contained within the intervals in both (41a) and (41b). In this case, however, the problem could presumably be resolved by redefining the intervals in question as open at both the upper and lower ends.
In (42), a standard of comparison is introduced by a *compared to* phrase; in (43), a prepositional *for* phrase serves to make the frame of reference explicit. As evidence that this material is in fact involved in setting the standard of comparison for the Q-adjective, note that sentences of this sort lend themselves to being used to clarify the standard that the speaker has in mind:

(44)  
SPEAKER A:  The Northgate College library owns many linguistics books  
SPEAKER B:  What do you mean? There can’t be more than 50 here on the shelves!  
SPEAKER A:  Yes, but compared to the Peoria public library, our library has many, indeed!

(45)  
SPEAKER A:  There are hardly any cars here today  
SPEAKER B:  But there must be over 100 here  
SPEAKER A:  Yes, but it’s usually really crowded on the weekend. For a Sunday, there are few cars in the lot

Note also that examples featuring overt *for* and *compared to* phrases also are characterized by an intermediate zone where neither *many* nor *few* can be said to apply (Kennedy 2007). For example, both (46a) and (46b) might be judged false, if the number of linguistics books owned by the Northgate library is more or less the same as the number owned by Peoria public library. Similarly, both (47a) and (47b) would be judged false if the number of cars in the lot is in the typical range for a Sunday.

(46)  
Compared to the Peoria public library…  
a.  …the Northgate College library owns many linguistics textbooks  
b.  …the Northgate College library owns few linguistics textbooks

(47)  
For a Sunday….  
a.  …there are many cars in the parking lot  
b.  …there are few cars in the parking lot
The implication is that these examples must also be analyzed with reference to a neutral range, and that the *compared to* or *for* phrase plays a role in determining where on the scale this range is situated.

Note that parallel examples of *for* and *compared to* phrases with gradable adjectives are discussed by Kennedy (2007):

\begin{enumerate}
\item Kyle’s car is expensive for a Honda
\item This book is long compared to that one
\end{enumerate}

I will propose a somewhat different analysis of these data than Kennedy, so before proceeding, let me take a brief look at his account, and summarize why I do not adopt it.

With regards to examples such as (48a), Kennedy rejects the common view that the *for* phrase introduces a comparison class in relation to which a standard of comparison is set, and argues instead that it introduces a restriction on the domain of the gradable adjective. His account is based on the crucial observation that an adjectival phrase of the form *Adj for a(n) NP* presupposes that its subject falls within the extension of NP, as evidenced by the anomalous nature of examples such as the following: only a Honda can be characterized as *expensive for a Honda*.

\begin{enumerate}
\item Kyle’s BMW is expensive for a Honda
\end{enumerate}

Based on this, Kennedy concludes that if *expensive* is a function from individuals to degrees of cost\textsuperscript{25}, *expensive for a Honda* is a function from *Hondas* to degrees of cost.

\begin{equation}
[(\text{expensive for a Honda})_{\text{ed}}] = \lambda x: \text{Honda}(x).\text{expensive}(x)
\end{equation}

The POS morpheme, which takes the gradable adjective as an argument, introduces a standard of comparison that is calculated as a function of the gradable predicate, namely

\textsuperscript{25} Kennedy’s analysis of gradable adjectives is different from the one assumed here, in that gradable adjectives are taken to denote measure functions of type $\langle \text{ed} \rangle$, with degree morphology responsible for turning them back to predicates over individuals.
the function that returns a value of the property in question that would be considered ‘significant’ for the purposes at hand.

\[
(51) \quad \text{[(POS(expensive for a Honda))(et)]} = \lambda x: \text{Honda}(x). \text{expensive}(x) > s(\lambda y: \text{Honda}(y). \text{expensive}(y))
\]

Thus after composition with POS, \textit{expensive for a Honda} is a (domain specific) function that is true of a Honda if its cost exceeds the cost that would be considered significant for a Honda. This analysis therefore captures both the presuppositional contribution of \textit{for} phrases and the intuition that they introduce a frame of reference within which the standard of comparison is calculated.

With regards to examples such as (48b), Kennedy proposes that the presence of the \textit{compared to} phrase introduces the requirement that the denotation of the predicate be calculated with respect to a domain of discourse containing just two individuals, in this case ‘this book’ and ‘that book’. On the assumption (Klein 1980) that the standard of comparison must be such that the predicate \textit{long} has non-empty positive and negative extensions, and that ‘this book’ is in the positive extension, it follows that ‘that book’ is in the negative extension, and therefore that ‘this book’ is longer than ‘that book’.

Appealing though this approach is, there are problematic cases for both of these analyses, both from the realm of ordinary gradable adjectives and especially when we expand into the territory of Q-adjectives. With regards to \textit{compared to} phrases, consider the following examples:

\[
(52) \quad \begin{align*}
\text{a. Compared to the average jockey, John is tall} \\
\text{b. The shirt was expensive compared to what I expected}
\end{align*}
\]

In (52a), it is not at all certain what sort of entity (if any) is denoted by the noun phrase \textit{the average jockey}, but it does not seem to be the sort of thing that could be part of a two-
membered universe the other member of which is John. In (52b), *what I expected* seems to introduce a cost, not an individual that could be compared to the shirt; to utter this sentence, I need only have a cost in mind (say, $20), not a shirt, or any other individual.

The notion that the relevant comparison is between two degrees rather than two individuals is even clearer in the case of Q-adjectives, where the compared to phrase can explicitly introduce a number or amount:

\begin{align*}
(53) & \quad \text{a. Few students attended the lecture, compared to the number of professors who were there} \\
& \quad \text{b. I don’t have much money, compared to the amount that Fred has}
\end{align*}

Here, there is an additional possibility that needs to be considered. On the present account, while gradable adjectives such as *tall* are predicates of individuals, Q-adjectives are predicates of scalar intervals. Perhaps, then, examples such as those in (53) still do involve the comparison of two entities within the domain of the gradable expression (e.g. in (53a), a scalar interval representing the number of students who attended the lecture, and an interval representing the number of professors there). But this would force us to conclude that the domain of discourse could contain only two sets of degrees, which strikes me as questionable.

As for *for* phrases, consider the examples in (54):

\begin{align*}
(54) & \quad \text{a. Freddy reads difficult books for an 8-year-old} \\
& \quad \text{b. For a politician, Frank doesn’t know many people}
\end{align*}

In both of these cases, there is a presupposition similar to that observed in (48a): Freddy must be an 8-year-old, and Frank must be a politician. But note that here, the subject (of whom the presupposition holds) is not an argument of the gradable expression, meaning that the presupposition cannot be readily captured as a restriction on the domain of that expression. A related problem arises in the following cases:
(55) a. The store is crowded for a Tuesday
b. For a Sunday, there are few cars in the parking lot

In these examples, the *for* phrase again introduces a presupposition, just as in the previously discussed cases, but again not one that can be readily analyzed as a domain restriction. Thus in (55a), what is presupposed is not something about the store itself, but rather that the time of evaluation is a Tuesday; as evidence for this, note that (55a) would be infelicitous if uttered as a comment on the state of the store on a Friday. A similar point can be made about (55b), with the Q-adjective *few*. Whatever is responsible for the presuppositional nature of the *for* phrase in these examples, it cannot universally be reduced to a domain restriction.

To summarize, if we broaden the range of data considered, the account of *compared to* phrases as involving a covert restriction on the domain of discourse becomes difficult to maintain, and the analysis of *for* phrases as introducing a domain restriction on the gradable expression no longer accounts for the crucial facts. I would like to propose instead that both of these types of phrases play a more direct role in setting the standard of comparison for gradable expressions. Specifically, both provide information that enables the neutral range $N_S$ to be determined, though they do so in different ways.

In what follows, I will provide an analysis of these two constructions as they relate to determining the neutral range introduced by POS. From there, I will extrapolate to the situation in which there is no overt standard of comparison introduced (i.e. no *for* or *compared to* phrase), which I propose involve the same mechanisms. My focus for obvious reasons will be on Q-adjectives, but I take it that the account could be extended to gradable adjectives more generally.
4.5.2 Compared To Phrases

Let us begin with compared to phrases, as in the following examples:

(56)  
   a. Compared to the Peoria public library, the Northgate College library owns many linguistics books  
   b. Compared to the Peoria public library, the Northgate College library owns few linguistics books  

Here, a reasonable position to take is that the compared to phrase sets the standard by which many or few is judged to the number of linguistics books owned by the Peoria public library. Then (56a) is true iff the number of such books owned by the Northgate College library exceeds this number by some ‘significant’ amount, while (56b) is true iff the number owned by Northgate falls significantly short of this number. But then this gives an indication of what the neutral range is: a range of values that are judged to be not significantly different from the point of comparison \( (PC) \) introduced by the compared to phrase, here the number of linguistics books owned by the Peoria public library. The truth conditions of the above examples can then be restated as follows:

(57)  
   a. \( \lbrack (56a) \rbrack = 1 \) iff \( N_\# \subseteq (0, \# \text{ linguistics books owned by NCL}] \)  
   b. \( \lbrack (56b) \rbrack = 1 \) iff \( N_\# \subseteq [\# \text{ linguistics books owned by NCL}, \infty) \),  
   where \( N_\# = \{d: d \text{ is not significantly different from the } \# \text{ of linguistics books owned by the Peoria public library}\} \)

I will have more to say below about the use of the term ‘significant’, but let me note here that there is presumably no sharp cut-off in when one number is considered ‘significantly different’ from another, even with respect to a particular context. As such, the representation in (57) implies the existence of borderline cases where there is uncertainty as to whether many or few applies.

The other examples introduced above can likewise be analyzed as involving a neutral range that consists of a range of values around the number introduced by the
compared to phrase. Based on this, I will propose the following refinement to the semantics of the positive morpheme:

\[
(58) \quad [\text{POS}] = \lambda I. \{d: p_c - d \text{ is not significant} \} \subseteq I
\]

The only difference between this definition and the earlier one given in (39) is that it explicitly defines the neutral range as the set of values not significantly different from some point of comparison \(p_c\).

In the example in (57), the compared to phrase introduces an alternative to the sentential subject. An examination of some further cases demonstrates that this is not the only possibility. To be concrete, imagine the following situation involving a college open house for prospective students and their parents, who have the opportunity to visit tables representing the different majors available at our college. Imagine further that there are many activities taking place simultaneously, and prospective students and parents are coming in and out of the room we are in, such that we are not able to get a sense of the total number of attendees (I add this condition to ensure that the situation favors a cardinal rather than proportional reading). In this situation, I might utter one of the following:

\[
(59) \quad \begin{align*}
\text{a. Compared to parents, few prospective students visited the linguistics table} \\
\text{b. Compared to the number visiting the psychology table, few prospective students visited the linguistics table} \\
\text{c. Compared to last year, few prospective students visited the linguistics table this year} \\
\text{d. Compared to what I expected, few prospective students visited the linguistics table}
\end{align*}
\]

In (59a), the comparison involves an alternative to the restrictor (parents rather than prospective students); in (59b), an alternative to the nuclear scope (visited the psychology table vs. visited the linguistics table); in (59c), an alternative time (last year vs. this year);
and in (59d), some alternative possible world (one consistent with my expectations).

These examples suggest that we can go in several different ‘directions’ to find an appropriate point of comparison $p_c$: into the space of alternatives to the restrictor or scope, to some other point in time, or to some alternate world.

Based on this, we can be a little more explicit about the nature of $p_c$. Consider a sentence of the form $\text{many}(A,B)$ or $\text{few}(A,B)$ evaluated with respect to a world $w$ and a time $t$:

\[
\text{(60) a. } \left\lceil \text{many}(A,B) \right\rceil_{w,t} = 1 \text{ iff } \{d: p_c - d \text{ is not significant}\} \subseteq \{d: \exists x[A(x) \wedge B(x) \wedge \mu(x) \geq d \text{ in } w \text{ at } t]\}
\]

\[
\text{b. } \left\lceil \text{few}(A,B) \right\rceil_{w,t} = 1 \text{ iff } \{d: p_c - d \text{ is not significant}\} \subseteq \text{INV}(\{d: \exists x[A(x) \wedge B(x) \wedge \mu(x) \geq d \text{ in } w \text{ at } t]\})
\]

Based on the above example, at least the following possibilities are available for $p_c$,

where the prime indicates an alternative to the element in the original formula.

\[
\text{(61) } p_c = \max(\lambda d.P(d)), \text{ where } P(d) \text{ has one of the following forms}
\]

\[
\text{a. } \exists x[A'(x) \wedge B(x) \wedge \mu(x) \geq d \text{ in } w \text{ at } t]
\]

\[
\text{b. } \exists x[A(x) \wedge B'(x) \wedge \mu(x) \geq d \text{ in } w \text{ at } t]
\]

\[
\text{c. } \exists x[A(x) \wedge B(x) \wedge \mu(x) \geq d \text{ in } w \text{ at } t']
\]

\[
\text{d. } \exists x[A(x) \wedge B(x) \wedge \mu(x) \geq d \text{ in } w' \text{ at } t]
\]

The following example suggests that more than one of these can be varied:

\[
\text{(62) Compared to the number who usually vote for third party candidates, many Americans voted for Ralph Nader in 2000}
\]

Here the point of comparison involves both a change of time and an alternative to the scope (though one that follows from the change of time).

Up to this point I have been considering examples in which the point of comparison is supplied by an overt $\text{compared to}$ phrase. Let us now turn to the case without an overt comparison phrase. For example:
(63) Few prospective students visited the linguistics table

I believe that this sentence can receive the same interpretations as the examples in (59): depending on what the speaker (or hearer) has in mind, (63) might be interpreted as meaning that that the number of prospective students who visited the linguistics table was significantly less than the number of some other group who did so, the number of prospective students who did something else, the comparable number at some other time, or the number that was expected (which might itself be based on one of the other factors). Thus examples such as these can likewise be analyzed via the positive form in (58).

Relative to the previously discussed cases, the only difference is in how $p_c$ is determined; rather than being made overt by a compared to phrase, it is supplied by the context.

On the surface, it might seem that this solution begs the question, replacing some fuzzy notion of a neutral range with an equally fuzzy notion of a point of comparison $p_c$. But I would argue that we have taken a step forward, in two respects. First, from a formal perspective, we now have a more precise characterization of what we must look for the context to provide: some degree $d$ that serves as an appropriate point of comparison. Second, at a deeper level, hypothetical dialogues of the sort in (44) and (45) – which I think are by no means atypical of the sort of conversations one actually hears – are indicative that speakers (and listeners) do have this sort of implicit standard of comparison in mind when using many and few (or any other gradable terms).

4.5.3 For Phrases

The discussion of the preceding section showed that the compared to phrase determines the neutral range by specifying a point of comparison $p_c$ that serves as its midpoint. Furthermore, I have proposed that in the absence of an overt compared to
phrase, the neutral range may be set in the same way, with reference to some implicit point of comparison derived via the context. Let us turn now to for phrases, which I propose work in a different way to determine the scalar position of the neutral range, and which I further take to represent the overt counterpart to another way in which the neutral range may be set implicitly. Consider the following examples:

(64)  a. For a Sunday, there are many cars in the parking lot
     b. For a Sunday, there are few cars in the parking lot

In non-technical terms, the truth conditions of (64a,b) seem to be something like the following: if we take the set of relevant Sundays, and determine the number of cars in the lot on each of these days, then the number of cars today (i.e. the time of utterance) is an outlier – on the high side in the case of many, on the low side in the case of few. That is, the situation is as in the frequency distributions in (65a,b):

(65)  a. For a Sunday, there are many cars in the parking lot

\[
\begin{array}{c}
\text{# of Sundays} \\
\hline
\text{# of cars in the lot today} \\
\end{array}
\]

b. For a Sunday, there are few cars in the parking lot

\[
\begin{array}{c}
\text{# of Sundays} \\
\hline
\text{# of cars in the lot today} \\
\end{array}
\]
As evidence that we need to take into account the distribution of relevant values as opposed to simply (say) the mean, consider the contrast represented below:

(66) For a Sunday, there are many cars in the parking lot

We would, I think, be more willing to accept this sentence as true if the situation was as depicted by the line (a) than if it were as depicted by line (b), despite the fact that both the mean number of cars in the lot on Sundays and the number there today are the same in the two cases. Thus for (64a) to be true, it is not sufficient that the number of cars in the lot today exceed the average (across Sundays) by some fixed amount; rather, the number there today must be greater than the number there on most Sundays. The more variation there is in the number of cars in the lot on Sundays, the more today’s number must exceed the mean in order for many to obtain.

This in turn suggests a way to view the neutral range $N_S$ in cases involving for phrases. As before, it is centered around a point of comparison $p_c$, which in this case is a mean over the comparison class (here, the mean number of cars in the lot on a Sunday); the range around this value is such that it encompasses the values corresponding to most members of the comparison class. The more ‘spread’ there is around the mean, the wider is the neutral range.
While the example above involves a comparison class over times (Sundays), the same points can be made about more typical examples involving comparison classes over individuals:

(67)  
a. For a market research firm, Analysis Inc. has many/few employees  
b. Fred owns many/few books for a plumber  
c. New Paltz has many/few bars for a college town  
d. For such an important book, few people have read *The Wealth of Nations*

(67a) is interpreted as saying as saying that the number of Analysis Inc.’s employees exceeds (for *many*) or falls short of (for *few*) the typical range for market research firms; (67b) that the number of John’s books exceeds/falls short of the typical range for plumbers; (67c) that the number of bars in New Paltz exceeds/falls short of the typical range for college towns; and so forth. Furthermore, the notion of dispersion around the mean is relevant here too: to take (67a) as an example, the more variation there is in the size of market research firms, the more Analysis Inc.’s number of employees must exceed/fall short of the mean across this class to establish the truth of the sentence.

Needless to say, this sounds quite a lot like mathematical statistics, and of course the field of statistics offers us a formal measure of dispersion that corresponds to the intuitive notion introduced above, namely the standard deviation. In statistics, the standard deviation is a measure of the dispersion of a set of values; in a normal distribution, roughly 2/3 of values fall within one standard deviation of the mean and roughly 95% within two standard deviations of the mean. We might borrow this concept to give a more formal characterization of the neutral range in sentences involving *for* phrases:
(68) \( N_S = \text{Mean}_{\text{comparison class}} \pm n \) standard deviations, where \( n \) is a small number

On this view, the use of the word ‘significant’ in the earlier definition of POS in (58) is apt, in that statistically speaking, what qualifies as a ‘significant’ difference from a mean is defined in terms of standard deviations.\(^{26}\)

The characterization of the neutral range sketched out here leaves one factor unspecified, namely the proportion of values that fall within the neutral range (to carry on with the statistical analogy, what is left unspecified is the level of significance, reflected by the value \( n \) in (68)). I do not believe that it is possible to be more specific here. For (67a) to be true, does Analysis Inc. need to have more employees than 2/3 of market research firms? 80%? 90%? There is no definitive answer. I take this to be the source of the borderline-case problem discussed Section 4.2: while some values are without doubt sufficiently different from the mean (given the spread of the curve) to qualify as falling outside the neutral range, and other values are without doubt within it, there will also be values for which there is indeterminacy, depending on the choice of \( n \).

Below, I give a more formal analysis of one of the earlier examples:

(69) \[ \text{[Analysis Inc. has many employees for a market research firm]} = 1 \]
iff \( N_\# \subseteq (0, \# \text{ of employees}] ), \text{where } N_\# \text{ is calculated relative to }\]
\[ \lambda x: \text{market-research-firm}(x) \lambda d. \exists y [\mu_\#(y) \geq d \land \text{employs}(x,y)] \]

Here, calculating \( N_\# \) relative to the specified lambda expression involves (in statistical terms) computing the mean and standard deviation over the set in question:

\(^{26}\) A curious and oddly satisfying aspect of the observations here is that while the average speaker does not understand the statistical concept of a standard deviation, it seems that we use very much the same notion in interpreting sentences involving for phrases.
\[(70) \quad N_\# \text{ calculated relative to} \]
\[
\lambda x: \text{market-research-firm}(x) \lambda d. \exists y[\mu_d(y) \geq d \land \text{employs}(x,y)]
\]

is defined as:

\[
 p_c = \text{Mean}_x: \text{market-research-firm}(x)(\text{Max}(\lambda d. \exists y[\mu_d(y) \geq d \land \text{employs}(x,y)]))
\]

\[
 N_\# = p_c \pm n \text{Stdev}_x: \text{market-research-firm}(x)(\text{Max}(\lambda d. \exists y[\mu_d(y) \geq d \land \text{employs}(x,y)]))
\]

Stripped of the statistical terminology, the formulae in (69) and (70) essentially say the following: the neutral range is a range of values such that for most market research firms, the number of the firm’s employees falls within the range; the sentence in (69) is true only if the number of employees at Analysis Inc. exceeds this range.

I will not attempt a compositional analysis of the \textit{for} phrase here. This task will be a non-trivial one, in that any semantic analysis must account for both the presuppositional nature of the \textit{for} phrase and its role in setting the standard of comparison. I leave this as a topic for the future.

But to return to the main question of interest, let us consider again examples in which the standard of comparison is not made overt. My claim is that sentences with \textit{for} phrases represent the overt counterpart of one of the ways in which the neutral range may be set covertly. Take for example the following:

\[(71) \quad \begin{align*}
\text{a.} & \quad \text{Fred owns many books} \\
\text{b.} & \quad \text{There are few cars in the parking lot}
\end{align*} \]

These sentences can, I believe, receive the same interpretations as the corresponding examples with \textit{for} phrases. Thus (71a) might be judged true if the number of books owned by Fred exceeds a range around the mean for some comparison class of which Fred is a member (plumbers, college professors, 10-year-old boys, or whatever), that range being set in such a way that the majority of values for the comparison class fall
within that range. Likewise, (71b) might be judged true if the number of cars in the lot falls short of some range around the mean over relevant times. This is therefore another strategy by which a neutral range can be determined contextually: we start with some relevant comparison class, which the subject belongs to (or in the case of (71b), which the time of interpretation belongs to), and then take the mean of values across this comparison class, and establish a range around this mean that encompasses the values associated with most members of the comparison class.

To conclude this section, let me as an aside suggest that the facts discuss here tend to support a semantic account of the gap between positive and negative antonyms (e.g. between many and few). Consider again the example and scenarios depicted in (66). As we move from the situation represented by the line (a) to that represented by the line (b), the boundary between many and not many seems to move upward. But at the same time, the boundary between few and not few moves downward. That is, the two boundaries are independent of one another. This effect does not seem easy to capture on a pragmatic account under which few is equated semantically with not many, and the illusion of a gap is the result of epistemic uncertainty as to the location of the boundary; but it is fully compatible with the view that the gap or neutral range is part of the semantics, and that factors relating to the context may determine its extent.

4.5.4 Summary

The present account of the vagueness of Q-adjectives is one in which their positive forms are taken to be underspecified. Bare many and few are interpreted with reference to a neutral range $N_b$ (introduced by the positive morpheme POS), which is structured as a range of values around some point of comparison $p_c$. The truth
conditional variability of these terms is analyzed as arising from variability in how this range is set. But the scalar position of the neutral range is not completely unconstrained. Data from the interpretation of overt compared to and for phrases give evidence for two mechanisms by which this range may be set. It may be a range of values considered not significantly different from some implicit point of comparison \( p_c \), which may itself be derived by substituting an alternative to the restrictor or the scope, or an alternate time or world. Or we might derive \( p_c \) as the mean across some implicit comparison clause, and take \( N_o \) to be a range around that number that includes the values associated with most members of the comparison class.

Having these various possibilities available, it seems that we should ask whether it is possible to reduce them to one. For instance, could all of the discussed cases be analyzed in terms of a comparison class, or a comparison involving an alternative to the restrictor, or to the scope? The answer, I believe, is no. While such an approach might be preferable in terms of simplicity, it does not fit the facts. For example, (72a) can readily be interpreted as involving an implicit comparison of today’s situation to the state of the mall on other days; we can capture this with a neutral range derived via the mean and ‘standard deviation’ of the number of people in the mall for some relevant set of days that serves as a comparison class.

(72)  
   a. There are few people in the mall today
   b. Few students attended the lecture

On the other hand, to interpret (72b), it is not necessary to have in mind a comparison class across which an average is computed; for example, we might judge (72b) true if the number of students who went to the lecture was substantially below the number who went to Friday Movie Night.
I would argue that as speakers, we have a range of possibilities to choose from, and as such, the semantics of the positive form must be able to accommodate this. The proposal put forth here, in contrast to some of those discussed in Section 4.4, offers this flexibility, without the need to posit an actual lexical ambiguity.

Finally, my focus in this section has been on Q-adjectives (in particular *many* and *few*), but I take it that the proposed semantics for the positive morpheme POS could be extended to ordinary gradable adjectives as well, thereby capturing the interpretive parallels between the two classes. In the next section, however, I discuss behavior of Q-adjectives that set them apart from ordinary adjectives.

**4.6 Scale Structure and the Cardinal/Proportional Distinction**

**4.6.1 The Issue**

I turn now to the proposed ambiguity of *many* and *few*, particularly the availability of distinct readings that (following Partee 1989) have come to be known as ‘cardinal’ and ‘proportional.’ Let us begin by recappping the issue. Recall that examples such as (72) have two readings. On one interpretation (the so-called cardinal reading), what is asserted is simply that the number of students registered for Semantics II is large (small); this is equivalent to the interpretation that would be given to the equivalent *there*-sentence (*there are many/few students registered for Semantics II*). On the other reading (the proportional one), the assertion is that a large (small) proportion of some contextually determined set of students (say, first-year students in our PhD program in linguistics) are registered for Semantics II.

(73) a. Many students are registered for Semantics II  
b. Few students are registered for Semantics II
As discussed above, Partee (1989) proposes that this ambiguity is the result of *many* and *few* having two distinct lexical entries of different semantic types: a quantifying determiner with proportional semantics, and a cardinality predicate with cardinal semantics (see also Diesing 1992 for a similar view). But as noted above, this proposal runs into difficulty with examples that demonstrate proportional readings to be available in contexts that on Partee’s account should allow only cardinal readings. For example, Partee proposes that the comparatives *more* and *fewer* are based on the adjectival cardinal entries. But examples such as the following are perfectly felicitous, and on the most natural reading (I believe) true, despite the fact that it is certainly not the case that the number of residents of Ithaca (population 20 thousand) who know their neighbors is greater than the number of residents of NYC (population 8 million) who do so (likewise for Brazilians and Swedes with college degrees). The true reading thus requires that *more* and *fewer* be interpreted proportionally (a greater proportion of the residents of Ithaca than NYC; a smaller proportion of Brazilians than Swedes).

(74)  
   a. More residents of Ithaca than New York City know their next door neighbors.
   b. Fewer Brazilians than Swedes have college degrees

More significantly, the attributive use of Q-adjectives is likewise proposed to involve the cardinality predicate entries, but as I noted above we can construct examples in which a proportional reading is available in this position:

(75)  The few children who like spinach do not need vitamin supplements

Whatever the source of the distinction between cardinal and proportional interpretations, it does not fall out directly from a cardinality predicate/quantifying determiner duality.
The account I develop holds that the distinction is in fact one of scale structure. Q-adjectives are predicates of intervals on a measurement scale. My claim is that the proportional reading arises when an upper bound to the scale is assumed, whereas the cardinal reading arises when there is no salient upper bound. Thus the two readings of (73a) correspond to the two situations depicted below:

(76) a. Many students are registered for Semantics II (Proportional)

\[
\begin{array}{c}
\text{# of students registered for Semantics II} \\
0 \quad N_s \\
\end{array}
\]

b. Many students are registered for Semantics II (Cardinal)

\[
\begin{array}{c}
\text{# of students registered for Semantics II} \\
0 \quad N_s \\
\end{array}
\]

On this view, the readings available to a Q-adjective is independent of the position in which it occurs, and thus it is not unexpected that we find proportional readings in the more adjective-like uses of Q-adjectives. Another nice benefit of this analysis is that it suggests a solution to one of the puzzles noted above, namely the availability of proportional readings in comparatives, as in (74). Specifically, an example such as (74a) can be analyzed as involving the comparison of intervals on two bounded scales whose endpoints are aligned:

(77) More residents of Ithaca than New York City know their next door neighbors

\[
\begin{array}{c}
\text{# who know neighbors} \\
\text{# of Ithaca residents} \\
\end{array}
\]

\[
\begin{array}{c}
\text{# who know neighbors} \\
\text{# of NYC residents} \\
\end{array}
\]
The implication here is that the scales involved in the proportional reading are in some sense scales of proportion. This is made more plausible by the generally overlooked fact that *many* and *few* can form the basis for overt comparisons of proportion:

\[
\begin{align*}
(78) & \quad \text{a. Fewer than half of Americans think the U.S. will win the war in Iraq} \\
& \quad \text{b. More than a quarter of adults in Ohio are obese}
\end{align*}
\]

The goal of this section is to motivate this analysis. The plan will be to explore the factors that determine the availability of cardinal versus proportional readings, and from this basis develop an analysis that associates different constructions with different scale structures.

Before proceeding, it bears noting that it can be very difficult to distinguish cardinal and proportional readings – or even to convince oneself that the difference exists. After all, a set of individuals whose number is judged ‘large’ relative to the context is also quite likely to represent a ‘large’ proportion of some second set (similarly for a ‘small’ number and a ‘small’ proportion). It is therefore helpful to have a test to distinguish the two readings. Such a test is introduced by Partee (1989), who notes that on the cardinal reading, *few* *N* can be *all* of the contextually relevant *Ns*. For example, (79a) might be judged true if there are only a small number of first year students and all of them are registered for Semantics II; this is made explicit by the possible continuation in (79b):

\[
\begin{align*}
(79) & \quad \text{a. Few first-year students are registered for Semantics II} \\
& \quad \text{b. …because there are few first year students}
\end{align*}
\]

Therefore, to test whether a cardinal reading is available, we can construct examples where the Q-adjective is *few*, and where it is plausible that the extension of the NP sister of *few* contain only a small number of individuals. If *few* in this case can be *all*, then a
cardinal reading is available; if few cannot be all, the cardinal reading is not available. (It is less easy to design a test to rule the proportional reading in or out; but this turns out not to be necessary.)

4.6.2 The Availability of Cardinal and Proportional Readings

Let us turn now to the data. As was discussed earlier, the availability of cardinal versus proportional readings is to a large extent grammatically constrained. The first case of this, and the most well known, has to do with predicate type. Specifically, only the proportional reading is available in the subject position of an individual-level predicate (ILP), while the subject position of a stage-level predicate (SLP) allows both cardinal and proportional readings. For example, consider the following:

(80)   a. Few Americans are over 7 feet tall  
       b. Many Swedes have blue eyes  
       c. Few children like spinach  
       d. Many lawyers are corrupt

(81)   a. Many cars are available  
       b. Many people are in the kitchen  
       c. Few tourists visited the exhibit  
       d. Many people arrived  
       e. Few people left the room  
       f. Many black holes exist

The sentences in (80) feature ILPs, and correspondingly require a proportional interpretation for the Q-adjective; those in (81), featuring SLPs, can be read both cardinaly and proportionally. To be certain, the ambiguities in (81) are somewhat difficult to detect, since one or the other of the two readings tends to be more salient. For example, few in (81e) tends to be interpreted proportionally (a small proportion of those in the room left); but it could also be used to describe a situation in which we are watching the door of the room from the outside and notice only a small number of people
exiting, without have in mind any superset of people to which the size of the group leaving is compared. Conversely, (81f) strongly favors a cardinal reading; but recall from Milsark’s unicorn example (discussed in Section 4.3) that even the predicate exist allows a proportional reading for many. Even clearer evidence of the difference between the two sets of examples is provided by the few-as-all test:

(82) a. Few French-speaking senators are over 7 feet tall  
b. Few French-speaking senators have blue eyes  
c. Few French-speaking senators like spinach  
d. Few French-speaking senators are corrupt

(83) a. Few French-speaking senators are available  
b. Few French-speaking senators are in the kitchen  
c. Few French-speaking senators visited the exhibit  
d. Few French-speaking senators arrived  
e. Few French-speaking senators left the room  
f. Few French-speaking senators exist

For (82a-d) to be true, it must be the case that a small proportion of French-speaking senators are over 7 feet tall, have blue eyes, etc.; in the case where there are only a small number of French-speaking senators and the sentential predicate holds true of all of them, the sentence is false. By contrast, the examples in (83) all have readings on which they are true if there are a small number of French-speaking senators, all of whom are available, are in the kitchen, visited the exhibit, and so forth. That is, the examples in (83) have cardinal readings, while those in (82) do not.

The distinction by predicate type in interpretations available to Q-adjectives of course mirrors a more well-known pattern in the interpretation of bare plurals: ILPs give rise to generic readings (as in (84)), while SLPs yield existential readings (as in (85)):

(84) a. Americans are over 7 feet tall  
b. Swedes have blue eyes  
c. Children like spinach  
d. Lawyers are corrupt
The implication here is that the predicate plays a role in determining the interpretation of the Q-adjective, and that its role here is in some way related to its role with respect to bare plural subjects.

The role of the predicate in establishing the availability of proportional versus cardinal readings for Q-adjectives carries over to their occurrence in positions argued to require adjectival semantics. *There*-sentences are one such case: while this context has been claimed to allow only cardinal readings for weak quantifiers (McNally 1998), when a relative clause featuring an ILP is present, a strong, proportional reading is available. For example, (86a), like (86b), contains the ILP *like spinach*; and correspondingly, *few children* in (86a) readily allows a proportional reading equivalent to that in (86b), according to which it is a small proportion of all children (though not necessarily a small number in the absolute) who like spinach.

(86) a. There are few children who like spinach
b. Few children like spinach

As evidence that this is not an isolated phenomenon, observe that a similar pattern obtains with the weak quantifier *some*. Typically only weak unstressed *some* (‘*sm*)’ can occur in *there* sentences (87a) while strong stressed *some* cannot; but in the presence of an ILP in a relative clause (87b), stressed *some* is required:

(87) a. There are *sm/*some children in the yard
b. There are *sm/*some children who like spinach
As mentioned briefly above, the same pattern can also be observed in the attributive use of Q-adjectives. Thus few in (75), repeated below, has the same proportional interpretation that it does in (86):

(75) The few children who like spinach do not need vitamin supplements

Cases similar to (86a) have been discussed previously in the literature (see especially Herburger 1997), although the link to the ILP in the relative clause has not been explicitly made. It has not been universally accepted that such examples in fact represent a proportional interpretation of the Q-adjective, as opposed to simply reflecting context sensitivity in setting the standard of comparison (McNally 1998). But a further example supports the availability of proportional readings in there sentences and with attributive Q-adjectives; specifically, we can find cases in which few in these contexts must have a proportional interpretation (here, I again use few rather than many because the cardinal/proportional distinction is clearer in this case). For example, (88a) is false in a situation in which there are only a small number of French-speaking senators and all of them have blue eyes. I would argue that (88b) is also false in this situation, and that (88c) similarly introduces a secondary backgrounded proposition (cf. Section 3.7) that is likewise false, evidence that few is receiving a proportional interpretation:

(88) a. Few French-speaking senators have blue eyes
    b. There are few French-speaking senators who have blue eyes
    c. The few French speaking senators who have blue eyes are corrupt

As a further example, consider the figure and statements in (89):
(89)

a. Few circles are black
b. There are few circles that are black
c. The few circles that are black are above the line

Sentence (89a) is false in this situation. This is exactly what we would predict: *black* is an ILP, requiring a proportional reading for *few*; in the figure, however, all of the circles are black. But crucially, the speakers I have asked agree that (89b) is also false in this situation, for the same reason, and that (89c) is likewise infelicitous. By contrast, though the distinction is subtle, it seems marginally possible to get a reading for (90) that is true in this situation, the result of replacing the ILP *black* with the SLP *above the line*.

(90) There are few circles that are above the line

The consequence of these observations is that we cannot seek an explanation for the cardinal/proportional ambiguity that relies on the proportional reading being linked to traditional quantificational semantics (i.e. one in which Q-adjectives are of type \( \langle et, (et,t) \rangle \)). Rather, our account must capture the role of the predicate in this distinction, regardless of the position (quantificational, attributive, etc.) of the Q-adjective.

The above discussion described one context in which only a proportional reading is available to Q-adjectives. There is a second context where the same is true, namely the partitive construction, as in (91):
Here, \textit{many} and \textit{few} must be interpreted proportionally: a large/small proportion of the/our/those students came to the party. Again, we can use the \textit{few-as-all} test to demonstrate that a cardinal reading is not available. That is, if there are only a small number of the/our/those students, and they all came to the party, (91a-c) are false, evidence that in this context, \textit{few} is necessarily interpreted proportionally.

Finally, a similar effect can be observed without overt partitive syntax, in the case where the domain of quantification has been previously mentioned, or is salient in the context. Consider the following example:

\begin{enumerate}[(92)]
\item Few problems were found in the inspection
\end{enumerate}

Suppose that there is a house that I am considering purchasing, and I have just received the findings from a building inspection conducted as a condition of purchase. Uttered in this context, (92) would tend to have a cardinal interpretation: the number of problems found was small in the absolute, but it is not necessarily true that only a small proportion of all problems were found (in fact, one hopes this is not the case!). But now imagine that a year has gone by, during which time I have purchased the house and subsequently discovered there to be a much larger number of problems. Uttered in this context, (92) takes on a proportional interpretation: a small proportion of the contextually relevant set of problems (i.e. the problems I am now aware of) were found in the inspection. There is then an obvious parallel between (92) on the proportional interpretation and the corresponding overt partitive:

\begin{enumerate}[(93)]
\item Few of the problems were discovered in the inspection
\end{enumerate}
In fact, (93) is slightly awkward when a proportional interpretation is intended (as on the second scenario), precisely because an overt partitive is preferable in this case. For this reason, we can think of cases such as (92) on their proportional readings as covert partitives (a similar point is made by Doetjes 1997).

4.6.3 Totalizing

Having introduced the facts, let us turn now to how they may be accounted for. What I propose is that the two environments in which Q-adjectives must receive a proportional interpretation – the subject position of ILPs and the partitive – share a property in common, which I will call ‘totalizing’. The claim will be that the necessarily proportional reading arises when the totality of the domain of quantification enters into the semantic derivation, which I will argue has consequence for the structure of the scale invoked in the measurement of members of this domain.

The concept of totalizing, and the term itself, I borrow (with some adaptation) from Fiengo (2007), who makes use of this notion in the analysis of every and each, and of wh-words. Fiengo proposes that while sentences involving every and each have identical truth conditions (outside of intensional contexts), they differ in how those truth conditions are derived. To illustrate with an example, the verification of (94a) involves first gathering up the totality of dogs, and then determining whether barks applies to that totality. By contrast, the verification of (94b) involves going individual-by-individual through the universe and inquiring of each individual we find “is it a dog?” and if so, “does it bark?”, the sentence being true if all the entities identified as dogs also bark.

(94) a. Every dog barks
b. Each dog barks
In Fiengo’s terms, *every* is totalizing, while *each* is individualizing. This distinction provides an explanation for contrasts such as the following:

(95)  
\[ \begin{align*}
\text{a. } & \text{You have every prospect of success} \\
\text{b. } & \text{#You have each prospect of success}
\end{align*} \]

I might utter (95a) to express my belief that there are no limitations to your potential for success, no counterexample to a universal statement about your potential, a sentiment that can be expressed via the totalizing *every*. The oddness of (95b), by contrast, derives from the fact that its verification would involve going through the universe to identify individual prospects of success; but prospects of success do not seem to be the sorts of things that can be individuated and counted.

I propose that the two environments of interest in the present discussion likewise involve something like totalizing. In (96a), we begin with the totality of French-speaking senators, and then determine whether *few* is true of the number of these who are literate; in (96b), we start with the totality of the (contextually relevant) students and determine whether *many* is true of the number who came to the party.

(96)  
\[ \begin{align*}
\text{a. } & \text{Few French-speaking senators have blue eyes} \\
\text{b. } & \text{Many of the students came to the party}
\end{align*} \]

Relative to the situation with *every* and *each*, the difference is that here, the totalizing effect comes not from the Q-adjective itself but from the construction in which it occurs, in a way that I will describe below.

I further propose that totalizing has a consequence for measurement, in that the measurement scale introduced by the functional head Meas is restricted to measuring the extent of that totality. For example, the scale invoked in (96a) extends from zero (noninclusive) to the number that corresponds to the cardinality of the totality of French-
speaking senators. This is, I think, sensible from an intuitive perspective: if our starting point is the totality of such senators, then the only numbers of interest to us are numbers that correspond to the cardinalities of subsets of this group. My proposal is that this intuitive result can be derived formally.

Let me start by justifying the claim that these are totalizing environments. This point is most easily made about the partitive. As is well known (Jackendoff 1972; Ladusaw 1982), only definite DPs are allowed in partitives:

\[(97)\]
\[
\begin{array}{ll}
\text{a. Many/few of the/our/those students came to the party} \\
\text{b. *Many/few of some/twenty students came to the party}
\end{array}
\]

Definite descriptions are referring expressions, with plural definite descriptions such as those in (97a) denoting the maximal contextually relevant set meeting the description (here, students, our students and those students, respectively). That is, in the partitive, some totality is introduced explicitly, and the result is a proportional interpretation.

It might be asked whether the necessarily proportional interpretation of Q-adjectives in the partitive is due to the definite description itself, or whether it derives from other aspects of the partitive construction. This cannot be tested directly with many and few, since they occur only in true partitives with a definite DP. But a relevant piece of data is provided by expressions formed with a lot, which has semantic content similar to many/much, but which can occur in both true partitives (98a) as well as pseudopartitives with a bare plural (98b):

\[(98)\]
\[
\begin{array}{ll}
\text{a. A lot of the students came to the party} \\
\text{b. A lot of students came to the party}
\end{array}
\]

The (a) example above has only a proportional reading, corresponding to that in (97a). But the (b) example readily allows a cardinal reading: some large number of students
attended. Thus it is the introduction of some maximal set that triggers the proportional interpretation.

Let us turn now to the subject position of ILPs. The proposal that this too is a totalizing environment is in fact not far removed from the spirit of several classic analyses. In his original exploration of the stage-level/individual-level distinction, Carlson (1977) proposes that individual-level predicates take kinds as arguments. While Carlson does not conceptualize kinds as totalities of individuals, this link is made more explicitly in the neo-Carlsonian theory of Chierchia (1998b), for whom a kind in any given world can be associated with the totality of its instances in that word.

From a different perspective, Ladusaw (1994) draws on earlier work by Brentano (1874/1973) and especially Kuroda (1972) to propose that ILPs are predicates in categorical judgments. A categorical judgment is a two-stage judgment, consisting first of “the act of recognition of that which is to be made the subject” and secondly of “the act of affirming or denying that which is expressed by the predicate about the subject.” A categorical judgment is therefore presuppositional, in that “the mind of the judge must be directed first to an individual”, before a predicate can be associated with that individual (Ladusaw 1994, quoting Kuroda 1972:154). Categorical judgments are contrasted with thetic judgments, which simply state the existence of an individual or eventuality meeting a certain description. Ladusaw argues that among other consequences, this view explains why only strong DPs can be subjects of ILPs (a pattern referred to as Milsark’s generalization), the reason being that weak indefinites are inconsistent with the presupposition on the subject of an ILP.
Ladusaw also notes the parallel between the tripartite structure associated with strong quantifiers and the categorical judgment, in that the restrictor plays a role equivalent to that of the subject, and the scope a role equivalent to that of the predicate. This aligns well with the long-standing observation that the restrictors of strong quantifiers are presuppositional in nature, like the subjects of categorical judgments. On this view, the interpretation of an example such as (99a) involves first defining or ‘directing the mind to’ the domain of quantification (people) and then subjecting its members individually to the scope (left), a two-stage process parallel to the categorical judgment.

(99)  
a. Most people left  
b. Three people left  
c. Three people are over 7 feet tall

While strong quantifiers such as most must express categorical judgments, Ladusaw proposes that weak quantifiers (a group that includes many and few) are ambiguous between categorical and thetic judgments. For example, (99b) can be read as involving some set of students the speaker has in mind, three of whom are stated to have left (categorical); alternately, it can be read as simply describing an eventuality involving the leaving of three students, without any particular set of students presumed (thetic). This distinction, of course, corresponds to the distinction between strong and weak readings discussed earlier in this chapter.

Curiously, Ladusaw does not make an explicit connection between the predicate type and the mode of judgment applied to a weak quantifier in its subject position. But note that (99c), with an ILP, is odd unless we have some set of people in mind, of whom it is asserted that three are over 7 feet tall (i.e. the categorical mode of judgment). It thus
is natural to connect the two threads of Ladusaw’s account as follows: when the predicate is an ILP, a weak quantifier in subject position must be interpreted as taking part in a categorical judgment, in that attention is first drawn to the restrictor, and then this set is checked for the number of members that satisfy the predicate.

Let us now return to the notion of totalizing. Ladusaw’s proposal, and the extension suggested above, can be restated in these terms. If the predicate is an ILP, then a bare plural noun phrase in its subject position, or a plural NP serving as the restrictor of a weak quantifier in that position, is interpreted by first ‘pulling aside’ the totality of individuals in its extension, and then subjecting them to the predicate. The parallel to Fiengo’s account of every is clear. Thus the subject position of ILPs is a totalizing environment.\footnote{I will not attempt to explain why ILPs have this property. The individual-level/stage-level distinction was originally conceptualized as difference between more permanent and more temporary properties (Milsark 1977). As far as I can see, there is no \textit{prima facie} reason for this distinction to go hand-in-hand with a distinction between modes of judgment (in Ladusaw’s terminology) or in the requirement for totalizing (to use the present terminology). I leave the investigation of this link as a topic for the future.} I build on this conclusion below. Here for obvious reasons my focus will be on weak quantifiers, and especially Q-adjectives. But I believe this framework would also prove relevant to the analysis of generic readings for bare plurals in this position.

4.6.4 Meas with Totalizing and the Derivation of the Proportional Reading

Let me build on the observations of the preceding section to develop an analysis of the proportional reading of Q-adjectives within the present degree-based framework. I begin with the ILP environment, in an example such as the following

\[(100) \text{Few French-speaking senators have blue eyes}\]

\[
\text{LF: } [\text{POS-few}_{i} [\text{IP}[\text{DP}[\text{MeasP} t_{i} \text{ Meas French-speaking senators}]] \text{ have blue eyes}]]
\]

Since the subject position of have blue eyes is a totalizing environment, the calculation of the truth conditions of (100) must involve first ‘pulling aside’ the totality of
the group denoted by the plural NP, here, French-speaking senators. To capture this formally, I propose that this NP first enters the derivation as denoting a group, as in (101a). It then shifts to set type, but in doing so maintains its presuppositional character in the form of a domain restriction, as in (101b):

\[(101)\]
\[
\text{a. } [[\text{French-speaking senators}]] = \sup(\lambda x. *\text{Fr-spk-sen}(x))
\]
\[
\text{b. } [[\text{French-speaking senators}]] = \lambda y : y \sqsubseteq \sup(\lambda x. *\text{Fr-spk-sen}(x)). *\text{Fr-spk-sen}(y)
\]

In simple terms, the structure of the set expression in (101b) maintains the information that the members of the set are individual parts of some previously introduced totality.

At this stage, the nominal expression composes with the Meas head. Here, I propose that the presupposition on the plural nominal is transferred to Meas, where it is expressed as a restriction on the range of the measure function $\mu_{DIM}$. That is, as the domain of individuals that are input to the measure function is restricted to individual parts of the totality of French-speaking senators, so is the range of that function restricted to values less than or equal to the measure of that totality. We thus have the following (where $t_i$ is the trace of the raised Q-adjective, per the LF in (100)).

\[(102)\]
\[
\text{t}_i \text{ Meas French-speaking senators } = \lambda y : y \sqsubseteq \sup(\lambda x. *\text{Fr-spk-sen}(x)) \lambda d : d \leq \mu_{DIM}(\sup(\lambda x. *\text{Fr-spk-sen}(x))). *\text{Fr-spk-sen}(y) \land \mu_{DIM}(y) \geq d
\]

The result of the range restriction on the measure function is that the set of degrees under consideration is restricted to the number corresponding to the totality of French-speaking senators; that is, we effectively have a scale that is bounded on the upper end.

From here, the derivation may proceed in the usual way: the expression in (102) combines intersectively with the predicate blue eyes; the individual variable is then bound
by Existential Closure, and the Q-adjective takes as argument the set of degrees formed by lambda abstraction over its trace. The result is the following:

\[
\left[ (100) \right] = \text{1 iff } \\
\forall d \in \mathbb{N}_{\#} [\neg \exists y : y \subseteq \sup (\lambda x. *\text{Fr-spk-sen}(x))[*\text{Fr-spk-sen}(y) \land \mu_{\#}(y) > d \land \text{blue-eyes}(y)],
\]

where the domain of degrees d is restricted to \( \mu_{\#}(\sup (\lambda x. *\text{Fr-spk-sen}(x))) \)

While I will have more to say below about the location of the neutral range \( \mathbb{N}_{\#} \) in this situation, note first of all that it must fall in the range of values from 0 to the number of French-speaking senators, since degrees exceeding this range have been excluded from consideration. The situation is thus that depicted in (104), where the scale has an upper bound. This necessarily corresponds to a proportional reading:

\[
(104) \quad \text{# of French-speaking senators who have blue eyes} \\
\text{# French-speaking senators} \\
\text{\# of French-speaking senators who are at the meeting}
\]

Let me now contrast the case involving an ILP with that involving an SLP. Here, there is no requirement that a totality be introduced into the semantic derivation, and therefore no restriction on the domain of degrees is derived. An example such as (105) thus receives the interpretation in (106), corresponding to the situation depicted in (107):

\[
(105) \quad \text{Few French-speaking senators are at this meeting.} \\
(106) \quad \left[ (105) \right] = \text{1 iff } \forall d \in \mathbb{N}_{\#} [\neg \exists x[*\text{Fr-spk-sen}(x) \land \mu_{\#}(x) > d \land \text{at}(x, \text{meeting})]]
\]

\[
(107) \quad \text{# of French-speaking senators who are at the meeting}
\]

This diagram in (107) depicts a cardinal reading. There is no indication of what point on the scale corresponds to the number of the totality of French-speaking senators; thus the
picture here is entirely compatible with the situation in which all French-speaking senators (a small group in total) are at the meeting.

Recall that examples such as (105), with an SLP, also allow proportional readings. There are two ways in which this could arise. The first has to do with the selection of the neutral range. On the present account, (105) is true if the number of French-speaking senators who attended the meeting falls short of some neutral range \( N_\# \).

As was discussed in the preceding section, \( N_\# \) is centered around a point of comparison \( p_c \), which can be selected in any number of ways. Suppose that \( p_c \) is set to some number much less than the total number of French speaking senators (say, the number of that group who attended our last meeting; or the number who we saw at the bar last night; or whatever), a very reasonable possibility. Then the resulting truth conditions will entail that a small proportion of all French-speaking senators attended the meeting, without the need for a bounded scale. But I believe there is also another mechanism in play. As was pointed out in the discussion of the home inspection example in Section 4.6.1 (few problems were discovered in the inspection), we tend to get a proportional reading when the domain of quantification is some set that we have in mind, or that has been part of the preceding discourse (resulting in a covert partitive). The implication is that a totality that is present in the discourse may also serve to limit the domain of the measure function, in the same way that one introduced by a totalizing construction can do.

The present analysis of the proportional reading of Q-adjectives in their quantificational use differs crucially from those of Partee (1989) and Diesing (1992), in that quantification over individuals still results from an operation of Existential Closure. There is no need to posit a separate lexical entry with quantificational semantics for
proportional *many* and *few*. Rather, the distinction between cardinal and proportional readings derives from a distinction in whether or not counting (and the judgment of quantities) makes reference to some pre-established upper bound that represents the cardinality of some totality.

This approach then leads us to expect that we will find proportional readings in non-quantificational uses of Q-adjectives, and as discussed earlier in this section, this is precisely what we do see. For example, (108), like (100), must be read proportionally; in the situation in which there are only a small number of French-speaking senators and they all have blue eyes, the sentence must be judged false.

(108) There are few French-speaking senators who have blue eyes

As I suggested earlier, I take the requirement for a proportional reading here to have precisely the same origin as it does in the quantificational example discussed above, namely the totalizing effect of the ILP in the relative clause.

I take (108) to have the LF in (109), where the Q-adjective *few* has raised to take sentential scope (just as in the quantificational case).

(109) [POS-few] [there are [DP[Measp ti Meas [NP French-speaking senators who have blue eyes]]]]

Let us follow the standard assumption that the relative clause has set-type semantics, as in (110a). Then this expression can combine intersectively with a nominal expression. But I propose that the totalizing effect of the ILP is in force in this environment as well, in that the derivation must proceed by first pulling aside the totality denoted by that nominal expression (here, French-speaking senators). Thus the NP in (108) acquires the same presuppositional interpretation given in (101a,b), yielding (110b):

(110) a. ［who have blue eyes］ = λx.blue-eyes(x)
b. \[ \text{[\text{NP French-speaking senators who have blue eyes}] = } \lambda y : y \sqsubseteq \text{sup}(\lambda x. \text{Fr-spk-sen}(x)).\text{Fr-spk-sen}(y) \land \text{blue-eyes}(y) \]

When the NP combines with Meas, this presupposition again translates into a range restriction on the measure function, restricting the set of degrees under consideration to those less than or equal to the measure of the totality of French-speaking senators.

Taking *there be*, for simplicity, to introduce existential quantification, the resulting truth conditions for (108) are identical to those derived in (103) for the quantificational case, which necessarily yields a proportional interpretation.

Finally, let us take a look at the partitive construction. As discussed above, the partitive involves the explicit introduction of a totality, and as such it is not surprising that we observe the same effects on scale structure (and resulting interpretive effects) that we do in the totalizing environment created by ILPs. Without attempting a detailed discussion of the syntax or semantics of the partitive (see Ladusaw 1982; Barker 1998; Ionin, Matushansky & Ruys 2006), I take (111) to be the structure of a relevant example:

(111) Few of the students went to the party

\[
\text{SS: [IP[DP[MeasP POS-few Meas [XP of [DP the students]]]] went to the party]}
\]

\[
\text{LF: [POS-few, [IP[DP[MeasP t, Meas [XP of [DP the students]]]] went to the party]]}
\]

I follow Ionin, Matushansky & Ruys (2006) in taking partitive *of* to map the plurality denoted by the definite DP (in this case, *the students*) to the set of entities that are individual parts of that plurality. I further take the presupposition on that plurality to be passed up to the set level as a domain restriction (much as in the case of the totalizing cases above), yielding the following as the denotation of the XP *of the students*.

(112) \[ \text{[of the students] = } \lambda y : y \sqsubseteq \text{the-students.} \star \text{student}(y) \]
Once again, on combination with Meas, the result is a range restriction on the measure function. Thus here too we end up with a scale that is bounded on the upper end by the number corresponding to the cardinality of the totality of students; the result is a necessarily proportional reading.

To summarize, in each of the cases discussed here (quantificational, there-insertion and partitive), the presence of a totalizing environment has the effect of constraining the set of degrees under consideration to just those less than or equal to the measure of the totality. The measurement scale is thus in effect bounded on the upper end, with the effect that a proportional reading necessarily obtains. Below, I will conclude this section with a brief discussion of how $N_\#$ is set in the proportional case.

4.6.5 $N_\#$ and Bounded Scales

The first point to note about the location of the neutral range in the bounded scale case is that it must be on the bounded segment of the scale, since these are the only values of cardinality under consideration. Beyond this, examples such as the following indicate that the same possibilities are available for $p_c$ in the proportional (bounded scale) cases as in the cardinal (unbounded scale) cases:

(113)  a. Few Americans speak Finnish, as compared to the number of Europeans who do
b. Few Americans speak Finnish, as compared to French
c. Compared to what we predicted, few Americans speak Finnish
d. Compared to the situation fifty years ago, today few Americans speak Finnish

Here we have points of comparison established with reference to an alternative to the restrictor (113a), an alternative to the scope (113b), an alternate possible world (113c), and an alternate time (113d); compare these to (59), which show the same options are possible in the cardinal case.
But on the proportional reading, where the scale is bounded on both ends, there is another possibility, namely that percentages may serve as points of comparison. We see this explicitly in examples such as the following:

(114) Few Americans speak French, as compared to the percentage of Germans who do

Furthermore, there even seems to be a sort of default possibility for the percentage that can serve as point of comparison on a bounded scale. Considered the following, where we have no contextual expectations as to the proportion of squares that are red:

(115) a. Many of these squares are red
    b. Few of these squares are red

With nothing else go on, there would, I believe, be an inclination to judge (115a) as true if the proportion of the relevant squares that are red at least approaches the halfway point on the scale; conversely, (115b) would be true if the proportion that are red is well short of half. This would suggest that something around 1/3 represents a default point of comparison on a bounded scale. We might think of a neutral range around this point as dividing the scale into those values that are ‘close to’ zero and those that are not.

In fact, in a bounded scale situation, it is difficult to interpret the neutral range as being far removed from this general area, unless this is in some way explicitly signaled. For example, (116a) is not really an appropriate way to describe a situation in which 70% of the students in my class are right-handed (compared to the expected 90%); rather, we need to say something along the lines of (116b-d):

(116) a. Few students in my class are right handed
    b. Relatively few students in my class are right handed
    c. Compared to what you’d expect, few students in my class are right handed
    d. Fewer students in my class than you’d expect are right handed
The use of *relatively* in (116b) is intriguing, since Q-adjectives are inherently interpreted relatively. Its use seems to signal that the standard of comparison is different from what one might expect by default, though the nature of its effect is somewhat murky. This would be an interesting topic for further exploration.

To summarize, the neutral range on a bounded scale is set in essentially the same manner as in the unbounded-scale case (as a range of values around some point of comparison). The chief differences are that the neutral range must be on the bounded portion of the scale, and the existence of an upper bound on the scale has the further effect of establishing a default option for the neutral range.

### 4.7 The Reverse Reading

In concluding this chapter, let me briefly return to a discussion of the so-called reverse reading of *many* and *few*, with the goal of showing that the degree-based analysis developed here yields insights into its correct analysis. As noted earlier, the classic example is the following, first discussed by Westerståhl (1985):

(117)  

a. Many Scandinavians have won the Nobel Prize in Literature  
b. Many winners of the Nobel Prize in Literature are Scandinavian

Westerståhl notes that (117a) has a reading that is equivalent to (117b), which would be judged true if Scandinavians make up a (relatively) large proportion of all winners. Suppose that 14 of out of the 81 Literature prizes awarded to date have been won by Scandinavians (the actual situation at the time of Westerståhl’s writing in 1985). While 14 is not a particularly large number of Scandinavians, and certainly not a large proportion of all Scandinavians, we are nonetheless tempted to judge (117a) true in this
situation, since a large (or rather, larger than expected) proportion of all winners are Scandinavian.

As noted earlier, the existence of the reverse reading is problematic for a GQT analysis of Q-adjectives, in that it violates conservativity. This is not an issue on the present account, in that Q-adjectives are not analyzed as quantifying determiners but rather as degree predicates. Nonetheless, it is of interest to inquire how the reverse reading arises.

One prominent account of these facts is that of Herburger (1997), who dubs this interpretation the focus-affected reading, noting that it tends to arise when the NP sister of many or few is focused. For example, (118a), with cooks focused, is interpreted as paraphrased in (118b):

\[(118)\]
\[
(118)\]
\[a. \text{ Few COOKS applied} \]
\[b. \text{ ‘A small proportion of those who applied were cooks’} \]

A similar effect obtains when a smaller constituent within the NP is focused; for example, Herburger suggests that (119a) has the interpretation paraphrased in (119b):

\[(119)\]
\[
(119)\]
\[a. \text{ Few INCOMPETENT cooks applied} \]
\[b. \text{ ‘A small proportion of the cooks who applied were incompetent} \]

On Herburger’s analysis, these facts are accounted for by allowing flexibility in the mapping between syntactic structure and semantic structure, with focus playing a crucial role in this interface. Specifically, Herburger proposes that the non-focused material is mapped to the quantifier’s restrictor (the first argument), leaving the focused material to constitute the scope (the second argument). On this analysis, (118a) and (119a) have the logical forms in shown below (expressed in the tripartite quantificational structures of Diesing 1992):
(120) a. [Few x: applied(x)][cook(x)]
b. [Few x: cook(x) ∧ applied(x)][incompetent(x)]

Other authors (Büring 1995; de Hoop & Solà 1996; Cohen 2001) have discussed examples that demonstrate that Herburger’s analysis cannot be maintained in the form stated, in that focal stress is neither necessary nor sufficient to produce a reverse reading.

Büring gives a different account of the role of focus in cases such as (119a). Drawing on Rooth’s (1985) analysis of focus as introducing sets of alternatives, Büring proposes that an example such as (121a) is interpreted as in (121b), or more simply, (121c):

(121) a. Few INCOMPETENT cooks applied
b. ‘It is incompetent, rather than any of the alternatives to incompetent, that makes it true that the number of such cooks that applied was smaller than expected’
c. ‘It is incompetent, rather than competent, that makes it true that the number of such cooks that applied was smaller than expected’

Although Büring himself does not comment on this, it is worth pointing out that if we assume that the same standard of comparison is invoked in judging the numbers of competent and incompetent cooks who applied, then Herburger’s analysis of the reverse reading follows from (121c). That is, if it is the case that few incompetent cooks applied, and it is not the case that few competent cooks applied, then the number of competent cooks applying must exceed the number of incompetent cooks applying. From this it in turn follows that the proportion of cooks applying who were incompetent is less than half. This is essentially equivalent to Herburger’s paraphrase in (119b), though on Büring’s account this is pragmatically derived, rather that being part of the truth conditions of the sentence in question.

Let us consider again the original Westerståhl example (117a). On closer consideration, it becomes apparent that this is a different sort of animal from Herburger’s
focus-affected example (118a); for one, as noted by Büring, Scandinavians does not need to yield the apparent reverse reading. Here, an interesting observation is made by Cohen (2001), who suggests that Herburger’s analysis does not adequately capture the truth conditions of this example. He notes a contrast between the following:

(122)  

a. Many Scandinavians have won the Nobel Prize in Literature  
b. Many Andorrans have won the Nobel Prize in Literature

Cohen argues that as few as two or three Andorran winners would be sufficient to establish the truth of (122b), whereas the same number of Scandinavian winners would not be enough for (122a) to be judged true, the difference being that there are many fewer Andorrans (roughly 60,000) than Scandinavians (roughly 25 million). Now, I do not agree with Cohen’s specific claim that two or three Andorrans would count as many Andorrans (or more generally, that two or three Ns of any sort could under any circumstances be considered many Ns). But the general point is, I believe, correct: the size of the restrictor set (here, Scandinavians or Andorrans) is relevant to the truth or falsity of examples such as these. In fact, this is implicit in Westerståhl’s original example. If it is the case that 14 out of 81 winners of the Literature prize are Scandinavian, that proportion is just 17% - certainly not a large proportion in an absolute sense. But what justifies the use of many here is that 17% is a much greater proportion of Scandinavian winners than we would expect, given the proportion that Scandinavians make up of the total world population (or perhaps, of the population of countries that we might expect to contribute Nobel prize winners).

Cohen’s own analysis of the reverse reading is to treat it as a version of the proportional reading, which he calls the relative proportional reading. On this analysis, (117a) is true if the following obtains:
That is, (117a) is true if the proportion of Scandinavians who have won the Literature prize exceeds the proportion of individuals of all nationalities who have won the prize.

In the form stated, Cohen’s analysis is, I think, implausible, for two reasons. If the reverse reading is a variant of the proportional reading, as Cohen proposes, we would expect it to have the same distribution as the latter. But as Herburger points out, reverse readings are only possible in contexts that allow cardinal readings, and in particular do not arise in contexts that require a proportional interpretation, the paradigm case of this being the subject position of ILPs. For example, as noted by Herburger, a reverse reading does not seem to be available for either the (a) or the (b) example below:

(124)  a. Few COOKS know how to make a soufflé  
       b. Many Scandinavians are Nobel Prize winners

At a more basic level, I do not believe that the formula in (123) reflects what we are doing when we assess a sentence such as (117a). Though differing by more than an order of magnitude, the proportion of Scandinavians who have won the Literature prize and the proportion of individuals of all nationalities who have done so are both miniscule (by my calculations, $5.6 \times 10^{-7}$ and $1.0 \times 10^{-8}$, respectively). The average individual is not terribly good at comparing very small percentages (witness the difficulties in assessing the relative risks of dying in a plane crash or a car accident); but despite this, given the facts as we have laid them out, it is quite straightforward to judge (117a) true.

However, a slight modification to Cohen’s formula yields an equivalent formula that is much more plausible:
That is, (117a) is true if the number of Scandinavians who have won the prize exceeds the number we get by taking the total number of prize winners and multiplying it by the proportion of Scandinavians in the world’s population. But this is just a variant of the cardinal reading in which the number of winners and the relative size of the population of Scandinavia are taken into consideration. This interpretation can be readily accommodated within the present account if we take the expression on the right side of the equation in (125) as the point of comparison $p_c$:

$\text{(126)} \quad \left[\text{many Scandinavians have won the Nobel Prize in Literature}\right] = 1 \text{ iff } \{d: p_c - d \text{ is not significant}\} \subset \{d: \exists x[\text{Scand}(x) \wedge \text{won-Nobel-Lit}(x) \wedge \mu(x) \geq d]\}$

where $p_c = \frac{|\text{Scandinavians}|}{|\text{All-Nationalities}|} \times |\text{All-Nationalities} \cap \text{Winners}|$

The representation in (126) in fact does a better job of capturing the truth conditions of the given sentence than that in (123) (or even 125)). I would argue that the number of Scandinavian winners must exceed the given point of comparison by some amount for (117a) to be judge true (just as in any case involving the positive forms of many and few). That is, the simple ‘greater than’ relationship reflected in (125) is not quite right. More significantly, taking the given value as the point of comparison is entirely consistent with the previous discussion on standard setting. How many Scandinavians might I expect to have won the Nobel Prize in Literature? In the absence of other information, a reasonable guess would be that they would be represented among prize winners in proportion to their presence in the world population as a whole. That is precisely what is
reflected in (126). In other words, the reverse reading is fully consistent with the analysis of the positive forms of *many* and *few* that I have developed in this chapter.

But a question also arises in this context. On the present account, the proportional reading of Q-adjectives differs from the cardinal reading only in making reference to a scale that is bounded rather than unbounded. We might then ask why the reverse reading seems to be absent in contexts that require a proportional interpretation. I would like to propose that this is entirely expected given the above-described perspective on the reverse reading, and the structure of the theory of standard-setting I have developed. Let us return to the examples purported not to allow a reverse reading:

(127) a. Few COOKS know how to make a soufflé
    b. Many Scandinavians are Nobel Prize winters

I will consider the two cases separately, since as discussed above they represent different phenomena. On Büring’s pragmatic analysis, the effect of focus in (127a) would be to give it the interpretation paraphrased in (128), which strikes me as more or less right:

(128) ‘It is *cooks*, rather than *non-cooks*, that makes it true that the number of such individuals who know how to make a soufflé is smaller than expected’

But since the example in (127a) features an ILP, *few* is interpreted proportionally, in that the number of individual of a certain sort (e.g. cooks, non-cooks) who know how to make a soufflé is evaluated with respect to a neutral range that is based on a scale bounded on the upper end by the cardinality of that group. Thus (128) might more aptly say ‘it is *cooks*, rather than *non-cooks*, that makes it true that the proportion of such individuals who know how to make a soufflé is smaller than expected’. From this it might follow that a smaller proportion of cooks than non-cooks know how to make a soufflé. But without knowing the ratio of cooks to non-cooks, we cannot deduce what proportion of
all individuals who know how to make a soufflé are cooks. (I leave it to the reader do the simple mathematics to prove this to her/himself.) In particular we do not get the implication that cooks make up a small proportion of the individuals who know how to make a soufflé (i.e. the reverse reading). Thus focus in (127a) has the same effect that it does in (118a) and (119a), but the mechanisms of interpretation in the bounded-scale situation do not allow an apparent reverse reading to be derived.

Now let us look at (127b), where focus does not play a crucial role. Here again we have an ILP, and therefore a situation in which the scale is bounded at the upper end by the total number of Scandinavians. Recall from the discussion in Section 4.6 that in the bounded-scale case, there is a default location for the neutral range (somewhere around the 1/3 point), and relevant examples are not readily interpreted as invoking a neutral range very far from this default, unless overtly signaled. Now observe what would be the case if we extended the approach to standard setting in (126) to the present case. We would derive a neutral zone that was centered around a point equal to the total number of Nobel prize winners multiplied by the proportion of Scandinavians in the world population. But this number represents far less than even 1% of Scandinavians, resulting in a neutral range at the very low end of the relevant scale, and in particular very distant from the default neutral range. I propose that the unavailability of this as a possibility for the neutral range is what blocks the apparent reverse reading for (127b). In this context, observe that the modified examples in (129) are perhaps awkward, but I believe interpretable as true in the situation previously described:

(129)  
a. Relatively many Scandinavians are Nobel Prize winners  
b. Compared to what you would expect (given how many such prizes have been awarded), many Scandinavians are Nobel Prize winners  
c. Compared to Spaniards, many Scandinavians are Nobel Prize winners
Thus when it is explicitly signaled that the standard of comparison is not what one would expect by default, it may be set in a way that allows a very small proportion to be many (which is what would be required to yield the proportional counterpart of a reverse reading).

To conclude, the present degree-based theory allows the derivation of reverse readings by the same mechanism of standard setting utilized more broadly, and also provides insight into the distribution of this reading. I have also highlighted the existence of a previously unnoticed parallel phenomenon in cases requiring a proportional reading of Q-adjectives, though one that does not give the appearance of a reverse reading.

4.8 Conclusions

The goal of this chapter was to show that the degree-predicate theory developed in this work is able to accommodate the range of interpretations available to Q-adjectives, and further to show that this theory provides a useful framework within which to investigate a number of known problems in this area. I approached this question by breaking down the interpretive variability of Q-adjectives into two dimensions: their vagueness (which positions them with gradable adjectives such as tall and expensive) and their apparent ambiguity (which represents one aspect of a broader pattern observed with other so-called weak quantifiers and bare plurals).

The vagueness of Q-adjectives has been addressed within a framework developed in the study of gradable adjectives, in which the interpretation of gradable expressions in their positive form arises via composition with a null positive morpheme POS. This approach of course has the nice consequence of unifying the analysis of the two sorts of gradable expressions. What is more significant is what we are able to learn about the
semantics of Q-adjectives, and about vagueness more generally, by approaching the interpretation of Q-adjectives within this framework. This work supports the view that the standard of comparison invoked by gradable expressions in their positive forms should be conceptualized as a range rather than a point, and further yields insights into how this range is set, with particular reference to the role of an implicit point of comparison or comparison class. Furthermore, the otherwise puzzling reverse reading of Q-adjectives emerges as the consequence of one natural possibility for the location of the neutral range.

It has further been shown that the distinction between so-called cardinal and proportional interpretations of Q-adjectives arises as a result of the interaction of the standard-setting mechanism described above with the structure of the measurement scale referenced. The proportional reading arises in what I have called totalizing environments, where some totality of individuals enters into the semantic derivation. Totalizing has the effect of restricting the range of the measure function introduced by the functional head Meas to produce a scale that is bounded on the upper end by the measure of the relevant totality; constraints on how the neutral range may be set on a bounded scale produce a necessarily proportional interpretation. This analysis has the benefit of unifying the treatment of the two environments that allow only a proportional interpretation, namely the subject position of ILPs and the partitive, while also allowing for the availability of proportional readings in the ‘non-quantificational’ uses of Q-adjectives. The proposal developed here also provides an explanation for the truth-conditional effect of the cardinal/proportional distinction, in contrast to the non-truth conditional nature of the strong/weak ambiguity found with other weak quantifiers: the
effect relates to the location of the neutral range, which is necessarily tied to the positive forms of Q-adjectives.

The general conclusions from this exploration of the interpretation of Q-adjectives thus support the relevance of scale structure to the understanding of natural language meaning. This theme will be continued in Chapter 5, where I show that issues of scale structure also can account for a completely different set of facts, namely the contrast between *a few and *a many.
Chapter 5
*A Few* and Related Matters

5.0 A Curious Contrast

The third puzzle that I introduced in Chapter 1 relates to the contrasts in distribution among individual Q-adjectives. The starting point for this chapter is one such often-overlooked contrast, exemplified below:

(1) a. Few students came to the party  
    b. A few students came to the party

(2) a. Many students came to the party  
    b. *A many students came to the party

While the issues surrounding the distribution and (especially) the interpretation of *many* and *few* have been well studied, one idiosyncrasy of *few* that has received little serious attention is that it forms a pair with the superficially similar expression *a few*, the only such pair in the English count noun quantifier system. In particular, while *few* and *many* otherwise exhibit very similar properties, there is no *a many* in parallel to *a few*.

In the mass domain, we similarly see a contrast between the grammatical (and entirely colloquial) *a little* and the ungrammatical *a much*.

(3) a. There is a little water in the bucket  
    b. *There is a much water in the bucket

It seems reasonable to suspect a common explanation in the two cases.

My goal in this chapter is to provide an account of the contrast exemplified in (1)-(3), and some related matters, and in doing so to explore the broader implications for the semantics of quantity expressions.
An immediate question that comes up in this context is whether there is a compositional relationship between *few* and *a few* (and similarly between *little* and *a little*). That is, is *a few* in some way derived from *few*? Within basic accounts of generalized quantifiers (e.g., Keenan & Stavi 1986) as well as introductory semantics texts (e.g., Gamut 1990), the standard if unspoken assumption would seem to be that *a few* is an idiom, that is, a fixed, unanalyzable unit. But on closer examination, it is clear that *a few* does not always function as a unit: *a* and *few* may be separated by an adverb (as in (4)) or even by an adjective modifying the head noun (as in (5)):

(4)  a. A very few students got perfect scores on the test  
     b. An incredibly few collectors have the good fortune to own one

(5)  a. A lucky few students will get fellowships  
     b. We spent a happy few days at John’s house in the country

The conclusion must be that *a few* is composed of an independent *a* and *few* which combine in the syntax; in light of this, a compositional semantic treatment is desirable as well.

As further evidence of the close relationship between *few* and *a few*, *a few* occurs in some contexts where we would expect *few*. For example, Klima (1964) notes that while *few* is typically interchangeable with *not many*, in certain contexts *not many* corresponds instead to *a few*:

(6)  a. Not many/few students came to the party  
     b. Not many/*(a) few years ago, I lived in Paris

In (6b), it is *a few*, not *few*, that seems to serve as the opposite of *many*.

Particularly telling evidence for the relationship between *few* and *a few* comes from the following data:
(7)  
a.  Not every student passed the test
b.  Not many students passed the test
c.  Not a student passed the test
d. Not twenty visitors came to the museum
e. Not a dozen visitors came to the museum
f.  Not five minutes later, the professor walked in
g.  Not a few students passed the test

In (7a-f), *not* + quantifier + N specifies a *smaller* number than quantifier + N. That is, *not many students* is *fewer* than many students, *not five minutes later* is *less* than five minutes later; *not twenty/not a dozen visitors* is *fewer* than twenty/a dozen visitors; and so forth. But curiously, in (7g) *not a few students* is *more* than a few students. In fact, *not a few* has precisely the interpretation that we would expect from *not few*. We might capture this interpretive fact by saying that in *not a few*, *not* is able to negate an underlying *few* that is part of the semantic content of *a few*. But this means that at some level, *a few* must contain *few*.

The conclusion we are led to is that *a few* is not a separate quantifier independent of *few*; rather, *a few* is in some way derived from *few*, in a way that leaves the underlying *few* available to interact with other sentential elements. But there is not, to my knowledge, a semantic account that is able to capture this relationship. My goal in the present chapter is to give such an account. Below, I show that the degree-based theory of Q-adjectives that I have developed in this work allows *a few* to be derived compositionally from *few*. I further show that a basic fact about the structure of the scale of cardinality, namely that it has a lower but not an upper bound, blocks the corresponding derivation of *a many*. Thus this analysis will further support the relevance of scale structure to the distribution and interpretation of quantity expressions.

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28 It is a perhaps related fact that *not few* (as in *not few students came to the party*) is ungrammatical. I do not at this point have an account for this gap.
The outline of this chapter is the following: Section 5.1 introduces some basic facts in the interpretation of *a few*, and its contrasts to *few*, that must be accounted for. Section 5.2 reviews what little has been said to date about this topic, with a focus on Kayne (2005). Section 5.3 develops an analysis of the facts. Some broader issues and extensions are discussed in Section 5.4, and conclusions are summarized in Section 5.5.

5.1 The Basics of *Few* and *A Few*

I begin with some basics about *a few*, with a focus on relating its distribution and interpretation to the patterns previously discussed for *few*. Recall first that *few* has ‘at most’ semantics: it is defined by its upper bound, and tests shows that it allows zero as a possibility (cf. Chapter 1). For example, (8a) may be followed by one of the continuations in (8b), but not those in (8c), evidence that the former but not the latter values are encompassed within the range of values describable as ‘few’:

(8) a. Few students wrote great papers; in fact…
   b. …hardly any/only three/fewer than five/only one/none did
   c. …*lots/*dozens/*more than twenty did

By contrast, *a few* is existential, and at least marginally allows an ‘at least’ reading. As evidence, (9a) cannot be followed by (9c), showing that *a few* cannot be none (or for that matter, one), but the continuations in (9b) are acceptable, evidence of ‘at least’ semantics (see Horn 1989 for related discussion):

(9) a. A few students wrote great papers; in fact…
   b. …lots/dozens/many/more than twenty did
   c. …*none/* one did

In simple terms, we might capture these observations by paraphrasing *few* as ‘at most a small number’ and *a few* as ‘at least a small number’. The two expressions therefore have essentially mirror image semantics.
In keeping with this, *few* and *a few* exhibit opposite formal properties. While *few* is monotone decreasing, and thus licenses negative polarity items, *a few* is monotone increasing, and does not license NPIs.

(10) a. Few students in the class own cars ⇒ Few students in the class own red cars
b. Few students in the class have ever owned a car

(11) a. A few students in the class own red cars ⇒ A few students in the class own cars
b. *A few students in the class have ever owned a car

Similarly, while *few* exhibits behavior characteristic of sentential negation (cf. Chapter 2), *a few* does not. For example, sentences with *few* (like other negative sentences) take *either* tags, but those with *a few* take *too* tags (like other positive sentences):

(12) a. The students didn’t come to the party, and the professors didn’t either/*too
b. No students came to the party, and no professors did either/*too
c. Few students came to the party, and few professors did either/*too
d. Many students came to the party, and many professors did too/*either
e. A few students came to the party, and a few professors did too/*either

But while *few* and *a few* in these respects act something like opposites of one another, there is a deeper difference between them. Specifically, although *a few* is in its way vague, it does not exhibit the context sensitivity seen with *few*, and in particularly has an exclusively cardinal (rather than proportional) reading. For example, the intuition seems to be that *few Americans* in (13a) could refer to a larger number of individuals than *few senators* in (13b), which in turn could be a larger number than *few students in my class* in (13c) (assuming a class of fifteen students or so):

(13) a. Few Americans voted for Ralph Nader in 2004
b. Few senators support the bill
c. Few students in my class speak French
In fact, (13a) is clearly true – and perfectly felicitous – in a situation where one hundred thousand Americans (out of millions) voted for Nader in 2004, evidence that few is interpreted proportionally. These facts can be captured with the previously introduced mechanism by which proportional readings for Q-adjectives are derived (cf. Chapter 4). By contrast, regardless of the context or the nominal expression with which it combines, a few specifies a small number of individuals in an absolute sense. Thus (14a-c) could all be judged true if the predicate was true of three or four individuals within the domain (Americans, senators or students in my class, respectively):

(14) a. A few Americans voted for Ralph Nader in 2004  
   b. A few senators support the bill  
   c. A few students in my class speak French

Furthermore, in the situation in which one hundred thousand Americans voted for Nader in 2004, (14a) is pragmatically odd if not actually untrue. The issue is one of informativeness (Grice 1975); if the speaker knows that the true number is in the thousands, then it would be massively underinformative to choose a few to describe the situation, just as it would be underinformative to use, say, five. But this can only be the case if a few is necessarily interpreted as a small number in the absolute, rather than relative, sense.

To put this differently, the few in a few could be considered ‘context insensitive’, in that regardless of the context it always seems to mean something like (at least) three or four. As evidence that this is not an idiosyncrasy of the collocation a few, note that there are other constructions in which few likewise receives a context-insensitive interpretation. For example, regardless of the context, (15a) must mean that the river floods every three to four years or so. We cannot create a context in which every few years means (say)
every hundred years, even if we are for example talking about time spans in millennia. A
similar point can be made about examples such as (15b,c):

(15) a. The river floods every few years
    b. The first few callers won tickets to the concert
    c. The same few students always come in late

Interestingly, there is a further commonality between the constructions exemplified in
(15) and a few, namely that we cannot replace few with many (as noted by Kayne 2005):

(16) a. *The river floods every many years
    b. *The first many callers won tickets to the concert
    c. *The same many students always come in late

Thus there is a few versus *many contrast that goes hand-in-hand with a context-
insensitive interpretation on the part of few.

The intuition that a few means something like ‘three or four’ regardless of the
context suggests that a few is in a sense a ‘fuzzy’ cardinal numeral. This is reinforced by
several further parallels between a few and cardinals. For example, a few does not
readily occur in predicative position; in this its behavior diverges from that of few, and
patterns instead with cardinal numerals:

(17) a. *Visitors to the new museum were a few
    b. *Visitors to the new museum were twenty
    c. Visitors to the new museum were few

A few is also like cardinal numerals in allowing a collective or cumulative interpretation
for the noun phrase; this is not possible with few. For example, (18a) and (18b) can be
interpreted collectively: it is a group of a few/three men who (together) lifted the rock.
But (18c) has only a distributive interpretation: the number of men who individually
lifted the rock was small.
(18)  a. A few men lifted the rock  
    b. Three men lifted the rock  
    c. Few men lifted the rock

Similarly, (19a) and (19b) can be interpreted cumulatively, as asserting that a few/three potatoes in total are enough to make a soup; (19c), by contrast, can only mean that few individual potatoes are sufficient for soup-making purposes:

(19)  a. A few potatoes are enough to make a soup  
    b. Three potatoes are enough to make a soup  
    c. Few potatoes are enough to make a soup

Finally, the ‘at least’ reading that characterizes a few is also parallel to that commonly ascribed to cardinal numerals (in that I have three dogs means, at the semantic level, that I have at least three dogs).

In summary, while we have concluded that a few is based on few, its behavior is that of a cardinal numeral, not an ordinary Q-adjective.

5.2 Previous Work

To my knowledge, the only attempt in the semantics literature to establish a compositional relationship between the semantics of few and a few is found in Barwise & Cooper (1981). B&C note that if we interpreted a few to mean ‘exactly a few’, the set of properties denoted by a few N is precisely the intersection of the sets denoted by some N and few N. They do not, as far as I can tell, intend this as a formal analysis of a few, and in any case it is unsatisfactory in that it does not capture the ‘at least a few’ reading that I have argued is required (and that B&C themselves indicate is the one that should be accounted for in the semantics). But I take this to be the first step towards the goal that I set out to accomplish in this chapter.
More notice has been paid to the pair *few* and *a few* in the syntactic literature. Jackendoff (1977) classifies the *few* in *a few* as a seminumeral (like *dozen* or *hundred*), and thus a noun. It is therefore distinct from ordinary *few*, which is a quantifier. However, Jackendoff acknowledges that this view cannot account for the grammaticality of *a very few*, where the purportedly nominal *few* is modified by a quantifier modifier.

Kayne (2005), which I have discussed in previous chapters, takes a more in-depth look at *few* and *a few*. Recall that Kayne proposes that *many* and *few* are modifiers of an unpronounced noun NUMBER, similar to the overt noun *number* (with *much* and *little* correspondingly modifiers of an unpronounced AMOUNT). A sentence such as (20a) thus has the actual form (20b), where the upper case indicates an element that is not phonologically realized.

(20)  
a. John has few books  
b. John has few NUMBER books

Kayne goes on to point out the contrast between (21), where *a* is required, and (20a), which is grammatical without *a*, a pattern that he argues is unexpected if NUMBER is taken to be a regular singular count noun, like overt *number*.

(21) John has *(a) small number of books

The explanation he puts forth is that in contrast to *number*, the null NUMBER is neither singular nor plural, and thus unlike singular count nouns does not require the presence of *a* (or any other overt determiner). But while *few* does not require an overt determiner (hence the grammaticality of (20a)), it does occur with *a*, in the sequence *a few*. This implies, Kayne argues, that the null noun NUMBER can be optionally single, in which case it requires *a*, just like any other singular count noun. A sentence such as (22a) thus has the form (22b):
The absence of the corresponding *a many* is then attributed to the fact that *many* denotes a larger number than *few*, and thus is less compatible with singular *NUMBER*, being ‘farther from’ singular in meaning.

I have already discussed some of the strengths of Kayne’s overall theory, particularly in explaining the distributional differences between Q-adjectives and ordinary adjectives (cf. Chapter 2). Kayne’s theory is also notable in taking seriously the surface similarity between *few* and *a few*, and offering perhaps the only serious attempt in the literature to establish a derivational relationship between the two, rather than (tacitly) assuming the latter to be some sort of fixed idiomatic unit.

But it is also worth pointing out that the central elements of Kayne’s account do not in and of themselves fully explain the facts of distribution and interpretation of *many* and *few*. Rather, a series of further stipulations are required, which Kayne introduces primarily in the form of additional null elements in the syntactic representation. In a point relevant to the current topic, to explain why *few* licenses negative polarity items while *a few* does not, Kayne proposes that bare *few* is preceded by a null *ONLY* or *NOT*, while this is not present in the case of *a few*. Similarly, to explain the contrast between the ungrammatical *a many* and the grammatical *a good many*, Kayne proposes that in the presence of singular *NUMBER*, *few* and *many* must be preceded by an additional adjective, which in the case of *many* must be overt (as in *a good many*), but which may be null in the case of *few* (as in *a few books*, for which he proposes the modified form *a GOOD few NUMBER books*). Clearly, an account under which these facts fell out from the basic analysis of *many*, *few* and *a few* would be more attractive.
Not surprisingly, Kayne does not address the semantics of the Q-adjectives he discusses, but here too questions can be raised. Most centrally, the semantic differences between *a few* and *few* discussed earlier in this chapter do not seem transparently related to Kayne’s singular vs. nonsingular/nonplural dichotomy, particularly since the purportedly singular *a few* cannot be one, while the nonsingular *few* can be:

(23)  

a. Few students wrote great papers; in fact, exactly one did  
b. *A few students wrote great papers; in fact, exactly one did  

Again, an analysis which captured both syntactic distribution and semantic interpretation would be preferable.

5.3 Analysis

5.3.1 *A Few* as Degree Denoting

The starting point for my analysis is the observation that *a few* behaves like a cardinal numeral. I have analyzed cardinals as directly denoting degrees (i.e., they are of type $d$):

(24)  

\[
\left[ \text{three} \right] = 3
\]

Assume that cardinal numerals, like Q-adjectives, are Quantity Phrases (QPs) situated in the specifier position of the functional head Meas (25a). On this analysis, the cardinal numeral can compose *in situ* with the measure function introduced by Meas, as in (25b), giving a sentence such as (26a) the semantics in (26b):

(25)  

a. \[ [\text{MeasP} \ [\text{QP} \text{three} \text{Meas} \text{NP students}]] \]  
b. \[ [\text{MeasP} \text{three Meas students}] \]  

\[ = \left[ \text{Meas students} \right] \left( \left[ \text{three} \right] \right) \]
\[ = \lambda d . \lambda x . *\text{student}(x) \land \mu_\theta(x) \geq d \ (3) \]
\[ = \lambda x . *\text{student}(x) \land \mu_\theta(x) \geq 3 \]

(26)  

a. Three students came to the party  
b. $\exists x [*\text{student}(x) \land \mu_\theta(x) \geq 3 \land \text{came-to}(x, \text{party})]$
Note that this gives us an ‘at least’ reading for the numerical noun phrase: it is a set of at least three students that is asserted to have come to the party.

Drawing on the parallel between a few and cardinal numerals, let us propose that a few is likewise degree-denoting:

\[ [a \text{ few}] = a \text{ few}_d \]

Here, we can take there to be some indeterminacy in the specific degree that is picked out by a few (is it three? four?), but the crucial point is that a few is of type \( d \). Then like cardinal numerals, a few can compose \textit{in situ} with the measure function Meas. Putting aside for now the internal syntactic structure of the XP a few, we have the following:

\begin{enumerate}
\item A few students came to the party
\item LF: \([ \text{IP}[\text{DP}[\text{MeasP [XP a few] Meas [NP students]]}] \text{ came to the party}] \]
\item \( \exists x[\text{*student}(x) \wedge \mu(x) \geq a \text{ few} \wedge \text{came-to}(x, \text{party})] \]
\end{enumerate}

Compare this to what we have with bare few. As argued in Chapter 3, few does not denote a degree but rather a quantifier over degrees, which cannot be interpreted \textit{in situ} but rather must raise for purposes of interpretability, binding a trace of type \( d \) in its base position:

\begin{enumerate}
\item Few students came to the party
\item LF: \([ \text{POS}_2[t_2 \text{-} \text{few}_1, [\text{IP}[\text{DP} [\text{MeasP t}_1 \text{ Meas [NP students]]}] \text{ came to the party}] \]
\item \( \forall d \in \mathbb{N}_0[\neg \exists x[\text{*student}(x) \wedge \mu(x) > d \wedge \text{came-to}(x, \text{party})] \]
\end{enumerate}

Importantly, with this approach we are already able to account for some of the facts discussed in Section 5.1. First, a few, like the cardinal numerals, receives an ‘at least’ interpretation: in (28), it is a set of students numbering at least a few that is asserted to have attended the party in question. Secondly, while the interpretation of bare few involves a negative element (few itself) with sentential scope (as in (29)), that of a few does not (per (28)). Hence a few does not license negative polarity items (vs. few, which
does), and sentences formed with *a few* do not exhibit the characteristics of sentential negation (in contrast to those formed with *few*, which do). Finally, recall that noun phrases with *a few* and the cardinal numerals allow collective and cumulative interpretations, while those formed with *few* do not. Without attempting a review of the literature on collective and cumulative interpretations (see e.g. Scha 1981; Link 1983; Reinhart 1997; Krifka 1999), we might hypothesize that a collective reading is available to a noun phrase only if it is semantically complete, in that it does not contain a variable that is bound externally to it. This condition is satisfied in the case of the cardinal numerals and *a few*; but it is not satisfied in the case of bare *few*, where a variable of type *d* in the noun phrase is bound by *few*, which has sentential scope at LF.

### 5.3.2 Deriving *A Few*

To summarize the preceding discussion, the type *d* analysis of *a few* looks to be a promising approach to capturing its differences from *few*, and its similar behavior to the cardinal numerals. But in Section 5.1 I presented evidence that *a few* is related to – in fact derived from – *few*. This suggests that the interpretation given in (27) is a derived one, and not the basic lexical entry for *a few*.

In exploring how to implement this compositionally, let us revisit the basic lexical entry for *few* that I proposed in Chapter 3:

(30)  \[ [\textit{few}] = \lambda d \lambda I#.d \in \text{INV}(I) \]

Here, *few* is a gradable predicate of scalar intervals, or equivalently, a gradable quantifier over degrees. I argued that the semantics of its positive form arise via composition with a null positive morpheme POS, also a degree quantifier, which introduces a contextually determined neutral range *N#* to serve as the standard of comparison.
\[ [\text{POS}] = \lambda (\text{do}) . \forall d \in \text{N}_\# [d \in I] \]

In a sentence such as (29), \textit{few} and POS apply successively to the interval (set of degrees) created by lambda abstraction over the trace in the base position of the QP POS-\textit{few}: first \textit{few} maps the interval to its join complementary interval, and then POS applies to the resulting interval, returning ‘true’ if it contains \( \text{N}_\# \). More simply, the combination POS+\textit{few} is true of an interval \( I \) iff the neutral range \( \text{N}_\# \) is contained within the join complementary interval of \( I \). But which specific intervals fall within the extension of POS+\textit{few} depends, of course, on how \( \text{N}_\# \) is set (cf. the discussion of standard setting in Chapter 4). Or to put it differently, whether a particular interval \( I \) is in the extension of \textit{few} depends on the scalar location of \( \text{N}_\# \). For example, in the situation depicted in (32a), the intervals \( I, J \) and \( K \) all fall within the extension of POS+\textit{few}; in this context, they all count as \textit{few}. By contrast in the situation depicted in (32b), \( I \) and \( J \) are in the extension of POS+\textit{few}, while \( K \) is not; finally, in (32c), only \( I \) is in the extension of \textit{few}.
But now note that as the neutral range $N_#$ moves lower on the scale (as in the steps from (32a) to (32c)), there are some intervals that tend to remain within the extension of $\text{POS}+\text{few}$, namely those that are ‘close to’ zero, the lower bound on the scale. Suppose that $N_#$ can approach but not reach the lower end of the scale; that is, its lower bound must be greater than zero. Such a situation would be consistent with the view (see e.g. Klein 1980) that for any given context, a gradable expression must divide its domain into positive and negative extensions; in the context of the present discussion, this is to say that there must always be some range of values that count as \textit{few}. There will then be some intervals that will be in the extension of \textit{few} regardless of how $N_#$ is set, namely those that fall below the lowest possible lower bound of $N_#$, a point I will call $d_{\text{min}}$. To borrow a term from the supervaluation theory of Fine (1975), we could say that \textit{few} is ‘supertrue’ of these intervals.

I propose that this aspect of standard setting allows us to derive a non-gradable, context-insensitive interpretation for \textit{few}. Specifically, the context-insensitive (CI) \textit{few} is true of intervals that would be considered \textit{few} regardless of how $N_#$ is set, i.e. those intervals that do not extend past $d_{\text{min}}$.

\[(33) \quad [\text{few}_{\text{CI}}] = \lambda I. \max(I) \leq d_{\text{min}}\]

The CI \textit{few}, as defined in (33) is not a gradable expression (as is the basic entry for \textit{few} given in (30)), but rather a non-gradable predicate of scalar intervals, type $\langle dt, t \rangle$.

Now observe that the definition in (33) also has the effect of dividing the scale into two regions: that extending from 0 to $d_{\text{min}}$, and that above $d_{\text{min}}$. 
The degrees less than or equal to $d_{\text{min}}$ are those that would be considered \textit{few} regardless of the context. This suggests that the type $\langle dt, t \rangle$ entry in (33) might have a type $\langle dt \rangle$ counterpart: a predicate of degrees that is true of a degree $d$ if it is less than or equal to $d_{\text{min}}$. In the literature on noun phrase semantics, it is common to view this sort of correspondence as the result of a type shifting operation (Partee 1986; McNally 1998; de Swart 2001), with the standard $\langle et, t \rangle$ to $\langle et \rangle$ shift being the BE operation of Partee (1986). Let us extend this approach to the domain of degrees with the following shift operation, which I model on Partee’s BE:

(35) SHIFT: $\lambda \mathcal{P}_{dt,t} \lambda d. \mathcal{P}(\lambda d'. d = d')$

In parallel to BE, SHIFT functions by identifying all of the singleton sets (i.e. intervals) within the extension of a generalized quantifier over degrees, and collecting their elements into a set. Applied to the CI entry for \textit{few} in (33), we have:

(36) $\text{SHIFT}(\text{few}_{\text{CI}}) = \lambda \mathcal{P}_{dt,t} \lambda d. \mathcal{P}(\lambda d'. d = d')(\lambda I. \max(I) \leq d_{\text{min}})$

$= \lambda d. d \leq d_{\text{min}}$

It should be clear that this expression describes the lower interval in the diagram (34).

Let us return now to the main focus of this section, namely the derivation of \textit{a few}. I propose that the interpretation of \textit{a few} is derived from the CI \textit{few} in (33), with the presence of \textit{a} signaling that the type shift in (36) has taken place. Syntactically, I take \textit{a} to be the head of a maximal projection that selects the QP headed by \textit{few}. In terms of the semantics, we then have:

(37) $\llbracket [XP \text{ a} [QP \text{ few}]] \rrbracket = \text{SHIFT}(\text{few}_{\text{CI}}) = \lambda d. d \leq d_{\text{min}}$

On this analysis, \textit{a few} denotes a set of degrees (type $\langle dt \rangle$), derived via a type shift from a generalized quantifier over degrees (type $\langle dt, t \rangle$). I remain neutral as to whether \textit{a} itself encodes the SHIFT operation, though I consider this to be a possibility.

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It is of course reasonable to ask whether there is a connection between the *a* of *a few* and the indefinite article *a* in, for example, *a student*. To be certain, the role I have proposed for *a* here is not the standard approach to the indefinite article. But in this context, I think it is appropriate to point out that within non-quantificational theories of indefinites (e.g. Heim 1982; Reinhart 1997; Landman 2004), there is not (to my knowledge) any commonly accepted view as to the semantic contribution, if any, of the indefinite article. Furthermore, the proposal that the indefinite article in the nominal domain is associated with a shift to a set type is not without precedent. Such a view is found in particular in Zamparelli’s (1995) analysis of the structure of DP. Zamparelli argues for a series of functional projections within the DP, each of which corresponds to a specific semantic type. On his account, nouns are not inserted from the lexicon as elements of type *〈et〉*, but rather as kinds (hence the name Kind-Denoting Phrase or KiP for the lowest functional layer). It is the role of an intermediate functional level, the Predicative Determiner Phrase (PDP), to turn kinds into properties, i.e. to derive a type *〈et〉* denotation for the nominal expression. Importantly, the PDP is the level at which the indefinite article *a* is located; that is, the indefinite article is associated with the layer of structure at which the nominal expression takes on a set type. Other non-quantificational theories of indefinites (e.g. Landman 2004) likewise hold that an expression such as *a student* is of set type (though without explicitly assigning the indefinite article a role in deriving this type). The role proposed above for *a* in *a few* would then in one respect parallel the role of *a* in the nominal domain, in that *a few* likewise is of set type (though in this case the shift is from a quantificational rather than referential type).
Now let us consider how the expression in (37) may enter into composition with other sentential elements. Building on the syntactic analysis I developed in Chapter 3, a nominal expression such as *a few students* has the structure in (38). Semantically, the plural noun *students* first composes with Meas via Variable Identification, as in (39). Now note that *a few* as defined in (37) shares an argument of the same type as the resulting expression Meas *students*. As a first attempt, let us then propose that *a few* likewise composes with the Meas *students* expression via Variable Identification, with both the individual and degree variables ultimately bound by Existential Closure. The result is (41) for a sentence involving *a few*:

(38)  
\[
\text{DP[MeasP [XP a [QP few]] Meas [NP students]]}
\]

(39)  
\[
\text{〚Meas students〛} = \lambda d \lambda x. *\text{student}(x) \land \mu d(x) \geq d
\]

(40)  
\[
\text{〚a few Meas students〛} = \lambda d \lambda x. *\text{student}(x) \land \mu d(x) \geq d \land d \leq d_{\text{min}}
\]

(41)  
\[
\exists d \exists x [*\text{student}(x) \land \mu d(x) \geq d \land d \leq d_{\text{min}} \land \text{came-to}(x, \text{party})]
\]

The formula in (41) asserts a group of students attended the party whose number was at least a value that qualifies as *a few*. This is a close approximation to my earlier attempt in (28).

Before discussing some necessary refinements to this analysis, let me point out one particularly nice consequence that it has, namely an account of the facts relating to the sequence *not a few*. Recall that *not a few* is interpreted as more than *a few*, a result that sets *a few* apart from any other negated quantity expression. Neatly, this interpretation falls out directly from the semantics for *a few* given in (37). I first of all take *not* to be adjoined to the XP headed by *a*:

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Then we derive (43) as the semantics of \textit{not a few}; when this composes with the remaining sentential elements by the procedure above, we have (44):

\[(43) \quad \left[ \text{not a few} \right] = \left[ \text{not} \right] \left( \left[ \text{a few} \right] \right)
\quad = \lambda d. d \leq d_{\text{min}}
\quad = \lambda d. d > d_{\text{min}}
\]

\[(44) \quad \exists d \exists x [\ast \text{student}(x) \land \mu(x) \geq d \land d > d_{\text{min}} \land \text{came-to}(x, \text{party})]
\equiv \exists x [\ast \text{student}(x) \land \mu(x) > d_{\text{min}} \land \text{came-to}(x, \text{party})]
\]

As desired, on this analysis (44) asserts that some group of students numbering more than \(d_{\text{min}}\) — that is, more than a few — attended the party. Hence the present approach, in which \textit{a few} has a denotation at set type (type \(\langle dt \rangle\)), allows the correct semantics for \textit{not a few} to be compositionally derived.

However, the reader might have noticed that the formula in (41) is not, in the form given, precisely correct. Simplifying slightly, on this analysis the sentence in question is true if the number of student party-goers was greater than or equal to some value within the extension of \textit{a few}. \textit{A few}, in turn, denotes the set of degrees ranging from 0 (noninclusive) to \(d_{\text{min}}\). Earlier I noted that \textit{a few} is typically interpreted as three or four, suggesting these values are in this range. But then one and two must also be within the extension of \textit{a few}, in which case the sentence in (42) would be true if just one or two students attended the party. This does not seem correct.

Presumably the possibility that \textit{a few} includes one could be ruled out by morphological pluralization: \textit{a few dogs} cannot be one dog because of the plurality on \textit{dogs}. As evidence for this, we see the same pattern with \textit{some}:

\[(45) \quad \begin{array}{l}
a. \text{some dog} \\
b. \text{some dogs}
\end{array}
\]
(45a) could be just one dog, but (45b) must be at least two, with this effect necessarily coming from the pluralization on the noun.

As for whether two can be a few, there seems to be some disagreement between speakers: for myself, a few must be three or more, but some speakers I have consulted tell me that they might use a few in the case of two (though feeling that it is prescriptively incorrect). One possibility is that two is actually included within the possible values specified by a few, but is typically ruled out on pragmatic grounds: if the speaker meant two, he should have said so, since this would be more precise and thus more informative. But this explanation strikes me as not strong enough: the effect feels semantic in nature, not pragmatic. An alternative is to propose that there is some conventionalization in what degree is picked out from the set a few. This can be formalized via a choice function (Reinhart 1997):

(46) \[
\exists x. *\text{student}(x) \land \mu(x) \geq f(\lambda d. d \leq d_{\text{min}})
\]

Here the function \(f\) picks a degree out of the set denoted by a few. We can further impose the requirement that the degree be greater than two. More specifically, it is tempting to require \(f\) to pick out the most informative degree in the set \(\lambda d. d \leq d_{\text{min}}\), namely \(d_{\text{min}}\) itself. In either case, a few acquires a type \(d\) denotation, parallel to that posited in (27).

Let me conclude this section by noting that nothing I have said here pertains specifically to the scale of cardinalities. As a consequence, this account can be extended from few/a few (which operate on cardinalities) to little/a little (which operate on dimensions other than cardinality. Thus the same approach used to derive a few from few can likewise be applied to derive a little from little.
5.3.3 Blocking *A Many

I turn now to the absent counterpart of *a few, namely *a many. My claim here is that the process by which *a few is derived cannot be applied to derive *a many, due to an inherent property of scale structure. Let us see why that is.

Recall that the derivation of *a few involves the creation of a type \( \langle dt \rangle \) interpretation via the application of the SHIFT operation. This in turn requires us to first derive from the basic gradable (type \( \langle d, \langle dt,t \rangle \rangle \)) interpretation of few in (30) a non-gradable (type \( \langle dt,t \rangle \)) interpretation, the reason being that this is the semantic type to which SHIFT applies. I proposed that we can in fact derive such an interpretation, namely the context insensitive (CI) few in (33), a predicate of scalar intervals that is true of an interval \( I \) if it would be considered few regardless of the context (that is, regardless of how \( N_{\#} \) is set). It is this step that fails in the case of *many, for the following reason: while there are some values (and thus some intervals) that would be considered few regardless of the context, there are no corresponding values (intervals) that would be considered many regardless of context. To put this in more technical terms, for any \( I \), the neutral range \( N_{\#} \) can always be set in such a way that \( I \) does not extend past \( N_{\#} \). This is illustrated below, where for each interval (here, \( I, J, K \)), \( N_{\#} \) can be set to extend past the upper bound of the interval:

(48) a. 

\[ \begin{array}{c}
\text{CARDINALITY} \\
N_{\#}
\end{array} \]

\[ \begin{array}{c}
I \\
J \\
K
\end{array} \]
The result is that from the basic entry for *many in (49), we cannot derive a context insensitive interpretation equivalent to (33) for *few.

(49)  \[
\left\llbracket \text{many} \right\rrbracket = \lambda \lambda I#.d \in \text{INV}(I)
\]

We are therefore unable to apply the type shifting operation SHIFT; since \(a\) is the signal that such a shift has applied, we as a result cannot derive *a many.

The crucial factor distinguishing *a few from *a many is thus scale structure. The scale of cardinality (and correspondingly the scales of other amount dimensions) is bounded at the lower end by zero, but there is no corresponding upper bound. The result is that there is a lower bound to the possible locations for \(N#\), but no upper bound. Or to put it differently, there is an ‘anchor point’ for the lower end of the scale, namely zero, and we can think of points being ‘close to’ this anchor point, i.e. close to zero. But there is no corresponding anchor point at the upper end of the scale that values could be compared to. The result is that there is *a few, but no *a many.\(^{29}\)

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\(^{29}\) Recall from Chapter 4 that there is one situation where the scale invoked by *many and *few does take on an upper bound, namely in what I have called totalizing environments. I assume the derivation of *a few to take place before composition with other sentential elements, and thus before totalizing takes effect. That is, even in totalizing situations, at the relevant stage of the derivation the scale is unbounded at the upper end, thereby ruling out *a many.
Before moving on, there is one anomaly that deserves comment. As pointed out by Kayne (2005), and briefly mentioned above, while *a many is ungrammatical, we do find the perfectly acceptable *a great many (and marginally, *a good many). This fact is on the surface problematic for the account I have developed here. But I believe a case can be made that *a great many and *a good many are actually idioms, the result of a particular historical fact of English. Specifically, Jespersen (1938) points out that in the Middle English period the Germanic quantifier many was influenced by (and perhaps confused with) a French borrowing meneye, a noun meaning ‘retinue’ or ‘troop’. According to the Oxford English Dictionary, the use of *a good/great many dates to around this time. It seems likely that this collocation originated as the modification of the noun meneye by an adjective good or great; while the nominal many/meneye has not persisted into Modern English, I suggests that a remnant of it is still found in the idioms *a good many and *a great many.

5.4 Extensions and Broader Implications

5.4.1 An Extension

In Section 5.3, I accounted for the contrast between a few and *a many as resulting from the availability of a context insensitive interpretation for few, but not many. Earlier I mentioned another similar contrast between few and many, that in (50):

(50) a. Every few days
    b. *Every many days
    c. Every three days

There are two notable aspects of the examples in (50), namely the presence of every with a plural noun (cf. *every days) and the contrast again between the grammatical (50a) with few and the ungrammatical (50b) with many. That these are related to the a few/*a many
contrast is suggested by the following observations: first, as noted above, \textit{few} in \textit{every few} \textit{N} has a context-insensitive interpretation, always seeming to mean something like three or four (just as in \textit{a few}); and secondly, \textit{few} in this construction patterns with the cardinal numerals, as in (50c), reminiscent of the parallels between \textit{a few} and cardinal numerals. From this, it is reasonable to hypothesize that this occurrence of \textit{few} likewise is based on the CI \textit{few} whose semantics are given in (33).

The first point to make about examples such as those in (50a,c) is that they are not uniformly felicitous, as seen below:

\begin{enumerate}
  \item John’s parents visit every few/three weeks
  \item There’s a gas station every few/three miles
  \item For every ten cups of coffee you buy, you get one free
  \item *Every two houses look alike
  \item *Every three/few students went to the party
  \item *John read every three/few books
\end{enumerate}

Zamparelli (2004) points out two partially overlapping constraints on the occurrence of \textit{every few/every n}. Only nouns whose denotations have a natural linear ordering (e.g. weeks, miles, cups of coffee purchased) allow this construction; and secondly, \textit{every few/n} is awkward in argument position. Note also that the felicitous examples have a very particular interpretation. Thus (51c) does not mean that for every combination of 10 cups of coffee I buy, I get one free. For example, I don’t get one free cup for cups 1-10 purchased, another free cup for cups 3-12 purchased, another free cup for cups 2,4,6,8-14 purchased, and so forth; rather, it means that if we partition the sequence of my coffee purchases into units of ten, then for every one of these units, I get a free cup.

This is, I believe, the key insight into the correct analysis of noun phrases formed with \textit{every n} or \textit{every few}. These cases involve a grouping of the domain into units of size \textit{n} (or \textit{few}), with \textit{every} then universally quantifying over these units. The requirement
that the domain have a natural linear ordering falls out from this; otherwise (as in (51d-f)), there is no unambiguous way to establish such a grouping.

We might represent this more formally by introducing the notion of an $n$-grouping of a set, building on the notion of a cover (Schwarzschild 1994):

\begin{equation}
\text{(52) Definition:}\quad \Gamma_n(G)(X) \quad (G \text{ is an } n\text{-grouping of the set } X) \text{ if the following conditions are met:}
\end{equation}

\begin{enumerate}
\item $G$ is a cover of $X$:
\[\forall x \in X [\exists Y \in G \{x \in Y\}]\]
\item The elements of $G$ are non-overlapping:
\[\forall x \in X \forall Y,Z \in G [(x \in Y \land x \in Z) \rightarrow Y = Z]\]
\item The elements of $G$ are of cardinality $n$:
\[\forall Y \in G, |Y| = n\]
\item $G$ respects the natural ordering $\succ$ of $X$:
\[\forall x,y,z \in X \forall Y \in G [(x \in Y \land y \in Y \land x \succ z \succ y) \rightarrow z \in Y]\]
\end{enumerate}

We then have the following for one of our earlier examples:

\begin{equation}
\text{(53) a. For every 10 cups of coffee you buy, you get one free}
\end{equation}

\begin{equation}
\text{b. For any } G \text{ such that } \Gamma_{10}(G) \text{ (cups of coffee you buy)}:
\forall Y \in G \text{ [you get 1 cup of coffee free]}
\end{equation}

As in the paraphrase given above, (53) states that when we group the total set of cups of coffee you buy into a set of (sequential) groups of 10, you get one cup free for each member of the resulting groups.

Note that in the definition of an $n$-grouping in (52), the value $n$ must be filled in, which in the example in (53) is done by a cardinal numeral. The possibility of few in this construction (e.g. every few weeks) suggests that few can likewise fill this slot. In light of the preceding discussion, I propose that this is possible on the basis of the context insensitive (CI) few in (33), from which a type $d$ interpretation can be derived that
behaves like a cardinal numeral (just as *a few). Again, the unavailability of a comparable CI *many blocks the derivation of the corresponding *every many N.

5.4.2 Broader Implications for the Semantics of Quantity

In this chapter, I have focused on the relationship between *few and *a few, and on the contrast between *a few and *a many, and some related contrasts. It is, of course, satisfying to be able to shed light on small mysteries such as these. But I would like to suggest that the analysis developed here has some broader implications for the semantics of quantity expressions.

I have proposed that ‘quantity nominals’ (such as *few dogs, *a few students, *three people or *little water) involve the mediation of a functional head Meas, whose role it is to introduce a degree argument and link it to the individual argument. We have now seen two ways in which the degree argument introduced by Meas may be saturated. It may be saturated directly by a QP of type *d in its specifier position; such is the case with cardinal numerals and, as I have argued here, *a few (54a). Or it may be saturated by the trace of a degree quantifier in that position, with the latter taking as argument a lambda abstract over this trace; such is what we find with Q-adjectives in general (54b).

(54)

a. \[\text{MeasP} \quad \text{QP} \quad \text{MeasP'} \quad \text{Meas} \quad \text{NP} \]

b. \[\text{IP} \quad \text{QP} \quad \text{QP1} \quad \text{MeasP} \quad \text{MeasP'} \quad \text{Meas} \quad \text{NP} \]
To give labels to these two possibilities, we can say that the quantity expression can either be interpreted ‘low’, as an expression of type $d$ that composes in situ (54a), or ‘high’, as a degree quantifier that has sentential scope (54b).

In comparing the behavior of the degree quantifier *few* on the one hand, and cardinal numerals and *a few* on the other, we see two semantic correlates of this distinction. First, when the degree expression is interpreted low, a collective or cumulative interpretation is possible (as with *a few* and the cardinal numerals), whereas when it is interpreted high a collective or cumulative interpretation is not possible (as seen with the basic Q-adjectives *many* and *few*). Secondly, focusing in particular on the negative expression *few*, when it is interpreted high, it licenses negative polarity items in the sentential predicate, while when it is interpreted low (in the sequence *a few*) it cannot.

If these two possibilities for the saturation of the degree argument are in fact available, we would expect that these patterns would be observed more broadly. Evidence that this is the case is provided by data involving comparatives formed from *few*. As is the case with bare *few*, comparatives of the form *fewer than n* license negative polarity items in the sentential predicate:

(55) Fewer than 10 students have ever come to one of Prof. Jones’ parties

As described above, this is consistent with *fewer than 10* being interpreted high. And this is precisely how I analyzed comparatives in Chapter 3: in (55), the degree quantifier *fewer than 10* raises at LF to take sentential scope (with the comparative morphology *-er than 10* itself subsequently raising).

Observe also *fewer than 10* in (55) is interpreted distributively: the number of students who, individually, have come to one of the professor’s parties is fewer than 10.
In some contexts, however, noun phrases formed with *fewer than n* allow a collective or cumulative interpretation. This is particularly evident in statements of statistical distributions:

(56) a. Fewer than 10 people drank more than 10 bottles of wine  
    b. Fewer than 10 people do over 90% of all the work around here

(56a) has a distributive reading, according to which there are fewer than 10 individuals who each drank more than 10 bottles (on which reading it is, one hopes, true). But it also has a cumulative reading, according to which there was some group of people numbering less than 10 who, in total, drank more than 10 bottles. Similarly, in (56b), the only sensible reading is the cumulative one: there is a set of people numbering less than ten who, as a group, do over 90% of the relevant work. A collective reading is even marginally available for examples such as the following:

(57) Fewer than 5 people lifted the rock

But crucially, when *fewer than n* is interpreted cumulatively or collectively, it no longer licenses negative polarity items. Thus the examples in (58) allow only the distributive readings:

(58) a. Fewer than 10 people ever drink more than 10 bottles of wine  
    b. Fewer than 10 people ever do over 90% all the work around here  
    c. Fewer than 5 people have ever lifted this rock

Both of these patterns – the collective/cumulative interpretations in (56) and (57) and the lack of NPI licensing on this reading – are consistent with *fewer than n* in these cases being interpreted low. The conclusion would appear to be that there is an alternate interpretation available to *fewer than n* that allows it to compose *in situ* in the noun phrase. I will not attempt a specific analysis of how this occurs, though an account parallel to that developed for *a few* is plausible. The key point, however, is that the dual
behavior of fewer than n provides further support for a system in which both high and low interpretations of degree expressions are possible, with the two options having different linguistic consequences.

As a further question, if fewer than n allows both high and low interpretations, with no morphological marking of the distinction, are there other expressions that do likewise? An obvious candidate would be bare cardinal numerals. I have analyzed numerals as denoting degrees (type d), but we might wonder whether they have an alternate interpretation as degree quantifiers (type ⟨dt,t⟩), under which they would be interpreted high rather than low. Such a possibility would be consistent with the parallel referential/quantificational duality in the nominal domain (Partee 1986). I do not have any direct evidence either for or against this possibility, but in this context I will mention Kennedy & Stanley’s (to appear) analysis of ‘average’ noun phrases (e.g. the average American), which rests on precisely this sort of referential/quantificational (i.e., d vs. ⟨dt,t⟩) duality on the part of cardinal numerals.

5.5 Conclusions

I began this chapter with a rather small problem, namely the contrast between the grammatical a few and the ungrammatical *a many (and correspondingly between a little and *a much). I have shown that the degree-based approach to Q-adjectives that I have developed in this work is able to account for this pattern, in that a few can be derived from a context insensitive interpretation of few, while a corresponding a many cannot be so derived. Let me conclude this chapter with a brief summary of the broader relevance of this analysis. First, we have further evidence of the importance of scale structure to the distribution and interpretation of linguistic items. Specifically, the distinction
between *a few and *a many derives from an essential fact about the scale of cardinalities (and the scales of other amount dimensions): there is a lower bound to the scale, such that we can identify values that would be considered small regardless of context, but no upper bound, with the result that there is no range of values that would be considered large independent of context. Secondly, we have seen some further implications of the semantic framework developed here, in which quantity words (Q-adjectives, cardinal numerals) occur in the specifier position of a functional head Meas whose role it is to introduce a degree argument and link it to an individual argument. That is, the quantity expression may either be interpreted low (i.e. in situ), or high (after raising at LF), with the two options having different semantic consequences. It seems likely that there are further correlates of this distinction, but I must leave this as a topic for future investigation.
Chapter 6
Conclusion

The central proposal developed in this thesis is that adjectives of quantity have the semantics of degree predicates: predicates of intervals on the scale associated with some dimension of measurement. They are not, therefore, either quantifying determiners or (ordinary) adjectives, though they share characteristics with both classes.

To be sure, the degree-predicate analysis is not, in itself, a completely revolutionary idea. While this is not the orthodox view of Q-adjectives, proposals similar to parts of this story are found in the work of Klein (1982), Kayne (2005), Heim (2006), and Schwarzschild (2006). The real contribution of the present work is in applying this one simple proposal to account for a very diverse range of facts, some well known, some previously unrecognized.

In Chapters 2 and 3, I focused on the distribution of Q-adjectives. An explicit goal of this section was to start with the broadest set of data possible, with the aim of developing a theory of Q-adjectives that provides a unified account of their semantics across the full range of positions in which they occur. It was shown that the degree-predicate analysis is able to achieve this goal. Of particular note, this approach can handle not just their occurrence in quantificational, predicative and attributive positions, but also their differential uses (many fewer than 100; much shorter than Fred), which are not readily accommodated by alternative theories. A necessary consequence of the analysis is that essentially all semantic content is stripped away from many and much; their resulting vacuous nature gives us an explanation for the occurrence of much as a
dummy element in so-called *much* support contexts, and conversely for the slight infelicity of bare *many* and especially *much* in quantificational uses.

Importantly, the very same semantic analysis required to capture the differential use of Q-adjectives also accounts for the operator-like behavior of *few* and *little*. The reason is that a predicate of scalar intervals (what is required for the analysis of differential Q-adjectives) can also be conceptualized as a generalized quantifier over degrees, that is, a quantificational element that would be expected to interact with other operators. At the same time, this same mechanism also allows us to overcome a serious challenge to the non-quantificational analysis of Q-adjectives (how to avoid generating spurious ‘at least’ readings for *few* and *little* on application of an existential quantifier), without the introduction of ad hoc assumptions or multiple type shifting rules.

The analysis in Chapter 3 also sheds light on a previously unrecognized aspect of the semantics of attributive Q-adjectives, namely that they are best aligned to the nonrestrictive, rather than restrictive, readings of ordinary prenominal adjectives. In doing so, this work extends recent research on nonrestrictive modification (notably Potts 2003) into new territory.

Finally, in Chapter 3 we see some of the first evidence that factors relating to the measurement scales invoked in the semantics of Q-adjectives are relevant to the distribution and interpretation of these words. Specifically, the contrast in acceptability between *many/few* and *much/little* in predicative position (*the problems were many* versus *the difficulty was much*) was shown to relate to the fact that *many* and *few* encode a dimension of measurement (cardinality), while *much* and *little* are unspecified for dimension.
The facts that motivated degree-predicate analysis of Q-adjectives were primarily related to the broad syntactic distribution of these words. In Chapters 4 and 5, I showed that the degree-predicate theory can be extended to account for entirely separate sets of facts. In Chapter 4, it was shown that the vagueness and apparent ambiguity of Q-adjectives can be analyzed via the manipulation of two aspects of the scalar representation that this approach invokes: the location of the neutral range that serves as a standard of comparison, and the structure of the scale itself, specifically whether or not an upper bound is assumed. On this approach, we are able to account for the truth-conditional difference between the so-called cardinal and proportional readings (which sets this case apart from the strong/weak ambiguities found with other weak quantifiers), and for the availability of proportional readings in ‘non-quantificational’ uses of Q-adjectives (which is unexpected under theories which hold Q-adjectives to be ambiguous between a cardinality predicate and a quantifying determiner with proportional semantics). In turn, the bounded-scale situation was shown to arise in what I called totalizing environments, where some totality of individuals enters into the semantic derivation, an approach that unifies the treatment of two contexts that allow only proportional readings, the partitive and the subject position of individual level predicates.

Finally, Chapter 5 demonstrates that the degree-predicate theory is also a productive framework in which to investigate the contrasts among individual Q-adjectives. Specifically, the contrast between the grammatical *a few and the ungrammatical *a many was shown to relate to a basic fact about the scale of cardinalities (and other amount scales): there is a lower bound but not an upper bound,
such that we can identify values that are small regardless of context, but not those that are large regardless of context.

Let me conclude with some remarks on the broader relevance of the findings here. Most centrally, this work illustrates the explanatory value of notions of degrees and scales in the semantic analysis of quantity expressions, and thus supports the introduction of degrees into the semantic ontology. One of the ways that natural languages encode quantity is via expressions that denote degrees and those that denote generalized quantifiers over degrees (or equivalently, predicates of scalar intervals). Beyond this, as discussed in the brief summary above, properties of measurement scales themselves – particularly the dimension of measurement involved and the presence or absence of endpoints – can be drawn on to explain the readings available to Q-adjectives, and some otherwise puzzling contrasts in distribution among individual members of this class. This work thus builds on recent findings by Kennedy & McNally (2005a) and others showing the relevance of scale structure to the distribution and interpretation of lexical items.

At the same time, degrees must be linked to individuals. On the present account, this role is played by the functional head Meas, which introduces degrees into the semantic representation and associates them with individuals. This work therefore supports a view of the syntax of the nominal domain in which functional elements contribute semantic content.

Finally, from another perspective, this dissertation contributes to two current areas of investigation in the study of quantificational expressions. First, the present treatment of Q-adjectives forms part of a growing body of research showing the need for fine-grained analyses of individual quantificational expressions (e.g. Geurts & Nouwen 2007),
as opposed to the unified but course-grained approach of Generalized Quantifier Theory. Secondly, this work continues a theme found particularly in Hackl (2000), in showing that complex expressions of quantity (e.g. *many fewer than 100, not a few*) can and indeed must be analyzed compositionally. I believe there is potential for the degree-based framework developed here to be applied to further problems in both of these areas.
References


