

Interpreting similarity using attribute spaces and generalized measure functions

Helmar Gust (University of Osnabrück)

Carla Umbach (ZAS Berlin)

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Similarity in natural language

- Similarity is a core concept in human cognition, e.g., in classifying objects / situations
 - Natural languages provide multiple ways to express similarity, e.g., adjectives *ein ähnliches Auto / a similar car*
demonstratives *so ein Auto / such a car, a car like this*
- Spell out the semantics of similarity expressions accounting for the results on similarity in Cognitive Science and AI without abandoning truth-conditional semantics

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The basic idea

- Multi-dimensional attribute spaces: feature structures + additional structure
- Generalized measure functions: many dimensional counterparts of one-dimensional measure functions in degree semantics (Kennedy 1999)
- Generalized measure functions map individuals to points in multi-dimensional attribute spaces
- Similarity is spelt out as indistinguishability with respects to a given set of attributes.

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Similarity demonstratives

Demonstratives expressing similarity (instead of identity):

- German *so, wie dies*
- English *such, like this*
- Turkish *böyle*
- etc.

(1) (speaker pointing to a car): *So ein Auto hat Anna.*
'Anna has a car like this.'

- Demonstration target: the car the speaker points at
- Referent of the NP: instance of the similarity class generated by the target, ≈ ad hoc kind

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Semantics of ad-nominal *so*

- (1) (speaker pointing to a car): *So ein Auto hat Anna.*
'Anna has a car like this.'

Similarity is a 3-place relation $SIM(x, x_{target}, \mathcal{F})$

- x NP referent
- x_{target} target of pointing
- \mathcal{F} representation, including a set of dimensions of comparison

$[_{NP} [_{DET} \textit{so ein} \textit{Auto}]]$ ('such a car')

- $[[\textit{so}]] = \lambda D. \lambda P. D(\lambda x. SIM(x, x_{target}, \mathcal{F}) \& P(x))$
- $[[\textit{so ein}]] = \lambda P. \lambda Q. \exists x. SIM(x, x_{target}, \mathcal{F}) \& P(x) \& Q(x)$
- $[[\textit{so ein Auto}]] = \lambda Q. \exists x. SIM(x, x_{target}, \mathcal{F}) \& \textit{car}(x) \& Q(x)$

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Semantics of ad-adjectival *so*

- (2) (speaker pointing at a person): *So groß ist Anna.*
'Anna is that tall.'

- nominals: multiple dimensions of comparison selected by the context, restricted by the noun, nominal dimensions may relate to ratio / ordinal / nominal scales
- adjectives: one dimension of comparison given by the adjective's meaning dimension has a ratio scales

- $[\textit{so groß}]$ ('this tall')

- $[[\textit{so}]] = \lambda f. \lambda x. SIM(x, x_{target}, \mathcal{F}(f))$
- $[[\textit{so groß}]] = \lambda x. SIM(x, x_{target}, \mathcal{F}(\textit{height}))$

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Generalized measure functions

- Measure function associated with *tall* (Kennedy 1999):

$$\mu_{\textit{height}}: D \rightarrow \mathfrak{R}$$

- Suppose, relevant dimensions of comparison are

- DRIVE_TYPE: {diesel, gasoline, natural gas, electric}
- HORSEPOWER: \mathfrak{R}^+
- DOORS: {1 ...5}
- EQUIPMENT: \wp {rear assistance, lane guide, park pilot, BLIS}
- IMMOBILIZER: {0, 1}

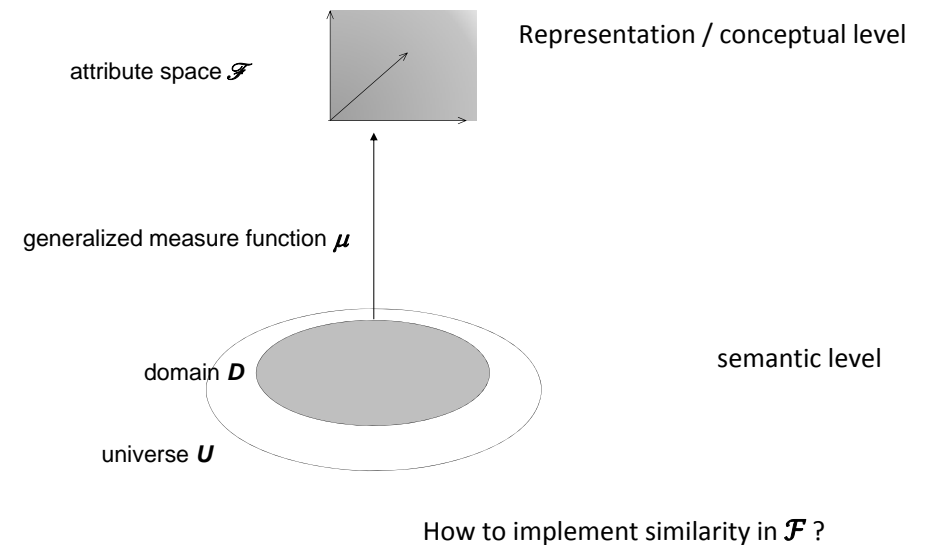
- Generalized measure function associated with *car* (in the context):

$$\mu_{\textit{CAR}}: D \rightarrow \text{DRIVE-TYPE} \times \text{HP} \times \text{DOORS} \times \text{EQUIPMENT} \times \text{IMMOBILIZER}$$

$$\mu_{\textit{CAR}}(x) = \langle \mu_{\textit{DRIVE-TYPE}}(x), \mu_{\textit{HP}}(x), \mu_{\textit{DOORS}}(x), \dots \rangle$$

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Interim result – half of the picture



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Classification functions

- Multi-dimensional attribute spaces are given by a representation \mathcal{F} specifying dimensions and corresponding measure functions and a set \mathbf{P}^* of classification functions.
- Classification functions approximate natural language predicates on a conceptual level yielding corresponding truth values.

*high-powered**($\mu_{HP}(x)$) iff *high-powered* (x) (for relevant x)

- Classification functions are defined, e.g., by a set of basic membership functions (crisp or fuzzy). The basic classification functions determine the maximal granularity.
- New classification functions are constructed from basis ones by conjunction, disjunction, negation and closure operators.

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Similarity as indistinguishability

- Similarity is defined making use of classification functions such that two individuals are similar (with respect to given \mathcal{F} and \mathbf{P}^*) iff the classification functions yield the same result when applied to corresponding points in \mathcal{F} :

(*) $\text{SIM}(x, y, \mathcal{F})$ iff $\forall p^* \in \mathbf{P}^*: p^*(\mu_{\mathcal{F}}(x)) = p^*(\mu_{\mathcal{F}}(y))$

- The similarity relation in (*) corresponds to the notion of indistinguishability in rough set theory (Pawlak 1998).
- The similarity relation in (*) is an equivalence relation.

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Integrate attribute spaces into truth-conditional semantics

- Recall:
Classification functions approximate natural language predicates on a conceptual level yielding corresponding truth values.

*high-powered**($\mu_{HP}(x)$) iff *high-powered* (x) (for relevant x)

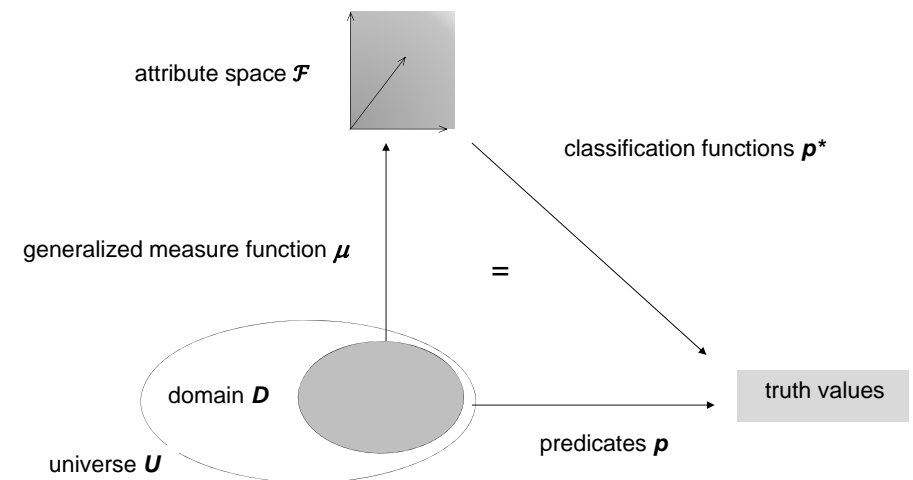
- Classification functions connect descriptions on the conceptual level to regular predicates:

(**) $\forall p^* \in \mathbf{P}^*: p^*(\mu_{\mathcal{F}}(x)) = p(x)$

- Classification functions warrant the integration of attribute spaces into truth-conditional semantics.

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The full picture



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Domains

A **domain** is a quadruple $\mathcal{D} = \langle D, .^+, .^-, P \rangle$ with:

- D a set
- $P = \{p_1, \dots, p_n\}$ a set of predicates over D
- $.^+ : \{p_1, \dots, p_n\} \rightarrow \wp(D)$ (positive examples)
- $.^- : \{p_1, \dots, p_n\} \rightarrow \wp(D)$ (negative examples)
- $p_i^+, p_i^- \subseteq D$
- $p_i^+ \cap p_i^- = \emptyset$ (consistency)

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Representations

A **representation** $\mathcal{F} = \langle F, \mu, .^*, \mathcal{D} \rangle$ of a domain $\mathcal{D} = \langle D, .^+, .^-, P \rangle$ is given by

- an **attribute space** F
- a **measure function**
 $\mu : D \rightarrow F$ with $\mu(p_i^+) \cap \mu(p_i^-) = \emptyset$
- **classification functions**
 $.^* : P \rightarrow \Omega^F$ we call p^* an **approximation** of $p \in P$
- $p^*(\mu(p_i^+)) = \{\text{true}\}$ (consistency)
 $p^*(\mu(p_i^-)) = \{\text{false}\}$

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Similarity

Similar as **undiscernable**

- $\mu(x) \sim_{p^*} \mu(y)$ iff $\forall p^* \in P^*: p^*(\mu(x)) \leftrightarrow p^*(\mu(y))$

We get an order on the P^* s:

- $P^* \leq P^{**}$ iff $\mu(x) \sim_{p^*} \mu(y) \rightarrow \mu(x) \sim_{p^{**}} \mu(y)$

Given a domain $\mathcal{D} = \langle D, .^+, .^-, P \rangle$ with representation $\mathcal{F} = \langle F, \mu, .^*, \mathcal{D} \rangle$

- $\text{sim}(x, y, \mathcal{F})$ iff $\mu(x) \sim_{p^*} \mu(y)$

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Similarity as equivalence relation ?

Tversky (1977) argued against a metrical notion of similarity / distance

- triangle inequality is hardly compelling
- minimality is problematic
- symmetry is apparently false.

Transitivity: fixed dimensions of comparison
 $\text{SIM}(x, y, \mathcal{F}) \ \& \ \text{SIM}(y, z, \mathcal{F}) \rightarrow \text{SIM}(x, z, \mathcal{F})$

Reflexivity: required when interpreting the demonstrative *so / such*,
blocked when interpreting the adjective *ähnlich / similar*,

Symmetry: Gleitman et al (1996) show that Tversky's results are due
to figure-ground effects.

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Graded similarity

While *so / such* are not gradable, *ähnlich / similar* are gradable

- (3) a. *Anna hat ein ähnlicheres Kleid.*
Anna has a more similar dress.
- b. * *Anna hat ein Kleid, das mehr so ist.*
* *Anna has a more such dress.*

Gradability of similarity:

x is more similar to z than y to z

iff there is a finer-grained classification system such that

x is similar to z but y is not similar to z

(**) more-sim(x, y, z, \mathcal{F}) iff $\exists \mathcal{F}' \text{ sim}(x, z, \mathcal{F}') \ \& \ \neg \text{sim}(y, z, \mathcal{F}')$

where \mathcal{F}' differs from \mathcal{F} only in $P^* \leq P^*$

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Conclusion

- Natural languages provide multiple ways to express similarity, e.g. demonstratives (*so / such*) and adjectives (*ähnlich / similar*)
- the demonstratives *so / such* generate similarity classes induced by the demonstration target, which serve as ad hoc kinds
- Similarity is spelt out using
 - multi-dimensional attribute spaces, and
 - generalized measure functions.
- Similarity is defined as indistinguishability with respects to a given set of attributes, where
 - *so / such* require reflexivity, and
 - *ähnlich / similar* require irreflexivity

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