VQ2
Berlin
9/12/2010

Is there a role for fuzzy logic in linguistics?

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Warning!

This is

- not a report on research results from LoMoReVI
- not about vague quantifiers or quantities
- not a "defense" of fuzzy logic in the context of reasoning about vagueness
- not even a systematic account of attempts to apply fuzzy logic in linguistics

Rather, I will attempt to

- highlight some facts about the troubled relationship between fuzzy logic and linguistics
- make methodological differences more explict
- hoping to clarify some misunderstandings

Puzzling facts

(F→L) Search results from google.scholar:

- `fuzzy linguistic`: 172.000 results
- `"fuzzy linguistic"`: 8.970 results

Keywords like

- `"linguistic hedges"` (1.710 google.scholar results)
- `"linguistic variable"` (17.600 google.scholar results)

abound in fuzzy logic papers and books.
Most of these papers explicitly mention vagueness.

(L→F) Linguistic papers on the semantics of vague expressions and related phenomena

- hardly mention fuzzy logic at all
- if they do, then only to dismiss FL explicitly

In semantics, fuzzy logic has been explored in the analysis of vagueness in the early 70s by Lakoff, but has been regarded as unsuitable for the analysis of language meaning since the influential work of Kamp in 1975.
(Uli Sauerland)

Some other work in linguistics referring to FL

- FL papers in the *Journal of Semantics*:
  18 JS-papers (incl. reviews) mentioning FL can be found, among them:
  - Lotfi A. Zadeh: A Fuzzy-Set-Theoretic Approach to the Compositionality of Meaning: Propositions, Dispositions and Canonical Forms (1983)
  - Francesco Paoli: Comparative Logic as an Approach to Comparison in Natural Language (1999)

  However, most of the papers by linguists indeed mention FL only in passing and/or deprecatingly.
- Matthias Gerner (City University of Hong Kong):
  A number of (very recent) papers explicitly employing FL as part of semantic and pragmatic models. (*Journal of Pragmatics*, *Linguistics & Philosophy*, etc.)
- (in addition to the classic papers by Lakoff and Kamp:)
  Manfred Pinkal: *Logic and Lexicon* (Kluwer, 1995)

Ignorance About FL?

Is this what we are talking about?

Ignorance About FL?

An important distinction:

- FL in the wide sense (FLw):
  fuzzy sets, fuzzy controllers, fuzzy neural networks, ...
- FL in the narrow sense (FLn):
  many valued, truth functional logics ("degrees of truth")

NB: The majority of the mentioned papers concerns FLw, while Kamp (et al.) targeted FIn.

Does this clarification help to solve our "puzzle"?

At most partially:

1. Many of the FLw papers are still explicitly about modelling the meaning of vague language.
   Why don't linguists find them interesting/useful?
2. FIn researchers often complain about ignorance regarding recent developments in their field.

Ignorance About Contemporary FLn?

In the last 10-15 years "mathematical fuzzy logic" has blossomed. Some monographs – most importantly Petr Hajek's *Metamathematics of Fuzzy Logic* – and hundreds of papers witness that the field satisfies all criteria of successful science:

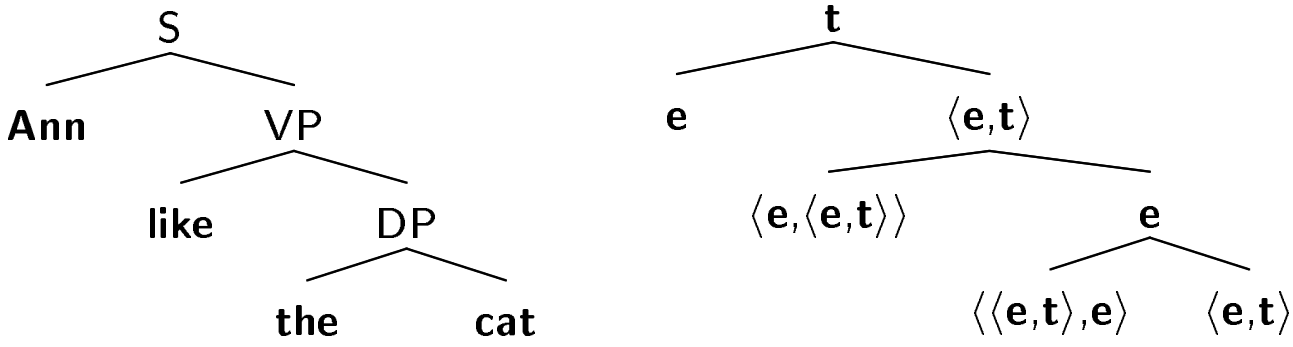
- deep mathematical results, major discoveries
- rich research agenda, wellconnected to traditional math. logic
- breadth: algebra, model theory, proof theory, complexity, etc.
- more and more refined picture about a complex realm of logics
- able to attract top scientists and gifted young researchers
- ...

But why should linguists care?

Hajek et al. do motivate FL as reasoning "vague notions and propositions". Moreover, they point to the same examples: *sorites* paradox, borderline cases, "very", "many", etc.

Generalizing Type t?

NB: Logicians and linguists seem to agree on Frege's principle – Syntax and semantics are compositional and interlocked:



Two basic semantic types:

t . . . truth values ($\{0,1\}$)

e . . . individuals (entities, objects of discourse)

Compound semantic types:

e.g., ⟨**e**,**t**⟩ . . . functions from **e** to **t** ($\sim$ sets)

Why not simply generalizing from **t**=$\{0,1\}$ to **t**=$[0,1]$?
(From set and relations to fuzzy sets and relations.)

One (of many) valid objections: we loose testability.

Attempts to Deflect Criticism of FL as ToV

- *FL is only about "gradedness", not about vagueness!*
  Many worries about linguistic adequateness persist
  (e.g., about "degree functionality").
- *You have to choose appropriate truth functions!*
  In judging *A and not A* to be wrong, one may refer to "strong Ł-conjunction" ($\max(0, x + y - 1)$). In judging *A and A* to be equivalent to *A* one refers to "weak conjunction" ($\max(x, y)$).
  But is conjunction really ambiguous in natural language?
  NB: It certainly is, according to modern logic (linear, etc.)!
  (Also compare: exclusive vs. inclusive disjunction.)
- *It is only a model!*
  (E.g., in reply to worries about the nature of degrees of truth, truth-functionality, comparability of all statements, etc.)

What is a Model?

- Linguistic models should always be empirically adequate: If a model does not make testable predictions or can be all too easily twisted to match any data it is useless. But note: features of a model often reflect modeling principles, rather than amounting to "predictions".
- Logicians often have prescriptive rather than descriptive adequateness in mind: Rather than explaining or simulating observable behavior, a logic may be understood as a model of correct reasoning even if no human being is expected to reason always in line with the model. (Cf. *probability theory*).
- In engineering logic based models are often understood as efficient tools for automatizing reasoning tasks: "correctness" and "adequateness" is heavily application-dependent and can amount to very different things.

Truth Functionality: Boon and Bane

NB: Many concepts in Fuzzy Set Theory (FLw) that are inspired by vagueness related phenomena in natural languages lead to non-truth-functional models of logical operators.

In Fln truth-functionality is called a "design choice" (e.g., Hajek). There are good reasons for this choice: mathematical elegance, computational efficiency, straightforward generalization of CL, ...

Analogy to classical logic:
The truth tables for $\wedge$, $\vee$, $\supset$ hardly correspond directly to natural language meanings. But violations of truth-functionality can be handled by extending the simple model; in particular to allow for various content-related connections between sub-sentences.
Cf., $A \Rightarrow B$ (conditional) modeled as $\Box(A \supset B)$, etc.
Claim: no difference between FLn and CL on that account!

Conclusions

- Fruitful interaction between linguistics and FL requires awareness about fundamental methodological differences.
- It is certainly appropriate to
  - start with criticizing a naïve translation of vague (natural language) sentences into particular FL-formulas.

  However, it is problematic to
  - stop at this very superficial glance at FL.
- Neither does linguistics need FL, nor does FL (w or n) need linguistics. Moreover, their aims are quite different.

  However, it were a pity if the outlined methodological discrepancies were preventing
  - linguists from learning more about a further tool generalizing classical logic that might be useful for various purposes, and
  - logicians working in FL from looking more carefully into proper linguistic modeling as a source of interesting logical challenges!